普通高等教育“十一五”国家级规划教材

# 数学专业英语（第2版）

吴炯圻　编著

高等教育出版社
HIGHER EDUCATION PRESS

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## 内容简介

本书以数学文献（主要是教材）的阅读理解为重点，讲授掌握数学专业英语的基本方法。

全书分六章。第一章介绍数学英语的特点和阅读翻译的基本方法；第二章为精读课程，分为12课，每课含3篇短文，附有生词与词组、预习要求、注释与说明和课外作业；第三章是阅读提高课程，根据内容分为6节，共含30篇短文，取材于各个数学分支英文版的本科、研究生教材和参考书；第四章是英语数学论文写作基础；第五章是查阅（包括上网查阅）英语数学文献的基本知识；第六章是数学文献常用词汇。

本书的科学性和实用性强，适应面较广且富有时代感。第二版对第一版做了局部修改和完善，特别在第二章增加了大量练习、扩充了词汇表并给独立单词附上了国际音标。

本书可作为数学学科各专业本科生和研究生的教材或参考书，也适用于其他相关学科领域的师生和科研人员阅读和参考。

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# 第二版前言

本书第一版自2005年4月出版以来，得到广大读者的厚爱和同行专家的认可，已被全国五十多所高等院校采用。本书第二版被列为普通高等教育“十一五”国家级规划教材。

这次出版前，作者根据自己多年的教学体会，参照多位读者（主要是教师与学生）的意见与建议，根据增强科学性和实用性的目标，对第一版各章节内容进行了重新审定，并在其基础上做了局部修改，其中在如下部分做了较大的变动：

第二章，为了强化精读部分，每节增加了中、英文互译练习的数量；增加了生词与词组表中的词汇量，把一表拆成两表，分别服务于课堂教学与课外练习，并给表中的每个独立单词附上了国际音标；另外，章末增加了附录“基本运算符号与算式的读法”。

第三章，对部分阅读材料做了增、减、改、换处理，增强了可读性、适用性和时代感。

第五章，根据目前网上资源的状况，更新了上网查找资料的部分内容。

第六章，对常用词汇表中的条目进行了局部增删，修改了少数条目的释义，并给表中的每个独立单词附上了国际音标。

尽管我们做了许多努力，但难免还存在一些不足之处，敬请读者不吝指教，多提宝贵意见与建议。顺便提一下，为满足广大读者需要，我们已另编一本《数学专业英语教学辅导与进阶训练》，既可作为本书的教学辅导、也可用于进阶提高的训练。

我校外语系欧阳耿教授仔细审阅了大部分修改材料并提出许多很有价值的意见和建议，莆田学院黄琴老师和其他兄弟院校多位老师对本书的修改也提出许多宝贵意见与建议，我系马米花、胡黎莉老师协助了校对工作，作者在此向他们表示衷心感谢。同时，作者诚挚地感谢高等教育出版社的有关人员为本书的出版所付出的努力。

吴炯圻

2008年11月于香港

# 第一版前言

20世纪90年代,计算机科学技术的迅速发展宣告了人类信息时代的到来。数学,这个古老而又优雅的学科获得了新的发展动力和发挥作用的舞台。她不仅是计算机科学技术的理论基础,而且是现代科学技术各个领域必不可少的研究工具,经济学与其他社会科学的发展都愈来愈多地需要数学。

伴随着人类社会进入21世纪,中国加入了世界贸易组织(WTO),国际数学教育与研究交流日益频繁,我国的教育也面临如何进一步与国际接轨的问题。教育部提出了高等学校各专业逐步使用英文教材,培养学生阅读英文版专业文献的能力的要求。为了适应新形势下教学改革的需要,我们认为有必要开设"数学专业英语"这门课程,并编写了这本教材——《数学专业英语》,其间道理应是容易理解的。因为只学过公共英语的学生,即使已通过六级英语考试,也常有不会读英文版数学教程和文献者。而有些学生逐字逐句英汉对译出来的文章谁(甚至包括他们自己)也无法理解。即使将来有一天出现了数学英语翻译软件,其翻译质量在许多情况下还得靠人来鉴别,况且目前的软件还不能支持数学翻译。

国内已有的数学专业英语教材甚少,一些版本要么内容陈旧,要么有深浅程度问题或有其他原因,不适宜一般高等院校本科生使用。

在多年教学改革的基础上,我们编写了此书,并已在福建省内五所高校试用了两、三轮,效果甚佳;同时也与省内外多所高等学校交流,获得了高度评价,不少中青年教师选用此书作为自学教材。这次的版本是在2004年第三次修订本的基础上重新修订、补充和完善的。作者力求突出如下主要特点:1. 重视基础,突出实用;2. 结构独特、合理(包含读、写、练与文献查阅,精读与泛读搭配);3. 内容新鲜活泼,信息量大;4. 解说评注深入浅出;5. 知识扩展循序渐进。

本书不仅可作为数学学科各专业的本科生、研究生的教材,也适合于其他学科领域的学生、教师和科研人员在阅读数学文献时参考或借鉴。

本书分为六章。第一章介绍数学专业英语的特点和阅读、翻译的基本方法,可谓导引。第二章是精读课程,共12课,每课含有3篇较为通俗的短文。第三章是阅读提高课程,包含30篇阅读材料。第二、三两章的66篇短文绝大多数直接取自英文版数学教程或参考书(包括部分论文、专著)。第四章介绍英语数学论文写作的基础知识,为初学者提供写作(包括数学论文的英文摘要写作)指

导。第五章介绍查阅(包括上网查阅)英语文献的基本知识。第六章列出了英语数学文献(主要是教材)中常见的词汇约2200条,旨在为读者学习第二、三章和查阅数学文献提供方便。

本书第二、三章的每篇短文均用脚注标明出处;第一、四、五章引用和参考的主要文献列在书末的“参考文献”中。对于摘录引用作为短文的材料,作者对其中的图号、表号、公式号按本书的章节进行了适应性的重排,对个别段落或字句作了必要的调整或更动,其余尽可能保持原貌。本人谨此对被引用和节录材料的各位作者表示衷心感谢,并对未能一一事先征得同意表示歉意。作者要特别感谢我的同事——本校英语系副教授、长期从事数学研究的欧阳耿先生,他多次认真阅读了本书的修改稿,并提出了许多宝贵的修改建议,特别是有关完善文字表达方面的意见。

限于作者水平,虽然做了许多努力,但错漏仍难免。欢迎专家和读者批评指正。

吴炯圻

2004年8月

# 目 录

# 第一章　数学专业英语的阅读和翻译初阶

对于学习数学的学生和准备从事数学研究的人员，在掌握了公共英语的基本知识的基础上，如果希望较快地掌握阅读英文版数学教程和科研资料的基本方法，进一步学习数学专业英语是必须的。在阅读英文版的数学教程和资料时，不仅会遇到许多从未见过的专业术语，而且还将遇到大量的、原先在学习公共英语时已多次见过且似乎较熟悉的所谓“半专业性术语”（如 set, power, function, 等等）。这时常常会出现这样的情况：读者要么不知所措（不能理解），要么只是按照学习公共英语时掌握的含义给出错误的翻译。这样一来，他们“译出来”的将是一篇无人能懂的“天书”。由于数学内容的英语表达有其特殊性，阅读时不仅要靠个人在公共英语上的基本功，而且还要靠数学基础知识的掌握及其运用的能力。因此，未具备一定数学基础的人是不可能翻译好数学教程和文献的；而有了一定数学基础，但未学习过数学专业英语课程或未进行过有关训练者，想要读懂英文数学教程、用好英文数学文献，也需要经过较长时间的艰苦摸索。这里顺便指出，指望不用学习，仅凭借软件就能翻译好英文数学文献的梦想，近期内似乎尚难实现。① 事实上，不论多好的软件所翻译出来的东西，最终还要靠人来判断是非、辨别优劣。因此，认真学习，提高自身的能力才是根本的。

本章简要介绍数学专业英语的基本特点和阅读与翻译的基本知识，以祈读者较快地进入本课程中心内容的学习。

## §1.1　数学专业英语的基本特点

为了学好数学专业英语，我们必须首先了解它的一些基本特点。

数学作为自然科学领域里的一个重要学科，其专业英语首先必具有科技英

① 进行中英文互译工作的困难包括如何处理一词多义、一义多词、惯用法的特殊性、语法的繁琐性以及文化差异等问题——由它们产生的复杂性是目前的计算机软件难以处理好的。因此机器的翻译离“准确、达意”的要求还有较大距离，译文中常出现形形色色的错误。

语的共性——科学内容的客观真理性与表达形式的完整性和简练性要求。但是,数学又有别于其他自然科学学科,这决定了数学专业英语的独特之处。事实上,数学的研究对象是空间的形、数、量以及它们的抽象和推广形式。用当今流行的语言来说,数学的研究对象是量化模式。其中,有的对象具有直观的背景(原型,如具体的图形和数值),更多的对象是抽象的概念、命题。其研究方法除了计算,更重要的是逻辑推理(当然也有少量的实验)。数学是既古老而又崭新的学科,它不仅历史悠久,而且至今仍在蓬勃发展,与其他自然科学分支相互渗透日益明显并获得广泛的应用。注意到这一点,我们就不难理解数学英语的专业特点。当然,我们这里简要介绍的只是基本特点而不是所有的特点,而且这种介绍只是初步的,并未深入展开,初学者也只需有个大致了解。要想对数学专业英语有较全面的认识和较好的掌握,必须不断学习和实践。相信读者在认真学完本书前三章之后,一定会有很大的进步。

**特点一:注意对客观事实与真理的描述**

1. 语句时态的使用上常用一般现在时

无论数学教程还是科研文献,通常以叙述的方式介绍概念和成果,并以论证的方法推导有关的结论,包括定理、引理和推论的证明。这些内容的正确性与特定的时间无关。因此尽管有些结论很早以前已经被发现或证明,在我们见到的材料中仍采用现在时态予以表述。

当然,在数学史以及专题报告和研究新闻中,为了强调某个历史事件发生的时间,也常根据实际需要,采用过去时或一般完成时。

2. 被动语态出现频率高,应用范围广

这是因为被动语态适合于强调客观事实和行为效果本身,而不强调行为的主体,正好与科学文献注重客观事实与真理性的要求相一致。

**例** The Fermat Conjecture has been proved to be true.(译文:费马猜想已被证明是正确的。)

这里强调猜想"已被证明是正确的",未指出是谁证明的,因为一般读者只关心该猜想的研究现状。

3. 主动语态句型也多数用于强调事实,而不是强调行为发出者及其情感

**例 1** Given $\varepsilon>0$, there exists a number $N>0$ such that $|a_n-a|<\varepsilon$ for all $n\geq N$.(译文:对给定的 $\varepsilon>0$,存在一个数 $N>0$ 使得 $|a_n-a|<\varepsilon$ 对所有的 $n\geq N$ 都成立。)

这种表示"存在"的句型显然不表示主语"a number $N$"发出什么行为,而表示满足的条件或具有某性质的主语的存在这一事实。

**例 2** Since $h(x)$ is harmonic on a neighborhood of $B(a,r)$, we have

$$\int_{\partial B} h(x)\,\mathrm{d}\sigma(x) = h(a).$$

（译文：因为 $h(x)$ 在某个邻域 $B(a,r)$ 内调和，故 $\int_{\partial B} h(x)\,\mathrm{d}\sigma(x) = h(a)$。）

这里“we have”并不强调“我们有”什么东西，而是说明“可以得出”什么结论而已，句中“we have”可以改成“one has”，或干脆省略掉，都不影响原意。

**特点二：科学内容的完整性与表达形式的精练性要求**

数学特别讲究严密性，每一个结论的成立都是有条件的，每一个结论的推导都有充分的根据。因此在叙述数学命题时必须把条件和结论不遗漏、不重复地准确表达出来；推导的过程要把每个论点的来龙去脉有条理地表述清楚。这种表述又必须是精练、明确和规范的，因此体现在数学英语的表达方式上有如下特点：

1. 长句比较多

主从复合句的大量使用是数学英语的重要特点之一；这类句子含有一个或多个从句，而有的从句本身也是复合句，因此许多句子较长。长句多的另一个原因是，不论是简单句或复合句，可能带有很多修饰语（定语、状语）、插入语等，而且句子成分常用各种短语或词组来充当。

2. 非限定动词使用频率高

非限定动词包括不定式，分词（现在分词、过去分词）和动名词，它们的使用频率都很高，经常用它们来代替从句，以达到简练的要求。

3. 名词化结构及其他简化表达的形式也较常出现

所谓名词化结构（Nominalization）就是一种以名词为中心词的短语，可以当名词用。

**例 1** The line rotates on $x$-axis, which forms a conicoid.（该直线绕 $x$ 轴旋转，形成了一个二次曲面。）

若把动词 rotate 换成名词 rotation，上面的句子可改写成：

The rotation of the line on $x$-axis forms a conicoid.

这时，The rotation of the line on $x$-axis 就是一个名词化结构。为了强调内容的客观性且使表达更简洁，多数英语的科技文献都较多地使用名词化结构。这一特点在数学英语中也有所体现，特别典型的是：一、数学文献的标题、小标题大都以名词化结构的形式出现（见§4.1），二、由于每个概念都有相应的性质，每种运算都有对应的“可运算性”，数学词汇中表示“-性质”的术语很多，以它们为中心词产生的名词化结构常出现在文献中。其中以“-ity”为后缀的单词如 positivity（正性），additivity（可加性），divisibility（整除性）等。

**例 2** Now we investigate whether the functions are integrable and why they are integrable or not.（译文：现在，我们来研究这些函数是否可积及它们可积或不可积的原因）。

若把形容词 integrable（可积的）换成名词 integrability（可积性），句子可改写成：

Now we investigate the integrability of the functions.（译文：现在，我们来研究这些函数的可积性）。

这时，名词化结构 integrability of the functions 代替了 whether 和 why 引起的从句，不仅使句子简洁，而且显得专业性更强。

关于“充分必要条件”表达方式的简化可参见下段。此外，在今后的课文中还可以见到多种其他的简化表达的形式。

**特点三：数学的专业性十分典型**

1. 数学符号、公式和图表到处可见

由于符号是表达数学内容的特殊而强有力工具，公式（包括算式）和图、表等既是数学的内容，也是数学的主要工具。因此它们到处可见，常和数学的论证与计算穿插出现。

2. 专业术语是构建数学大厦的砖瓦

数学专业术语，简称数学术语，通常以单词或词组的形式出现，也可以是短语，在数学中具有特定含义。它们是数学语言的重要组成部分，是构建数学大厦的砖瓦。

以单词形式出现的术语最基本。例如：integer（整数），diameter（直径），differential（微分），triangle（三角形），parallelogram（平行四边形），fractal（分形），sheaf（层，簇），continuum（连续统），probability（概率），capacitable（可定容的），homeomorphic（同胚的），homologous（同调的），holomorphic（全纯的）。

可能有好几个同义词对应于同一个概念，例如表示“计算”的词有：

count　计数、按次序数（动词）；calculate 计算、算出（动词）；calculation 计算（名词）；compute 计算（动词）；computation 计算（名词）。

同一词根的词很多（词性、词义可能不同），用它们可以生成一系列以词组形式或短语形式出现的术语。如：

integrability 可积性（名词）

integrable 可积的（形容词）

integral 积分（名词），或积分的（形容词），整数的（形容词）

integral calculus 积分学

integralization 整化（名词）

integraph 积分仪（名词）

integrate 积分（动词）

integrated circuit 集成电路

integrated curve 积分曲线

integrating factor 积分因子

integration 积分、积分法(名词)

integration formula 积分公式

integration by parts 分部积分

integrand 被积函数(名词)

mean value theorem of integral 积分中值定理

上述列举的是一些"专用性强"的数学术语,其中词组和大部分单词基本上只用于数学表达;其余单词(如 count、integral 等)虽然可应用的范围较大,但也较常用于数学表达,且用于他处时其词义与数学含义的关联性较强或易于辨别。实际上,数学术语还包括了其他类型的术语,如下面介绍的"半专业性术语"。

3. 半专业性术语穿插频繁、词义多变

这里所说的半专业性术语,除了在数学中使用时是具有特定含义的数学术语外,还常(更多地)在其他学科和日常用语中使用,且其含义与作为数学术语的含义有明显差异(甚至毫无关联)。它们的出现频率很高;其中的"独词语"(指仅由一个单词构成的半专业性术语)更活跃,含义多且用法复杂,应该特别注意其使用场合不同时的区别。

例如,function 常作"机能、作用"解,也表示"职务、任务、职责",还表示"仪式、典礼、社交的集会"。但它也是数学中常用的单词之一,常作"函数"解。它的派生词 functional 在公共英语中表示"功能的、起作用的"(形容词),但在数学中却常常不表示"函数的",而作"泛函"(名词)解。

又如 power 一词,在日常用语中常表示"能力、体力"等;在电力学中表示"动力、电力";在物理学中表示"功率"或显微镜的"倍率","度";在政治学中表示"权力";而在数学中它作"幂"或"乘方"解。

又如"set"一词,在数学中也非常活跃,作名词解是"集"或"集合",作动词以命令式出现时表示"令"、"假定";它在日常用语中可作动词、名词、形容词,词义更是繁多。

多数的半专业性术语(尤其是独词语)不仅使用面广、词义多,而且通过与适当的单词结合,可生成一系列专用性强的数学术语。比如,用 function 生成的数学术语仅常见者就不少于 40 条。因此,如能准确地掌握半专业性术语的数学含义,则阅读数学文献时可以事半功倍;反之,则会因其含义繁多,可能导致读者不知如何选择、无所适从。

4. 数学词汇的形式多样、数量庞大

"古代用语(包括拉丁语)"占了一定比例, 新创词语不断涌现,包括合成

词、派生词、以数学家名字命名的定理、公式等,数量可观;此外,还有大量的词组。

古希腊的欧几里得几何是数学的基础,在将其翻译成英文时采用了较多的拉丁文,它们至今还在使用。这一类词的拼写和发音的方式常和一般的英语单词有明显差异,给学习和记忆带来一定困难。如 icosahedral(二十面体的)、heptagon(七边形)、helicoids(螺旋面)等。但这类词常局限于初等数学的几何、三角及其相关的内容,初学者只要先掌握数十个较常用的术语,如 algebra(代数)、geometry(几何)、trigonometry(三角学)、isosceles triangles(等腰三角形)等即可。

所谓合成词,指的是由两个或更多的词合成的词,如:right-handed(system)右手系,joint-observation 联合观测,jump-function(跳跃函数)、four-color-problem(四色问题)。其中连字符有时不出现,如 nonnegative(非负的)。

派生词指的是通过对一个词加前缀或后缀构成的词,如:irregularity(非正则性)、interdependence(互相依存的)、inhomogeneous(非齐次的)、idempotence(幂等性)、hypergeometric 超几何的。

以数学家名词命名的术语很多,如:Jacobian(雅可比行列式)、Laplacian(拉普拉斯算符)、Cauchy inequality(柯西不等式)、Euler's equation(欧拉方程)、Gauss formula(高斯公式)、Hilbert problem(希尔伯特问题)、Perron method(佩龙方法)等。

数学词组数量大,其构成方式多种多样,较常见的是:形容词+名词,如 absolute error(绝对误差);名词+名词,如 balance equation(平衡方程);动词+副词,如 converge uniformly(一致收敛);副词+形容词,如 uniformly bounded(一致有界的)等。

由于 20 世纪新产生的数学文献比此前几千年数学文献的总和还多得多,因此与新内容相关的新单词、词组很多,但多数限于较专门的小分支领域内使用。对此,我们也只要抓住其中最基本的词汇加以记忆即可,因为其中许多合成词、派生词和词组可从其构成的基本形式猜测出大体的含义。作为数学英语的基础,只要先掌握本书附录列出的两千多个单词与词组即可。

5. 表示条件、推理根据的句型相对固定

常见的表示条件、推理根据的句型虽然较多,但大多已经形式化。如:

(1) 用 if、when、as 为连接词的条件从句。

**例** The function $f(x)$ approaches infinity as $x$ tends to zero.(译文:当 $x$ 趋于 0 时,函数 $f(x)$ 趋于 $\infty$。)

(2) 用 with 短语表示条件或补充条件。

**例** Suppose $D$ is an open set with its closure in $G$.(译文:假定 $D$ 是一个开集,且其闭包在 $G$ 中。)

(3) 用 such that 为连接词的从句表示条件或补充条件。

**例** Suppose $f(x)$ is a function on a domain $D$ such that $|f(x)|<M$ for all $x\in D$, where $M$ is a constant.（译文：假定 $f(x)$ 是区域 $D$ 上的一个函数，使得对所有 $x\in D$，$|f(x)|<M$ 成立，其中 $M$ 是一个常数。）

可用分词“satisfying”代替上述“such that”。

(4) 用特殊动词（如 suppose、let、set、assume）的命令式表示假定、设（大前提）等意义。

(5) 表示原因的句型，常用 since、as、for 等引导的从句，其中 since 从句最通用；虽然有时也出现 because 引起从句，但频率很低，通常只在非常强调因果关系时才用。

(6) 表示推理的根据常用“by 短语”，有时也用“according to”。

**例** By Lemma 2 we have $x\geqslant y$.（译文：根据引理 2 可推出 $x\geqslant y$。）

(7) 有时用现在分词表示“经过……而得到……”（推论）。

**例** Integrating the above inequality twice, we see that

$$y'(t)\geqslant c_0 t\log t.$$

（译文：将上一不等式两次积分得到 $y'(t)\geqslant c_0 t\log t$.）

(8) 采用反证法论述时，开头常用虚拟语态给出假设，结束时用“This contradicts the hypothesis”之类句子表示推导的结果与原假设矛盾。

其余情况，可参见本书第四章“英语数学论文写作基础”和其他文献。

6. 形成了一批数学专业性很强的特殊记号和表述方式

(1) 表示充分必要条件

**例** The sufficient and necessary condition for the equality is $\alpha>0$ and $p\geqslant 3$.（译文：该等式成立的充分必要条件是 $\alpha>0$ 且 $p\geqslant 3$。）

同一句子可改用如下形式表达：

The equality is valid when and only when $\alpha>0$ and $p\geqslant 3$.

其中“when and only when”可用“if and only if”替代或更简单地用“iff”替代，但用 iff 时应事先作出说明。

(2) 表示事先任意取定的量

**例** For any number $\varepsilon>0$ there exists a number $\delta>0$ such that $|f(x)-f(x_0)|<\varepsilon$ whenever $|x-x_0|<\delta$.（译文：对任意数 $\varepsilon>0$，存在一个数 $\delta>0$，使得只要 $|x-x_0|<\delta$，就有 $|f(x)-f(x_0)|<\varepsilon$。）

其中“For any number $\varepsilon>0$”可改为“Given $\varepsilon>0$”，意思基本一样（见下一条）。

(3) 表示某一个结论成立的范围

**例** (2)中的例子可改为：

Given $\varepsilon>0$, there exists $\delta>0$ such that $|f(x)-f(x_0)|<\varepsilon$ for all $x$ with $|x-x_0|<\delta$.
(译文：对取定的 $\varepsilon>0$，存在 $\delta>0$ 使得 $|f(x)-f(x_0)|<\varepsilon$ 对所有满足 $|x-x_0|<\delta$ 的 $x$ 都成立。)

(4) 逻辑符号的使用

**例** 对上面的例子，在适当的场合可用逻辑符号表成：

$\forall\varepsilon>0\ \exists\delta>0(|x-x_0|<\delta\Rightarrow|f(x)-f(x_0)|<\varepsilon)$.

其中"$\forall$"表示"对任意"或"对所有"，放在 $\varepsilon>0$ 前面表示"对任意取定的 $\varepsilon>0$"；而"$\exists$"表示"存在"，"$\Rightarrow$"表示"推出"(imply)。又，"$\Leftrightarrow$"表示"等价于"或"当且仅当"，例如：

$f$ is continuous at $x_0\Leftrightarrow f^{-1}(B)$ is a neighbourhood of $x_0$ for every ball $B=B(f(x),r)$.

(译文：$f$ 在 $x_0$ 连续等价于对每个球 $B=B(x_0,r)$，$f^{-1}(B)$ 都是 $f(x_0)$ 的一个邻域。)

**总结** 以上所介绍的只是数学专业英语的一些较常见的基本特点，更深刻和细致的语言规律有赖于读者在实践中不断学习、探索与积累。不过，据作者本人的学习和教学经验，初学者只要先抓住以上基本特点，就可以大大提高数学专业英语的阅读速度和理解的准确性。

## §1.2 数学专业英语的阅读与翻译

阅读是为了获取信息，理解和掌握专业的内容。翻译是把原文的内容用中文准确、清楚地表达出来。翻译的前提是通过阅读获得正确的理解，而翻译的过程有助于加深阅读理解。阅读未必是为了翻译，但由于多数中国读者都不是以英语为母语，更未从小学开始就学习数学英语，因此阅读理解的过程经常是：至少先在头脑中作了某种程度的"翻译"，然后才理解清楚的。因此，翻译与阅读二者紧密关联，相辅相成。

### §1.2.1 数学专业英语的阅读

词汇与语法基本知识是阅读者必备的基础，但仅有基础未必能做好阅读。要做好阅读理解，必须具备运用语言知识和数学知识的能力。这种能力包括：

(1) 根据上下文来确定词义或猜测词义；

(2) 正确理解句与句、段与段之间的逻辑关系；

(3) 对数学内容做必要的、基本的逻辑推理；

(4) 归纳段落大意和全文主题。

为了提高阅读能力，应该从以下三个方面加强训练：

1. 努力练好英语的基本功，掌握基本语法、习惯用法和常用单词的基本用法，逐步扩大常用数学单词和词组的识记和使用范围。

2. 多读多练，在阅读专业英语的过程中，逐步掌握数学专业英语的特点，不断总结提高。对于有一定基础的读者，可以进一步注意不同地区、学派、语言（特别是美国英语与英国英语）的作者在表达方式与风格上的差异（参见第四章的附录4）。

3. 努力掌握正确的阅读方法。

(1) 必须以意群为单位来阅读而不是以单词为单位来阅读；

(2) 既要学会精读，也要学会略（泛）读和查阅；

(3) 阅读时要抓住三个步骤：先粗、后细、再加深。

**先粗**：指要尽快抓住阅读材料的主旨和大意。数学文献大都是叙述性和论证性文体。为了尽快抓住论点、主要论据、中心内容与段落大意，读者可先抓大小标题，关键词，根据已有的专业知识来理解并做大体估计。

**后细**：指在先对主旨有了基本了解的基础上，进一步弄清阐述主旨（中心意义）的论据的具体事实和细节；既要根据英语知识努力做好每个句子的语法分析，又要按照上下文所表述的数学内容，尽可能地对有关词语的含义做出正确的判断，辨明意群与句子的意思及它们之间的关系。

**再加深**：对于长句，必须先抓句子主要成分，必要时进行适当分解，弄清每个意群（从句或短语）的含义与相互关系，再综合成全句的意思。在此基础上，要再加深理解，综观全文，进一步理清段落与段落之间的关系，论据与论点的关系，进行一定的判断、推理和引申，必要时要通过进一步的查证以澄清此前存在的疑惑、纠正原先理解得不正确或不确切的部分。最后做到尽可能深刻地理解作者的观点、意图，准确地了解全文的意义。

在上述每个步骤中都应做必要的逻辑分析，弄清逻辑关系，这是提高阅读质量的重要手段。

### §1.2.2　专业英语翻译的要求与基本方法

1. 数学专业英语翻译的要求

翻译的基本要求有两个①：

第一个要求也是最重要的要求就是*准确*，即*忠实于原文*；

第二个基本要求是译文的表达必须*通顺且符合专业规范*，包括词语的选用要得当，意义要明确，句子要流畅，让读者易读，并力求简练。

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① 翻译的第三个要求是，译出原文的风格。对于初学者，特别是数学英语的初学者一般不要求。

为了达到这两个要求,有的地方可直译,必要时应进行意译。

所谓直译,就是在译文语言条件许可时,在译文中既保留原文的内容,又保留原文的形式——特别指保留原文的比喻、形象和民族色彩等。由于每个民族都有自己的词汇、句法结构和表达方法,当原文的思想内容与译文的表达方式有矛盾而不宜采用直译时,就应该采用意译,即按原文的意思(含义)翻译,而不拘泥于原文的形式,有时需根据中文的表达习惯增加或减少一些词语或作其他必要的变动(意译的办法可参考本节第 4 点,即翻译中的若干技巧)。应注意,直译不是“死译”或“硬译”,意译也不是“乱译”。

有的翻译专家特别强调,翻译有三忌:

- 一忌错误理解原文或随意添加或遗漏;
- 二忌逐词死译,勉强凑合;
- 三忌文句不符合中文表达习惯和专业规范。

在实际应用中,若能通过直译把原文的意思表达清楚,当然最省事。但经常发生的是,必须或多或少地进行意译。本书稍后的章节将有大量的实例,这里先给出几个例子作为说明。

**例 1** 日常会话中说:“Take your time!”如果直译成:“拿走你的时间。”那就闹笑话了。数学英语中“$A$ takes its value 3.”中 take 不译成“拿”,takes value 意思为“取值”。

**例 2** Long ago, when people had to count many things, they matched them against their fingers. 通常“match sth. against sb.”指“和某人比赛某项技术或别的东西”。这里应按意思译成“匹配”:远古时,当人们必须数东西时,就得在那些东西和自己的手指之间进行配对。

**例 3** To be more accurate, we must measure what fractional part of the book the desk exceeds 5 book-lengths. 这句显然不能直译。应按原文的意思译成:为了得到更准确的值,我们要测量出课桌超过 5 倍书长的部分是(那本)书的几分之一。

2. 翻译的步骤

第一步:正确地理解原文(见上一节)。

第二步:恰当地表达成中文。翻译是一种创造性的工作,译者必须在正确理解原文的基础上选用恰当的中文词句将原文的意思既准确又通顺地表达出来,让它符合汉语的表达习惯和专业的规范。应该避免逐句对译,即看一句原文译一个句子的做法,而应在全文通读、逐段细读,完全或基本理解的基础上才可动手翻译。再者,理解清楚的东西未必能表达清楚,中国人有句俗语叫做“只可意会,不可言传”,在某种意义上就是对不知如何用语言恰当地表达意思这一尴尬情景的描述。这里既有技巧问题(见下一段的例子),更与译者的中文功底和数

学水平有深刻的关系，需要初学者在实践中不断学习和提高。

第三步：认真地进行校对。为了力求译文的表达准确无误，需要译者将译文与原文作认真的比较和校对，逐句推敲；并把公式、图表等作仔细对比，对于无十分把握的句子、词语或有疑惑之处千万不可潦草从事，一定要通过各种方式进行查实并作必要的纠正。同时，对中文的表达也常要进一步整理和完善，力争通顺易读，精益求精。因此，校对是理解和表达的进一步深化，是使译文符合要求的一个必不可少的步骤。

3. 长句翻译的实例分析

一般说来，短的句子的句型和成分相对简单，单词之间的关系较容易搞清楚。其难点主要在于理解，其次是翻译过程中常常要对词义做必要的转换和引申。我们将在下一段中介绍有关技巧。本段着重研究长句子的处理。

翻译长句的一般程序是“总—分—合”，即先做总体认识，辨别句型，发现特点；然后逐步分解，理解每个意群的含义，理清意群之间的关系；最后综合前两个步骤的分析，形成全句的译文。下面给出一例作为说明。

**例** The use of logarithms has decreased the labour of computing to such an extent that many calculations, which would require hours without the use of logarithms, can be performed with their aid in a small fraction of that time.

**分析**：首先判断出它是一个主从复合句。对这种句型可按以下步骤来分析：

（1）先抓主句，了解中心大意

在对整个句子的轮廓大致了解的基础上，先对主句做个基本分析，以了解全局的中心大意。容易看到，这个主句是一个“主—谓—宾”型的句子，其中主语是“The use of logarithms”，谓语是“has decreased”，宾语是“the labour of computing”。此外，还有一个程度状语“to such an extent”。

于是我们得到全句的中心大意是：“对数的应用在一定程度上减少了计算的劳动量”。这就为全面理解整个句子奠定了基础。

（2）次抓从句，理清主从关系

根据连接词易知，这里有两个从句。这时不仅要对每个从句的结构和含义做基本分析，还要理清从句与主句及从句之间的关系。

由“such…that”引导的从句是结果状语从句，用于修饰主句。该从句的主语是“many calculations”，谓语是“can be performed”（被 which 引导的从句隔开）。

由 which 引导的从句是非限制性定语从句，其主语是“which”，谓语是“would require”；修饰“many calculations”。

结合第一步的分析我们得知，这两个从句用具体例子进一步说明计算量减少所产生的结果。

（3）逐步细化，分析破解疑难

上两个步骤我们只要求先做“基本分析”，目的在于抓住要点和重点，避免因小失大；而为了全面理解掌握，我们还要进一步细化，才能提高准确性和完整性。

细化的任务很繁杂，如把语法分析细化，把词语的理解准确化，把意群间及其与句子的关系清晰化，把各种疑难问题化解等。通常根据句子难易状况来决定细化的程度。由于这个例子不是十分复杂，这里仅对步骤(2)做部分细化，即给出简要的补充说明。

that 从句的主要成分是 many calculations can be performed. 其中介词短语 with their aid 是条件，in a small fraction of that time 表示时间范围。

which 引导的定语从句采用虚拟语气，说明对 calculations 的假定（用介词短语 without the use of logarithms 表示条件），即“如果不用对数，可能要花上好几个小时的计算”。

这里较难翻译的是“fraction”这个单词，不宜直译为“分数”，应放在意群“in a small fraction of that time”中来处理，初步译成“那段时间的很小一部分”，到最后综合时再做文字上的调整。

最后，对全句综合处理、完善理解与表达，即在上述分析的基础上进行整合处理（必要时要对(2)中出现的错误或不完整的理解作出纠正），理顺各意群的表达，给出全句较为准确通顺的译文。

全句大体可译成：

“对数的使用把计算的劳动量减少到这样的程度，即许多计算当未采用对数时需耗费数个小时，而现在由于借助对数只要用原来的很少的一部分时间就能完成。”

而后是校对和改善，文句上还可进一步修饰和改善。

这一句之所以不算难，是因为句子成分还不够复杂，且涉及的数学知识较少。不过，上述的分析方法是普遍适用的。只要多作练习，就可不断提高。

4. 翻译中的若干技巧

*初学者对本小节的内容只需有大体了解即可，等到第二章学完后，再回过头来仔细阅读。*

在前面几节的例子中我们可以看到，在翻译时，并非逐一对应、逐句对译。由于汉语和英语的表达方式和语法结构有很大差别，因此在翻译时，译者要根据实际需要来考虑词语的必要增减、语序的改变、词义的适当选择和引申、词性和句子成分的必要转换。下面分别举例加以说明。

(1) 词义的选择

由于英语常一词多义，特别是半专业术语，含义更多，翻译时需对照上下文和全句的意思，认真选择适合的词义。

例如，前面谈过的 power 在不同处含义不同。

Energy is the power to do work.(译文:能是指做功的能力。)

The logarithm of a given number is the power to which another number, called the base, must be raised in order to equal the given number.(译文:一个已知数的对数是另一个称为底的数为了等于该已知数而必须自乘的幂次。)

同一个词,有时在同一个句子的不同处出现,其内涵和外延也发生变化,因此必须译成两个不同的概念。例如:

Equations are of two kinds — identities and equations of condition, the latter is called equations for short.(译文:等式分为两类——恒等式和条件等式,后者简称为方程。)

这时,如把第一个 equation 译成方程,那么恒等式就变成了方程中的一个子类,这与数学知识相违背。因此,这个句子的前两个 equation 都译作等式,而最后一个 equation 译成方程。

(2) 词义的引申

据上下文的联系和逻辑关系,将词义加以必要的引申,选择比较恰当的汉语词汇来表达。

**例1** Two and three make five.(译文:二加三等于5。)

原文 make 本来的意思是"制造",现引申为"等于"。

**例2** Year after year and century after century the moon goes through its cycles of changes.(译文:月亮的盈亏变化,一年又一年,一世纪又一世纪,周而复始。)

这一句改动较大,其中盈亏变化,周而复始都是采用中国的传统说法来表达原文的意思的。

(3) 词语的增减

依据汉语的习惯和数学专业的规范对原文的词语作适当的增删。

**例1** An arithmetic or an algebraic identity is an equation.(译文:代数和算术的恒等式都是等式。)

在译文中,英文的三处不定冠词"a"或"an"都不予译出,且按汉语习惯,把"or"译为"和"(一般情况下"or"译为"或者"),与此相应,后面添上"都"字。

**例2** The formulas differ only in the signs preceding their left numbers.(译文:这两个式子不同之处仅在于加在它们左边的数前面的符号之差异。)

这里"两个"是根据上下文推理出来的;"之差异"是根据中文的习惯添加的。

词的添加的一个重要理由是需要重复译出。

在英语并列句中,若后面一句的有些成分与前一句相同时,相同的成分可以省略。同样,复合句从句中与主句相同的成分也可省略。但译成汉语时,被省略的成分常要重复译出。

**例 3** Thus, $\sqrt{-5}$ means something very different from $\sqrt{5}$, and $\sqrt{-n}$ from $\sqrt{n}$. (译文:于是,$\sqrt{-5}$是与$\sqrt{5}$完全不同的东西,同样,$\sqrt{-n}$是与$\sqrt{n}$不同的(东西)。)这里,原文省略了"means something very different",在翻译时重复译出。

**例 4** After checking, we see −3 satisfies the original equation, but 3 does not. (译文:经检验得知,−3满足原方程,但 3 不满足原方程。)根据上下文得知,does not 后面省略了"satisfy the original equation",汉译时应补译出。

(4) 词序的变动

在按自然语序(非倒序)写出的英语陈述句中,主语、谓语、宾语的排列次序与汉语基本相同(主语—谓语—宾语),只需依次译出即可。但在一些情况下采用倒序的英语句子,汉译时常按自然语序译出。

**例 1** Such is the case. (译文:情况就是这样。)

此外,英语的各种短语或从句做定语时,一般都后置(放在被修饰词——中心词之后),而汉语的定语一般都前置。因此翻译时一般应按汉语习惯改变顺序。英语中有些单个分词或形容词当定语时也常后置,翻译时可据具体情况前置或后置,但以 able, ible 结尾的形容词若后置,通常应译成前置。

**例 2** The square root of a negative number is a pure imaginary(译文:负数的平方根是纯虚数。)这里,短语 of a negative number 作为 the square root 的定语。

**例 3** An equilateral triangle is one with all its sides equal. (译文:"各边都相等的三角形叫做等边三角形。"或"等边三角形是各边都相等的三角形。")这里短语 with all its sides equal 修饰 one。

**例 4** Something new(某种新东西);
the element known(已知的元素);
the conclusion required(所需要的结论)。

**例 5** The values of the constructed function should not exceed the maximum permissible. (译文:构造的函数的值不应超过所允许的最大数值。)

**例 6** The function differentiable on [a, b] is continuous. (译文:在[a, b]上可微的函数连续。)

(5) 词性的转换

由于英汉两种语言在表达习惯和语言结构上有很大差别,因此翻译时,如果不考虑具体情况,刻板地将名词一律译成名词,动词译成动词,则有可能使译文更含糊不清,不够通顺达意。因此,有时必须把词类作适当的转换。例如可把动词译成名词,名词译成动词,形容词译成副词等等。

**例 1** Applied mathematics aims at achieving the optimization. (译文:应用数学的目的是达到最优化。)这里把动词 aims 译成了名词。

**例 2** The application of computers makes for a tremendous rise in calculation speed.（译文：使用计算机可以大大提高计算速度。）这里把名词 application 和 rise 均译成动词。

英语中一些形容词在句子中作表语时，可译成汉语中的名词或动词。

**例 3** Rational number can be represented by the quotient of two integers, which is different from irrational one.（译文：有理数可以表示成两个整数的商，这是它与无理数的区别。）这里把形容词 different 译成名词。

This strait line is perpendicular to the known plane.（译文：这条直线垂直于已知平面。）这时把形容词 perpendicular 译成动词。

(6) 句子成分的转换

在汉译时，为了符合汉语习惯和专业要求，可将英语句中的一些成分译成汉语句中的其他成分。比如状语译成主语，定语译成谓语等。

**例 1** The same signs and symbols of mathematics are used throughout the world.（译文：全世界都使用同样的数学记号和符号。）这句把地点状语译成主语。其实，在"there+be+…"句型中也常把地点状语译成主语。

**例 2** The statement of the Gauss Theorem is as follows.（译文：高斯定理叙述如下。）这里把主语 statement 译成了谓语。

由(5)知，在词性转换时也常把句子的成分作了转换。

(7) 语态与人称的改变

根据需要也可以把被动语态译成主动语态或反过来把主动语态译成被动语态。

**例 1** Recently, The International Conference of Mathematicians 2002 was held in Beijing.（译文：最近，北京举办了 2002 年国际数学家大会。）本句把原文的初动语态译成主动语态，原来的地点状语译成了主语。

**例 2** We call a triangle an obtuse triangle when one angle is an obtuse angle.（译文：有一个角为钝角的三角形被称为钝角三角形。）这里把原文的主动语态译成被动语态，条件从句译成定语。

**例 3** A great deal of practical problems can be solved with the differential equations.（译文：使用微分方程，(人们)可以解决大量的实际问题。）

无人称语句"It….."可照样译成无人称句，必要时也可以译成人称句，其中人称可用"人们"、"有人"、"我们"等词译出。

以"It"为形式主语的句型(包括含主语从句的复合句)可译成无人称句；必要时也可译成人称句，这时人称可采用主句的逻辑主语；如无逻辑主语，可据需要补上"人们"、"有人"、"我们"等。

**例 4** It should be noted that equations(1) and(2) are merely two different ways of expressing precisely the same relations.（译文：应当指出，方程(1)和(2)仅是同一种关系的两种不同的表达方式而已。）

**例 5** It was found that the dimension of a fractal might be not an integer.（译文：人们发现，分形的维数可以不是整数。）

**例 6** It is necessary for college students to understand the basic concepts of set theory.（译文：大学生有必要了解集合论的基本概念。）这里，逻辑主语 college students 译成主语。

(8) 数量表达方式的变更

英语中数量的增加(减少)和倍数的表达方式有多处与汉语不同，翻译时应做相应的调整和更动。

**例 1** This year the output of our city has increased by three times as compared with that of 1990.（译文：今年我市的产值比 1990 年增加 3 倍。）在 increase(be, go up)+by+数字或倍数+…的结构中。by 后表示的是净增加数和倍数，所以应照译不改。

**例 2** This year the output of our city has increased three times as compared with that of 1990.（译文：今年我市的产值比 1990 年增加 2 倍。）此句与上句相比仅少了一个"by"，但应将增加的倍数减 1。

**例 3** The cost of MP4 decreased by 60%.（译文：MP4 的成本降低了百分之六十。）在"decreased(fall, drop, lower, decrease…)+by+数字"的结构中，by 后的数量是纯减少的量，所以照译不减。

**例 4** The cost of MP4 decreased to 40%.（译文：MP4 的成本降到了原来的 40%。）在"decreased(drop, decrease…)+to+数字"的结构中，"to"表示"到"的意思，所以常译成"降到……"，"减少到……"。

**例 5** With a new algorithm the computing time of the problem is shortened tree times.（译文：采用新的算法，该问题的计算时间缩短到(原来的)1/3。）在英语中如果减少的是倍数，汉译时应换算成分数或百分数来表示，换算办法是：把倍数 $n$ 当成分母，用 1 作分子，表示减少后的结果，译成"减少到(原来的) $n$ 分之一"或"减为 $n$ 分之一"。

本小节讲的主要是翻译时一般需做变动的情况。想了解其他更细致的方法的读者可参阅由西北工业大学外语教研室编的《科技英语翻译初步》[5]。凡此种种，仅供读者学习使用时借鉴。读者应在实践中加深认识和理解，并不断地发现总结规律，才能提高翻译水平。应该说，法无常法，要掌握好是不容易的。但可以肯定的总原则始终是：

在忠于原文和符合专业表达规范及汉语习惯的大原则下，可以且必须在英译汉过程中作适当的变动，以达到准确，通顺、简练、易读之目的。坚决反对"死

译"硬凑等不负责任的做法。

**附：人名的翻译**

外国数学家的名字常以两种形式出现在英文版文献中。一种是以本国文字出现，通常是拉丁文语系（包括英语）的国家的数学家名字，如 Newton（牛顿，英文），Cauchy（柯西，法文），Poison（泊松，法文），Hilbert（希尔伯特，德文）等；另一种是按读音翻译（简称"音译"）成英文，如 Urysohn（乌雷松，俄文 Урысон 的音译）。不论上述哪一种形式，译成中文时均采用音译。不过，音译成中文时由于译者不同，常出现不同的译法（例如：有人把 Cauchy 译成"哥西"或"科希"等），可能造成混乱。

现在国内翻译英文文献时基本上采用如下方法：（一）著名的数学家的名字按《数学名词》或《中国大百科全书》数学卷的规定统一译法；（二）其他数学家的名字一般不予译出，即把姓名原文抄录在译文中（姓的部分全写出，名的部分可缩写），如果要译出，必须在译名后加括号注明原文；（三）在同一篇译文中数学家的姓名要么全部不予译出，要么全部译出且对其中不大为人所知的人，特别是不在中国大百科全书数学卷出现的姓名后加括号注上原文。

**例 1** David Hilbert posed 23 mathematical problems in 1900.

译成：(i)1900 年大卫·希尔伯特提出 23 个数学问题。（因希尔伯特是著名的数学家，不必加括号注上原文。）

也可译成：(ii) 1900 年 David Hilbert 提出 23 个数学问题。

或译成：(iii) 1900 年 D. Hilbert 提出 23 个数学问题。

**例 2** We may use Z. Opial's formula to solve this problem.

译成：我们可利用奥皮尔（Z. Opial）公式来解这个问题。（因 Z. Opial 是人们不太熟悉的数学家，故加括号注上原文。）

或译成：我们可利用 Z. Opial 公式来解这个问题。

**插页：名家谈翻译**

鲁迅先生在谈及翻译工作的艰巨性时说过："我向来总以为翻译比创作容易，因为至少无须构想。但到真的一译，就会遇着难关。譬如一个名词或动词，写不出，创作时候可以回避，翻译上却不成，也还得想，一直弄得头昏眼花，好像在脑子里面摸一个急于要开箱子的钥匙，却没有。严又陵①说，'一名之立，旬月踌躇'，是他的经验之谈，的的确确的。"

① 严又陵即严复，是我国 19 世纪末 20 世纪初的伟大翻译家，他提出了"信、达、雅"的翻译标准，长期被奉为准则。

# 第二章 精读课文——入门必修

本章课文较通俗易懂，内容安排由浅入深，是学习数学专业英语的基础。全章共十二课，每课有 A，B，C 三篇课文，A，B 两篇供课堂使用或选用，C 篇可当课外练习。每课配有

(1) 两个生词与词组表(主要列举数学术语)，其中表(一)为课文 A、B 的学习提供方便、表(二)为课文 C 的学习和英译汉练习提供方便；

(2) 预习要求，包括在课文中出现的重要单词、常用词组和短语的学习与掌握；

(3) 课文的注释与说明，包括课文 A、B 中一些难句的翻译举例；

(4) 课外作业，包括汉译英与英译汉等练习。

另外，本章末附有《基本运算符号与算式的读法》，可供教学参考。

初学者应认真做好预习和复习工作，努力掌握数学英语阅读与翻译的基本知识和方法，为下一步的提高奠定良好的基础。

## §2.1 数学、方程与比例
## (Mathematics, Equation and Ratio)

### 课文 1-A What is mathematics[1]①

Mathematics comes from man's social practice, for example, industrial and agricultural production, commercial activities, military operations and scientific and technological researches[2]. And in turn, mathematics serves the practice and plays a great role in all fields. No modern scientific and technological branches could be regularly developed without the application of mathematics.

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① 本节短文 1-A 是作者综合了文献[1]的部分材料编写的。本章各节的短文凡未在题头处标注出处者皆取材于 T. M. Apostol, Calculus, Vol. 1, New York: John Wiley & Sons Inc., 1967。

From the early need of man came the concepts of numbers and forms. Then, geometry developed out of problems of measuring land, and trigonometry came from problems of surveying. To deal with some more complex practical problems, man established and then solved equation with unknown numbers, thus algebra occurred. Before 17th century, man confined himself to the elementary mathematics, i. e., geometry, trigonometry and algebra, in which only the constants were considered.

The rapid development of industry in 17th century promoted the progress of economics and technology and required dealing with variable quantities. The leap from constants to variable quantities brought about two new branches of mathematics — analytic geometry and calculus, which belong to the higher mathematics. Now there are many branches in higher mathematics, among which are mathematical analysis, higher algebra, differential equations, function theory and so on.

Mathematicians study conceptions and propositions. Axioms, postulates, definitions and theorems are all propositions. Notations are a special and powerful tool of mathematics and are used to express conceptions and propositions very often. Formulas, figures and charts are full of different symbols. Some of the best known symbols of mathematics are the Arabic numerals 1,2,3,4,5,6,7,8,9,0, and the signs of addition "+", subtraction "−", multiplication "×", division "÷" and equality "=".

The conclusions in mathematics are obtained mainly by logical deductions and computation. For a long period of the history of mathematics, the centric place of mathematical methods was occupied by the logical deductions[3]. Now, since electronic computers are developed promptly and used widely, the role of computation becomes more and more important. In our times, computation is not only used to deal with a lot of information and data, but also to carry out some work that merely could be done earlier by logical deductions, for example, the proof of most of geometrical theorems.[4]

## 课文 1-B Equation[1]①

An equation is a statement of the equality between two equal numbers or number symbols[5].

Thus $a(a-5)=a^2-5a$ and $x-3=5$ are equations.

Equations are of two kinds — identities and equations of condition.

① 方括号[ ]中的数字表示引用或摘录的文献在书末参考文献表中出现的序号。

An arithmetic or an algebraic identity is an equation. In such an equation either the two members are alike, or become alike on the performance of the indicated operation[6].

Thus $12-2=2+8$, $(m+n)(m-n)=m^2-n^2$ are identities.

An identity involving letters is true for any set of numerical values of the letters in it[7].

Thus the identity $x(a+2)=ax+2x$ becomes $3(7+2)=21+6$ or $27=27$, when, for example, $x=3$, and $a=7$.

An equation which is true only for certain values of a letter in it, or for certain sets of related values of two or more of its letters, is an equation of condition, or simply an equation[8]. Thus $3x-5=7$ is true for $x=4$ only; and $2x-y=10$ is true for $x=6$ and $y=2$ and for many other pairs of values for $x$ and $y$.

A root of an equation is any number or number symbol which satisfies the equation.

To obtain the root or roots of an equation is called solving an equation.

There are various kinds of equations. They are linear equations, quadratic equations, etc.

To solve an equation means to find the value of the unknown term. To do this, we must, of course, change the terms about until the unknown term stands alone on one side of the equation, thus making it equal to something on the other side. We then obtain the value of the unknown and the answer to the question. To solve the equation, therefore, means to move and change the terms about without making the equation untrue, until only the unknown quantity is left on one side, no matter which side[9].

Equations are of very great use. We can use equations in many mathematical problems. We may notice that almost every problem gives us one or more statements that something is equal to something; this gives us equations, with which we may work if we need to.

**生词与词组(一)**

addition[əˈdiʃən] n. 加,加法

algebra[ˈældʒibrə] n. 代数学

  higher algebra 高等代数

algebraic[ˌældʒiˈbreiik] adj. 代数的

arithmetic[əˈriθmətik] adj. 算术的

axiom[ˈæksiəm] n. 公理

branch[brɑːntʃ] n. 分支

calculus [ˈkælkjuləs] n. 微积分[学](the differential and integral calculus 的简写)

chart[tʃɑːt]n. 图表,表格
computation[ˌkɔmpjuˈteiʃən]n. 计算
concept[ˈkɔnsept]n. 概念
conception[kənˈsepʃən]n. 概念,观点
constant[ˈkɔnstənt]n. 常数
deduce[diˈdjuːs]v. 推导
deduction[diˈdʌkʃən]n. 推导,推理
logical deduction 逻辑推理
definition[ˌdefiˈniʃən]n. 定义
division[diˈviʒən]n. 除,除法
equality[iːˈkwɔliti]n. 等式,相等
equation[iˈkweiʃən]n. 方程,等式
*equation of condition* 条件等式
differential equation 微分方程
linear equation 线性方程
quadratic equation 二次方程
figure[ˈfigə]n. 图,图形
form[fɔːm]n. 形,形状;v. 形成
formula[ˈfɔːmjulə]n. 公式
function[ˈfʌŋkʃən]n. 函数
function theory 函数论
geometrical[dʒiəˈmetrikəl] adj. 几何的
geometry[dʒiˈɔmitri]n. 几何学
identity[aiˈdentiti]n. 恒等式
indicate[ˈindikeit]v. 指明,指出,指定
involve[inˈvɔlv]v. 包含
mathematical analysis 数学分析
mathematics[mæθiˈmætiks]n. 数学
higher mathematics 高等数学
elementary mathematics 初等数学
measure[ˈmeʒə]v. 测量
multiplication[ˌmʌltipliˈkeiʃən]n. 乘,乘法
notation[nəuˈteiʃən]n. 符号,记法
number[ˈnʌmbə]n. 数,号码
numeral[ˈnjuːmərəl]n. 数字;adj. 数的
numerical[njuːˈmerikəl]adj. 数值的,数字的
obtain[əbˈtein]v. 得到,获得
operation[ˌɔpəˈreiʃən]n. 运算,运作
military operations 军事行动
performance[pəˈfɔːməns]n. 执行,操作
postulate[ˈpɔstjuleit]n. 公设
proof[pruːf]n. 证明
proposition[ˌprɔpəˈziʃən]n. 命题
reasoning method 推理方法
root[ruːt]n. 根
set[set]n. 集,组,套;v. 令
sign[sain]n. 信号,记号
statement[ˈsteitmənt]n. 陈述,表述方式
subtraction[səbˈtrækʃən]n. 减,减法
symbol[ˈsimbəl]n. 符号
term[təːm]n. 项,术语;v. 命名
change the terms about 把这些项变形
theorem[ˈθiərəm]n. 定理
trigonometry[trigəˈnɔmitri]n. 三角学
true[truː]adj. 真的,成立
unknown[ˈʌnˈnəun]a. 未知的
untrue[ˈʌnˈtruː]adj. 非真的,不成立的
variable[ˈvɛəriəbl]adj. 变化的;n. 变量
variable quantity 变量

**预习要求**

1. 预习生词与词组(一),浏览课文 A、B,并在疑难处做上记号。

2. 复习或借助词典学习以下单词、词组与短语的含义及用法:

(1) alike
(2) bring about
(3) carry out
(4) come from sth.
(5) deal with sth.
(6) express change the terms about
(7) be equal to sth.
(8) be full of sth. /sb.
(9) in turn
(10) make sth. equal to sth.
(11) no matter
(12) occupy
(13) occur
(14) on the performance of sth.
(15) promote
(16) resulting method

**注**:本书用 sth. 表示 something,用 sb. 表示 somebody(ies),而 sth. /sb. 表示该处既可用 something,也可用 somebody(ies)。

3. 复习被动语态和动词不定式的用法。

**注释与说明**

1. 本节课文 A 是开场白,列举了许多最基本的数学用词和术语,其中一部分是读者在公共英语课程中已学过的功能词(半专业词汇)。读者应加以分门别类,尽快掌握它们。

2. man 用单数表示人类。operation 作为数学用语常译成"运算",但词组"operation research"译为运筹学。

3. For a long period of the history of mathematics, the centric place of mathematical methods was occupied by the logical deductions. 这句翻译成主动式较为符合中文的习惯:在数学史的很长的时期内,逻辑推理一直占据数学方法的中心地位。

4. In our times, computation is not only used to deal with a lot of information and data, but also to carry out some work that merely could be done earlier by logical deductions, for example, the proof of most of geometrical theorems. 这句可译成:现在,计算不仅用来处理信息与数据,而且用来完成一些在以前只能靠逻辑推理来做的工作,例如证明大多数的几何定理。

5. An equation is a statement of the equality between two equal numbers or number symbols. 注意,equation 有两个意思:方程(常用)和等式(少用);equality 也有两个意思:等式(少用)和相等(常用)。statement 也有多种含义,应据上下文和数学内容的含义与逻辑关系而选用适当的译法。整句可译成:等式是关于两个数或数的符号相等的一种陈述。

6. In such an equation either the two members are alike, or become alike on the

performance of the indicated operation. 这里 two members 表示(等号的)两端;alike 意思是相同的或一样的。on the performance of…中的 on 引导一个介词短语,作状语修饰 become alike。应注意 either…or…的用法。整句可译成:这种等式的两端要么一样,要么经过执行指定的运算后变成一样。

7. An identity involving letters is true for any set of numerical values of the letters in it. 这里 in it 是介词短语,修饰 letters。句中 set 作量词用,意为组、套等。因为数学命题有真假之分,把 is true 直译为"是真的"也可以,但只在特定场合才那么说。在一般情况下应译为"成立"。整句可译成:含有字母的恒等式对其中字母的任何一组数值都成立。

8. An equation which is true only for certain values of a letter in it, or for certain sets of related values of two or more of its letters, is an equation of condition, or simply an equation. 注意这句开头的 equation 和结尾的 equation 含义不同。主语 An equation的修饰成分很长,其实是说明 An equation 必须满足的条件。因此,整句可译成:一个等式若仅仅对其中一个字母的某些值成立,或者对其中两个或多个字母的若干组相关的值成立,则它是一个条件等式,或简称方程。

9. To solve the equation, therefore, means to move and change the terms about without making the equation untrue, until only the unknown quantity is left on one side, no matter which side. 这句的主语和表语都是动词不定式。to move and change the terms 是数学术语:移项和变形。without making the equation untrue 意思是保持方程同解,不使原方程成立的条件破坏了。no matter which side 后面省略了 it is。整句可译成:因此,解方程就意味着进行一系列移项和同解变形,直到未知量被单独留在等式的一边,不论是哪一边。

**课外作业**

1. 把下列各组的单词、词组与短语译成英语,并按它们的相关性联想记忆:

(1) 数学分支、算术、几何学、代数学、三角学、高等数学、初等数学、高等代数、数学分析、函数论、微分方程。

(2) 命题、公理、公设、定义、定理、引理、推论。

(3) 形、数、数字、数值、图形、公式、符号、记法/记号、图表。

(4) 概念、相等、成立/真、不成立/不真、等式、恒等式、条件等式;项/术语、集、函数、常数;方程、线性方程、二次方程。

(5) 运算、加法、减法、乘法、除法;证明、推理、逻辑推理。

(6) 测量土地、推导定理、指定的运算、获得结论、占据中心地位。

2. 汉译英:

(1) 数学来源于人类的社会实践,包括工农业的劳动,商业、军事和科学技

术研究等活动。

(2) 如果没有运用数学,任何一个科学技术分支都不可能正常地发展。

(3) 符号在数学中起着非常重要的作用,它常用于表示概念和命题。

(4) 17 世纪之前,人们局限于初等数学,即几何、三角和代数,那时只考虑常数。

(5) 方程与算术的等式不同在于它含有可以参加运算的未知量。

(6) 方程又称为条件等式,因为其中的未知量通常只允许取某些特定的值。

(7) 方程很有用,可以用它来解决许多实际应用问题。

(8) 解方程时要进行一系列移项和同解变形,最后求出它的根,即未知量的值。

3. 英译汉:

(1) Algebra has evolved from the operations and rules of arithmetic. The study of arithmetic begins with addition, multiplication, subtraction, and division of numbers:

$$4+7,\ 37\times682,\ 49-22,\ 40\div8.$$

In algebra we introduce symbols or letters—such as $a$, $b$, $c$, $d$, $x$, $y$, $z$—to denote arbitrary numbers and, instead of special cases, we often consider general statements:

$$a+b,\ cd,\ x-y,\ x\div a.$$

(2) The language of algebra serves a twofold purpose. First, we may use it as a shorthand to abbreviate and simplify long or complicated statements. Second, it proves a convenient means of generalizing many specific statements.

(3) Many expressions involve two or more operations. Grouping symbols tell us which operation is to be done first. The common grouping symbols are parentheses, ( ), brackets. [ ], and the fraction bar, —. For example, in the expression 2(3+4), we do the addition first and then we do the multiplication:

$$2(3+4)=2(7)=14.$$

4. 借助下面的生词与词组(二)自学课文 1-C,并把它译成汉语。

## 课文 1-C Ratio and measurement①

The communication of ideas today is often based upon comparing numbers and quantities. When you describe a person as being 6 feet tall, you are comparing his

① 课文 1-C 摘自:E. M. Hemmerling. College Plane Geometry, John Wiley & Sons. 1986。

height to that of a smaller unit, called the foot. When a person describes a commodity as being expensive, he is referring to the cost of this commodity as compared to other similar or different commodities. If you say that the dimensions of your living room are 18 by 24 feet, a person can judge the general shape of the room by comparing the dimensions. When the taxpayer is told that his city government is spending 42 per cent tax dollar for education purposes, he knows that 42 cents out of every 100 cents are used for this purpose.

The chemist and the physicist continually compare measured quantities in the laboratory. The housewife is comparing when measuring quantities of ingredients for baking. The architect with his scale drawings and the machine draftsman with his working drawings are comparing length of lines in the drawings with the actual corresponding lengths in the finished product.

DEFINITION. *The ratio of one quantity to another like quantity is the quotient of the first divided by the second.*

A ratio is a fraction and all the rules governing a fraction apply to ratio. We write a ratio either with a fraction bar, a solidus, division sign, or with the symbol " : " (which is read "is to"). Thus the ratio 3 to 4 is $\frac{3}{4}$, 3/4, 3÷4, or 3 :4. The 3 and 4 are called terms of the ratio.

It is important for the student to understand that a ratio is a quotient of like quantities. The ratio of a line segment to an angle has no meaning; they are not quantities of the same kinds. We find the ratio of one line segment to a second line segment or the ratio of one angle to a second angle. This we do by measuring them and then finding the quotient of their measurements. The measurements must be expressed in the same units.

A ratio is always an abstract number; i. e. , it has no units. It is a number considered apart from the measured units from which it came. Unless there is an important reason to the contrary, a ratio should be expressed in its simplest form. In the previous example where the dimensions of a living room are 18 by 24 feet, the final ratio of width to length is 3 :4, but not 18 :24.

**生词与词组(二)**

abbreviate[əˈbriːvieit] v. 缩短,省略
arbitrary[ˈɑːbitrəri] adj. 任意的
bracket[ˈbrækit] n. 括号,方括号
　round bracket 圆括号
commodity[kəˈmɔditi] n. 日用品
dimension[diˈmenʃən] n. 大小,维数

evolve[i'vɔlv]v. 发展;展开
expression[iks'preʃən]n. 表达[方]式
fraction['frækʃən]n. 分数,分式
fraction bar 除线,分数线
generalize['dʒenərəlaiz]v. 推广
grouping symbols(=sighs of grouping) 分组记号(用于表示优先计算的部分)
measurement['meʒəmənt] n. 测量,观测
parenthesis[pə'renθisis]n. 圆括号(=round bracket)
prove[pruːv]v. 证明,被证明是
ratio['reiʃiəu]n. 比例
shorthand['ʃɔːthænd]v. 速记
simplify['simplifai]v. 简化
solidus['sɔlidəs]n. 斜线分隔符,即"/"
twofold['tuːfəuld]adj. 双重的

# §2.2 几何与三角
# (Geometry and Trigonology)

## 课文 2-A Why study geometry?①

Why do we study geometry? The student beginning the study of this text may well ask, "What is geometry? What can I expect to gain from this study?"

Many leading institutions of higher learning have recognized that positive benefits can be gained by all who study this branch of mathematics[1]. This is evident from the fact that they require study of geometry as a prerequisite to matriculation in those schools.

Geometry had its origin long ago in the measurements by the Babylonians and Egyptians of their lands inundated by the floods of the Nile River. The greek word *geometry* is derived from *geo*, meaning "earth," and *metron*, meaning "measure." As early as 2000 B. C. we find the land surveyors of these people re-establishing vanishing landmarks and boundaries by utilizing the truths of geometry[2].

Geometry is a science that deals with forms made by lines. A study of geometry is an essential part of the training of the successful engineer, scientist, architect, and draftsman. The carpenter, machinist, stonecutter, artist, and designer all apply the facts of geometry in their trades. In this course the student will learn a great deal about geometric figures such as lines, angles, triangles, circles, and designs and

① 课文 2-A 和 2-B 摘自:E. M. Hemmerling. College Plane Geometry, John Wiley & Sons. 1986。

patterns of many kinds.

One of the most important objectives derived from a study of geometry is making the student be more critical[3] in his listening, reading, and thinking. In studying geometry he is led away from the practice of blind acceptance of statements and ideas and is taught to think clearly and critically before forming conclusions.

There are many other less direct benefits the student of geometry may gain. Among these one must include training in the exact use of the English language and in the ability to analyze a new situation or problem into its basic parts[4], and utilizing perseverance, originality, and logical reasoning in solving the problem. An appreciation for the orderliness and beauty of geometric forms that abound in man's works and the creations of nature will be a byproduct of the study of geometry. The student should also develop an awareness of the contributions of mathematics and mathematicians to our culture and civilization.

## 课文 2-B Some geometrical terms

1. **Solids and planes**. A *solid* is a three-dimensional figure. Common examples of solids are cube, sphere, cylinder, cone and pyramid.

A cube has six faces which are smooth and flat. These faces are called *plane surfaces* or simply *planes*. A plane surface has two dimensions, length and width. The surface of a blackboard or of a tabletop is an example of a plane surface.

2. **Lines and line segments.** We are all familiar with lines, but it is difficult to define the term. A line may be represented by the mark made by moving a pencil or pen across a piece of paper. A line may be considered as having only one dimension, length. Although when we draw a line we give it breadth and thickness, we think only of the length of the trace when considering the line. A *point* has no length, no width, and no thickness, but marks a position. We are familiar with such expressions as pencil point and needle point. We represent a point by a small dot and name it by a capital letter printed beside it, as "point $A$" in Fig. 2-2-1.

The line is named by labeling two points on it with capital letters or one small letter near it. The straight line in Fig. 2-2-2 is read "line $AB$" or "line $l$". A straight line extends infinitely far in two directions and has no ends. The part of the line between two points on the line is termed a *line segment*. A line segment is named by the two end points. Thus, in Fig. 2-2-2, we refer to $AB$ as a line segment of line $l$. When no confusion may result, the expression "line segment $AB$" is often replaced by segment $AB$ or, simply, *line AB*.

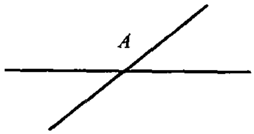


Fig. 2-2-1

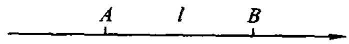


Fig. 2-2-2

There are three kinds of lines: the straight line, the broken line, and the curved line. A curved line or, simply, curve is a line no part of which is straight. A broken line is composed of joined, straight line segments, as *ABCDE* of Fig. 2-2-3.

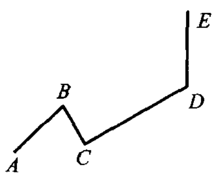


Fig. 2-2-3

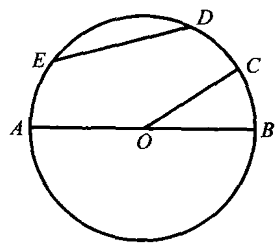


Fig. 2-2-4

3. **Parts of a circle.** A circle is a closed curve lying in one plane, all points of which are equidistant from a fixed point called the center (Fig. 2-2-4). The symbol for a circle is ⊙. In Fig. 2-2-4, *O* is the center of ⊙*ABC*, or simply of ⊙*O*. A line segment drawn from the center of the circle to a point on the circle is a radius (plural, radii) of the circle. *OA*, *OB*, and *OC* are radii of ⊙*O*. A diameter of a circle is a line segment through the center of the circle with endpoints on the circle. A diameter is equal to two radii. A chord is any line segment joining two points on the circle. *ED* is a chord of the circle in Fig. 2-2-4.

From this definition it should be apparent that a diameter is a chord. Any part of a circle is an arc, such as arc *AE*, which is denoted by $\overset{\frown}{AE}$. Points *A* and *E* divide the circle into *minor arc AE* and *major arc ABE*. A diameter divides a circle into two arcs termed *semicircles*, such as $\overset{\frown}{AB}$ and $\overset{\frown}{BCA}$. The *circumference* is the length of a circle.

**生词与词组(一)**

abound[ə'baund]v. 大量存在

angle['æŋgl]n. 角

appreciation [ əˌpriːʃi'eiʃən ] n. 鉴赏能力

arc[ɑːk]n. 弧
major arc 优弧
minor arc 劣弧
architect['ɑːkitekt]n. 建筑师,设计师
breadth[bredθ]n. 宽度
byproduct['baiˌprɔdʌkt]n. 副产品
center['sentə]n. 中心,圆心
chord[kɔːd]n. 弦
circle['səːkl]n. 圆周,圆
circumference[sə'kʌmfərəns]n. 周长
cone[kəun]n. 圆锥
critical['kritikəl]adj. 批评的,挑剔的,临界的
cube[kjuːb]n. 立方体
curve[kəːv]n./v. 弯曲
curved line 曲线
cylinder['silində]n. 柱体
define[di'fain]v. 对……下定义,界定
diameter[dai'æmitə]n. 直径
dimension[di'menʃən]n. 维数,大小
draftsman['drɑːftsmən]n. 制图员
endpoint['endpɔint]n. 端点
equidistant[ˌiːkwi'distənt] adj. 等距离的
geometrical[dʒiə'metrikəl] adj. 几何的
infinitely['infinitli]adv. 无限地
institution[ˌinsti'tjuːʃən] n. 学会、名人
inundate['inəndeit]v. 淹没
label['leibl]v. 标记;n. ,标号,标签
line [lain]n. 线,直线
line segment 直线段
broken line 折线
straight line 直线
machinist[mə'ʃiːnist]n. 机械师,机工
matriculation[məˌtrikju'leiʃən] n. 入学考试,考取大学
orderliness['ɔːdəlinis]n. 井然有序
originality[əˌridʒi'næliti]n. 创造力
perseverance[ˌpəːsi'viərəns]n. 毅力
plane[plein]n. 平面
position[pə'ziʃən]n. 位置,状态
positive['pɔzətiv]adj. 正的,正面的,肯定的
prerequisite[ˌpriː'rekwizit] adj. 须先具备的,n. 先决条件
pyramid['pirəmid]n. 棱锥,金字塔
radius['reidjəs]n. 半径(复数为 radii ['reidiai])
ray[rei]n. 射线
semicircle['semiˌsəːkl]n. 半圆
side[said]n. 边
solid['sɔlid]adj. 立体的 n. 立体
sphere[sfiə]n. 球,球面
stonecutter['stəunˌkʌtə]n. 采石者
surface['səːfis]n. 面,曲面
surveyor[sə'veiə]n. 测量者
thickness['θiknis]n. 厚度
triangle['traiæŋgl]n. 三角形
vanish['væniʃ]v. 消失,变成零

**预习要求**

1. 预习生词与词组(一),浏览课文 A、B 并在疑难处做上记号。
2. 复习或借助词典学习以下单词、词组与短语的含义及用法:

(1) appreciation of(for)sth./sb.

(2) awareness of sth./sb.

(3) blind acceptance of sth.

(4) *change the terms about*

(5) be composed of sth./sb.

(6) be derived from sth.

(7) divide sth. into sth.

(8) be equidistant from sth.

(9) expect to do sth.

(10) be familiar with sth./sb.

(11) gain

(12) be led away from sth.

(13) prerequisite to sth.

(14) refer to sth./sb. as sth./sb.

(15) treatment of sth.

(16) work with sth.

3. 复习现在分词和过去分词的用法。

**注释与说明**

1. Many leading institutions of higher learning have recognized that positive benefits can be gained by all who study this branch of mathematics. 其中 higher learning 表示"高学识"、"博学";positive 在数学中常表示"正的",这里表示"确实的"或"肯定的"。整句可译成:许多居领导地位的学术机构承认,所有学习这个数学分支的人都将得到确实的收益。

2. …we find the land surveyors of these people re-establishing vanishing landmarks and boundaries by utilizing the truths of geometry. 这里 find 意思是发现,后接宾语 the land surveyors, these people 和 re-establishing 引起的分词短语都是修饰它的定语,但译为汉语时,可把分词短语译成宾语的补语。整句可译成:我们发现这些民族的土地测量者利用几何知识重新确定消失了的土地标志和边界。

3. 本节 critical 是褒义的,意指"带批评观点的","不盲目相信的","审慎的"。

4. …training in the exact use of the English language and in the ability to analyze a new situation or problem into its basic parts…可译成:训练英语的准确使用及在分析新情况与新问题时直达基本要素的能力。

**课外作业**

1. 把下列各组的单词、词组与短语译成英语,并按它们的相关性联想记忆:

(1) 学会、建筑师、机械师、制图员、测量者、木匠。

(2) (i) 点、端点,线、直线、线段、曲线、折线、射线,平面、曲面;

(ii) 立体、柱体、立方体、球、棱锥、圆锥;

(iii) 圆、圆心、直径、半径、半圆、弦、弧、优弧、劣弧;

(iv) 角、边、三角形、直角三角形、斜边、直角边;

(v) 长度、宽度、厚度、位置;

(vi) 几何的、立体的、弯曲的、等距离的、无限的。

(3) 培养创造力、必需的毅力、提高鉴赏能力。

(4) 消失了的边界、有序性和优美感、几何图形大量存在、定理成立的先决条件。

2. 汉译英:

(1) 许多专家都认为数学是学习其他科学技术的必备基础和先决条件。

(2) 西方国家的专家认为几何起源于巴比伦和埃及人的土地测量技术,其实中国古代的数学家对几何做了许多出色的研究。

(3) 几何的学习使学生在思考问题时更周密和审慎,他们将不会盲目接受任何结论。

(4) 数学培养学生的分析问题的能力,使他们能应用毅力、创造性和逻辑推理来解决问题。

(5) 几何主要不是研究数,而是形,例如三角形,平行四边形和圆,虽然它也与数有关。

(6) 一个立体(图形)有长、宽和高;面(曲面或平面)有长和宽,但没有厚度;线(直线或曲线)有长度,但既没有宽度,也没有厚度;点只有位置,却没有大小。

(7) 射线从某个点出发无限延伸;两条从同一点出发的射线构成了角。这两条射线称为这个角的两边,当这两边位于同一直线上且方向相反时,所得的角是平角。

(8) 平面上的闭曲线当其中每一点到一个固定点的距离均相等时叫做圆。这个固定点称为圆心,经过圆心且其两个端点在圆周上的线段称为这个圆的直径,直径的一半叫做半径,这条曲线的长度叫做周长。

3. 英译汉:

(1) In geometry an angle is defined as the set of points determined by two rays $l_1$ and $l_2$ having the same endpoint $O$.

(2) In trigonometry we often interpret angles as rotations of rays. To obtain an angle we may start with a fixed ray $l_1$ having endpoint $O$, and rotate it about $O$, in a plane, to a position specified by ray $l_2$. We call $l_1$ the ***initial side***, $l_2$ the ***terminal side***, and $O$ the ***vertex*** of angle.

(3) A ***right angle*** is a 90° angle. An angle $\theta$ is acute if $0° < \theta < 90°$ or **obtuse** if $90° < \theta < 180°$. A ***straight angle*** is a 180° angle. Two acute angles are ***complementary*** if their sum is 90°. Two positive angles are ***supplementary*** if their sum is 180°.

4. 借助下面的生词与词组(二),自学课文 2-C,并将它译成汉语。

## 课文 2-C Trigonometric function and solution of right triangles[1]

The sides and angles of a triangle are mutually dependent. We know this from geometry. Trigonometry begins by showing the exact nature of this dependence between the sides and angles of triangles. For this purpose trigonometry employs the ratios of the sides. These ratios are called trigonometric functions. The six trigonometric functions of any acute angle in a right triangle, as $A$, are denoted as follows:

sin $A$, read "sine of $A$";

cos $A$, read "cosine of $A$";

tan $A$, read "tangent of $A$";

csc $A$, read "cosecant of $A$";

sec $A$, read "secant of $A$";

cot $A$, read "cotangent of $A$".

These trigonometric functions (ratios) are defined as follows (see Fig. 2-2-5):

(1) $\sin A=\frac{\text{opposite side}}{\text{hypotenuse}}\left(=\frac{a}{c}\right)$;

(2) $\cos A=\frac{\text{adjacent side}}{\text{hypotenuse}}\left(=\frac{b}{c}\right)$;

(3) $\tan A=\frac{\text{opposite side}}{\text{adjacent side}}\left(=\frac{a}{b}\right)$;

(4) $\csc A=\frac{\text{hypotenuse}}{\text{opposite side}}\left(=\frac{c}{a}\right)$;

(5) $\sec A=\frac{\text{hypotenuse}}{\text{adjacent side}}\left(=\frac{c}{b}\right)$;

(6) $\cot A=\frac{\text{adjacent side}}{\text{opposite side}}\left(=\frac{b}{a}\right)$.

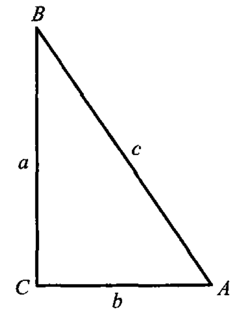


Fig. 2-2-5

These functions (ratios) are of fundamental importance in the study of trigonometry. They must be memorized.

One of the most important applications of trigonometry is the solution of triangles. Let us now take up the solution of right triangles. A triangle is composed of six parts, three sides and three angles. To solve a triangle is to find the parts not given. A triangle may be solved if three parts (at least one of these is a side) are given. A right triangle has one angle, the right angle, always given. Thus a right triangle can be solved when two sides, or one side and an acute angle, are given.

The general directions for solving right triangle are as follows.

(1) Draw a figure as accurately as possible representing the triangle in question.

(2) When one acute angle is known, subtract it from 90° to get the other acute angle.

(3) To find an unknown part, select from (1) to (6) trigonometric functions (ratios), a formula involving the unknown part and two know parts, and solved for the unknown part.

(4) Check the values found, when they satisfy relations different from those used in the last step, they are correct. A convenient numerical check is the relation

$$a^2 = c^2 - b^2 = (c+b)(c-b).$$

**生词与词组(二)**

acute angle 锐角
adjacent side 邻边
complementary[ ˌkɔmpli'mentəri] adj. [互]余的
determine[ di'təːmin ] v. 确定
hypotenuse[ hai'pɔtinjuːz ] n. 斜边
initial side of an angle 角的始边
mutually dependent 相互依赖的,互相关联的
obtuse angle 钝角
right angle 直角
right triangle 直角三角形
rotation[ rəu'teiʃn ] n. 旋转,转动
solidus[ 'sɔlidəs ] n. 斜线分隔符,即"/"
specify[ 'spesifai ] v. (具体)指定
straight angle 平角
supplementary [ ˌsʌpli'mentəri ] adj [互]补的
the terminal side 终边
trigonometric[ ˌtrigənə'metrik] adj. 三角[学]的
vertex[ 'vəːteks ] n. 顶点

# § 2.3 集合论的基本概念
# (Basic Concepts of the Theory of Sets)

## 课文 3–A Notations for denoting sets

The concept of a set has been utilized so extensively throughout modern mathematics that an understanding of it is necessary for all college students[1]. Sets are a means by which mathematicians talk of collections of things in an abstract way[2].

Sets usually are denoted by capital letters: $A, B, C, \cdots, X, Y, Z$; elements are designated by lower-case letters: $a, b, c, \cdots, x, y, z$. We use the special notation

$$x \in S$$

to mean that "$x$ is an element of $S$" or "$x$ belongs to $S$." If $x$ does not belong to $S$, we write $x \notin S$. When convenient, we shall designate sets by displaying the elements in braces; for example, the set of positive even integers less than 10 is denoted by the symbol $\{2,4,6,8\}$ whereas the set of all positive even integers is displayed as $\{2,4,6,\cdots\}$, the three dots taking the place of "and so on." The dots are used only when the meaning of "and so on" is clear. The method of listing the members of a set within braces is sometimes referred to as *the roster notation*.

The first basic concept that relates one set to another is *equality* of sets:

DEFINITION OF SET EQUALITY. *Two sets $A$ and $B$ are said to be equal (or identical) if they consist of exactly the same elements, in which case we write $A = B$. If one of the sets contains an element not in the other, we say the sets are unequal and we write $A \neq B$.*

EXAMPLE 1. According to this definition, the two sets $\{2,4,6,8\}$ and $\{2,8,6,4\}$ are equal since they both consist of the four integers 2, 4, 6, and 8. Thus, when we use the roster notation to describe a set, the order in which the elements appear is irrelevant.

EXAMPLE 2. The sets $\{2,4,6,8\}$ and $\{2,2,4,4,6,8\}$ are equal even though, in the second set, each of the elements 2 and 4 is listed twice. Both sets contain the four elements 2, 4, 6, 8 and no others; therefore, the definition requires that we call these sets equal[3]. This example shows that we do not insist that the objects listed in the roster notation be distinct. A similar example is the set of letters in the word *Mississippi*, which is equal to the set $\{M,i,s,p\}$, consisting of the four distinct letters $M$, $i$, $s$, and $p$.

## 课文 3-B Subsets

From a given set $S$ we may form new sets, called subsets of $S$. For example, the set consisting of those positive integers less than 10 which are divisible by 4 (the set $\{4,8\}$) is a subset of the set of all even integers less than 10. In general, we have the following definition.

DEFINITION OF A SUBSET. *A set $A$ is said to be a subset of a set $B$, and we write*

$$A \subseteq B,$$

*whenever every element of $A$ also belongs to $B$. We also say that $A$ is contained in $B$ or that $B$ contains $A$. The relation $\subseteq$ is referred to as set inclusion.*

The statement $A \subseteq B$ does not rule out the possibility that $B \subseteq A$. In fact, we may have both $A \subseteq B$ and $B \subseteq A$, but this happens only if $A$ and $B$ have the same elements. In other words,

$$A = B \text{ if and only if } A \subseteq B \text{ and } B \subseteq A.$$

This conclusion is an immediate consequence of the foregoing definitions of equality and inclusion. If $A \subseteq B$ but $A \neq B$, then we say that $A$ is a proper subset of $B$, we indicate this by writing $A \subset B$.

In all our applications of set theory, we have a fixed set $S$ given in advance, and we are concerned only with subsets of this given set. The underlying set $S$ may vary from one application to another; it will be referred to as the *universal set* of each particular discourse. The notation

$$\{x \mid x \in S \text{ and } x \text{ satisfies } P\}$$

will designate the set of all elements $x$ in $S$ which satisfy the property $P$. When the universal set to which we are referring is understood, we omit the reference to $S$ and write simply $\{x \mid x \text{ satisfies } P\}$. This is read "the set of all $x$ such that $x$ satisfies P." Sets designated in this way are said to be described by a defining property. For example, the set of all positive real numbers could be designated as $\{x \mid x>0\}$; the universal set $S$ in this case is understood to be the set of all real numbers. Similarly, the set of all even positive integers $\{2,4,6,\cdots\}$ can be designated as $\{x \mid x \text{ is a positive even integer}\}$. Of course, the letter $x$ is a dummy and may be replaced by any other convenient symbol. Thus, we may write

$$\{x \mid x>0\} = \{y \mid y>0\} = \{t \mid t>0\}$$

and so on.

It is possible for a set to contain no elements whatever. This set is called the empty set or the void set, and will be denoted by the symbol $\varnothing$. We will consider $\varnothing$ to be a subset of every set. Some people find it helpful to think of a set as analogous to a container (such as a bag or a box) containing certain objects, its elements[4]. The empty set is then analogous to an empty container.

To avoid logical difficulties, we must distinguish between the element $x$ and the set $\{x\}$ whose only element is $x$. (A box with a hat in it is conceptually distinct from the hat itself.) In particular, the empty set $\varnothing$ is not the same as the set $\{\varnothing\}$. In fact, the empty set $\varnothing$ contains no elements, whereas the set $\{\varnothing\}$ has one element, $\varnothing$. (A box which contains an empty box is not empty.) Sets consisting of exactly one element are sometimes called *one-element sets*.

Diagrams often help us visualize relations between sets. For example, we may think of a set $S$ as a region in the plane and each of its elements as a point. Subsets of $S$ may then be thought of as collections of points within $S$. For example, in

Figure 2-3-1 the shaded portion is a subset of *A* and also a subset of *B*. Visual aids of this type, called *Venn diagrams*, are useful for testing the validity of theorems in set theory or for suggesting methods to prove them. Of course, the proofs themselves must rely only on the definitions of the concepts and not on the diagrams.

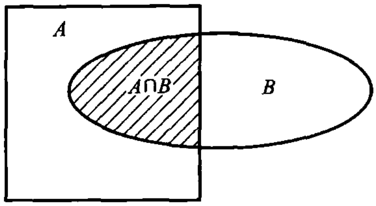

Fig. 2-3-1

**生词与词组(一)**

brace[breis]n. 大括号(常用复数 braces)
consequence['kɔnsikwəns]n. 结论,推论
denote[di'nəut]v. 表示,记
designate['dezigneit]v. 标记,指定,命名
diagram['daiəgræm]n. 图形,图解
  Venn diagram 文氏图
distinct[dis'tiŋkt]adj. 互不相同的
distinguish[dis'tiŋgwiʃ]v. 区别,辨别
divisible[di'vizəbl]adj. 可被除尽的
dummy['dʌmi]adj. 哑的;n. 哑变量
  dummy index 哑标,跑标
even integer 偶数(=even number)
if and only if 当且仅当
  only if 仅当
irrelevant[i'relivənt]adj. 没关系的,无关紧要的
Mississippi[ˌmisi'sipi]n. 密西西比河
positive number 正数
prove[pruːv]v. 证明,被证明是
roster['rəustə]n. 名册
  roster notation 枚举法
rule out 排除,否决
subset['sʌbset]n. 子集
the underlying set 基础集,底集
universal set 全集
validity[və'liditi]n. 有效性
vary['vɛri]v. 变化
visual['viʒuəl]adj. 可视的,看到的
visualize['viʒuəlaiz]v. 可视化,把……形象化
void set 空集(=empty set)

**预习要求**

1. 预习生词与词组(一),浏览课文 A、B 并在疑难处做上记号。
2. 复习或借助词典学习以下单词、词组与短语的含义及用法:

(1) analogous to sth./sb.
(2) be concerned with sth./sb.
(3) consist of sth./sb.
(4) be contained in sth.
(5) distinguish between sth./sb. and sth./sb.
(6) be divisible by sth.
(7) be referred to as sth./sb.
(8) rely on sth./sb.
(9) be said to be sth./sb.
(10) take the place of sth./sb.
(11) think of sth./sb. as sth./sb.
(12) vary from sth./sb. to sth./sb.

3. 复习定语从句的用法。

**注释与说明**

1. 本句中以 so…that…为连接词构成一个结果状语从句。这种从句通常译成:如此……以至于……。

2. Sets are a means by which mathematicians talk of collections of things in an abstract way. 这句中 by which 引起一个定语从句,其中 by which 和 in an abstract way 都是谓语 talk of 的行为方式状语。talk of 原意为谈论,这里可据上下文转译为"表述"。整句可译成:集合是数学家们用抽象的方式来表述一些事物的集体的工具。

3. therefore, the definition requires that we call these sets equal. 注意动词 call 的用法,它常带两个宾语或复合宾语。这句如果采用直译,则显得别扭。改用意译,可译成:因此,根据定义我们认为这两个集合相等。注意 call 和本节的一些动词如 denote, say, refer 等都是常用词,要记住它们的用法。

4. Some people find it helpful to think of a set as analogous to a container( such as a bag or a box) containing certain objects, its elements. 句中 helpful 是动词 find 的形式宾语 it 的补语,其后的动词不定式是真正的宾语。全句可译成:一些人认为这样比喻是有益的,即集合类似于容器(如背包和盒子)装有某些东西那样,包含它的元素。或译成:一些人认为这样比喻是有益的,即类似于容器(如背包和盒子)装有某些东西那样,集合含有它的元素。

**课外作业**

1. 把下列各组的单词、词组与短语译成英语,并按它们的相关性联想记忆:

(1) 集、子集、真子集、全集、空集、基地集。

(2) 正数、偶数,图形、文氏图,哑标,大括号。

(3) 可以被整除的、两两不同的、确定的、无关紧要的。

(4) 一样的结论,等同的效果、用大括号表示集、把这个图形记作 $A$、区别对象、证明定理、把结论可视化。

2. 汉译英:

(1) 由小于 10 且能被 3 整除的正整数组成的集是整数集的子集。

(2) 如果方便,我们通过在括号中列举元素的办法来表示集。

(3) 用符号 $\subseteq$ 表示集的包含关系,也就是说,式子 $A\subseteq B$ 表示 $A$ 包含于 $B$。

(4) 命题 $A\subseteq B$ 并不排除 $B\subseteq A$ 的可能性。

(5) 基础集可根据使用场合不同而改变。

(6) 为了避免逻辑上的困难,我们必须把元素 $x$ 与仅含有元素 $x$ 的集 $\{x\}$ 区

分开来。

(7) 图解法有助于将集合之间的关系形象化。

(8) 定理的证明仅仅依赖于概念和已知的结论,而不依赖于图形。

3. 英译汉:

(1) If $A$ is the set of all the letters of the alphabet, then listing each of elements would be tedious. So we write $A=\{a,b,c,\cdots,z\}$.

(2) In the set $A$, the last element is $z$. Many sets do not have last elements. Two important sets are **N**, the set of natural numbers, and $W$, the set of whole numbers. To list all the elements in these sets would be impossible because they go on forever. So we use three dots and write $N=\{1,2,3,\cdots\}$, $W=\{0,1,2,3,\cdots\}$.

(3) The whole numbers have many important subsets. A whole number is said to be ***even*** if it is divisible by 2; 2, 6, and 18 are examples of even numbers. A whole number is said to be ***odd*** if it is not divisible by 2; 1, 7, and 13 are examples of odd numbers. The natural numbers greater than 1 are called ***prime*** or ***composite***. A number is prime if it is divisible only by 1 and itself. A number is composite if it is divisible by a natural number other than 1 and itself.

4. 借助下面的生词与词组(二),将课文 3-C 译成汉语。

## 课文 3-C Introduction to set theory

In discussing any branch of mathematics, be it analysis, algebra, or geometry, it is helpful to use the notation and terminology of set theory. This subject, which was developed by Boole and Cantor in the latter part of the 19th century, has had a profound influence on the development of mathematics in the 20th century. It has unified many seemingly disconnected ideas and has helped to reduce many mathematical concepts to their logical foundations in an elegant and systematic way. A thorough treatment of the theory of sets would require a lengthy discussion which we regard as outside the scope of this book. Fortunately, the basic notions are few in number, and it is possible to develop a working knowledge of the methods and ideas of set theory through an informal discussion. Actually, we shall discuss not so much a new theory as an agreement about the precise terminology that we wish to apply to more or less familiar ideas.

In mathematics, the word "set" is used to represent a collection of objects viewed as a single entity. The collections called to mind by such nouns as "flock", "tribe", "crowd", "team", and "electorate" are all examples of sets. The individual objects in the collection are called elements or members of the set, and they are said to belong to or to be contained in the set. The set, in turn, is said to contain or be composed of its elements.

We shall be interested primarily in sets of mathematical objects: sets of numbers, sets of curves, sets of geometric figures, and so on. In many applications it is convenient to deal with sets in which nothing special is assumed about the nature of the individual objects in the collection. These are called abstract sets. Abstract set theory has been developed to deal with such collections of arbitrary objects, and from this generality the theory derives its power.

(George Boole (1815–1864) was an English mathematician and logician. His book, An Investigation of the Laws of Thought, published in 1854, marked the creation of the first workable system of symbolic logic.

Georg F. L. P. Cantor (1845–1918) and his school created the modern theory of sets during the period 1874–1895.)

**生词与词组(二)**

composite [ˈkɔmpəzit] n. 合数 a. 合成的
comprise [kəmˈpraiz] v. 组成
correspondence [ˌkɔrisˈpɔndəns] n. 对应
crowd [kraud] n. 人群
electorate [iˈlektərit] n. 选举团
flock [flɔk] n. (羊、鸟等的)群
individual [ˌindiˈvidjuəl] n. 单一的,个别的
investigation [inˌvestiˈgeiʃn] n. 研究,探讨,调查
noun [naun] n. 名词
odd number 奇数
primarily [ˈpraimərili] adv. 主要地
prime [praim] n. 素数
profound [prəˈfaund] adi. 深远的,深邃的
reduce to 化简为,归结为
symbolic logic 符号逻辑
tribe [traib] n. 部族,种族
whole number [非负]整数
workable [ˈwəːkəbl] adj. 可运转的

# § 2.4 整数、有理数与实数
# (Integers, Rational Numbers and Real Numbers)

## 课文 4–A Integers and rational numbers

There exist certain subsets of **R** which are distinguished because they have special properties not shared by all real numbers. In this section we shall discuss two such subsets, the *integers* and the *rational numbers.*

To introduce the positive integers we begin with the number 1, whose existence is guaranteed by Axiom 4. The number $1+1$ is denoted by 2, the number $2+1$ by 3, and so on. The numbers $1,2,3,\cdots$, obtained in this way by repeated addition of 1 are all positive, and they are called the *positive integers*[1]. Strictly speaking, this description of the positive integers is not entirely complete because we have not explained in detail what we mean by the expressions "and so on", or "repeated addition of 1". Although the intuitive meaning of expressions may seem clear, in a careful treatment of the real-number system it is necessary to give a more precise definition of the positive integers[2]. There are many ways to do this. One convenient method is to introduce first the notion of an *inductive set.*

DEFINITION OF AN INDUCTIVE SET. *A set of real numbers is called an inductive set if it has the following two properties:*

(a) *The number* 1 *is in the set.*

(b) *For every $x$ in the set, the number $x+1$ is also in the set.*

For example, **R** is an inductive set. So is the set $\mathbf{R}^+$. Now we shall define the positive integers to be those real numbers which belong to every inductive set.

DEFINITION OF POSITIVE INTEGERS. *A real number is called a positive integer if it belongs to every inductive set.*

Let **P** denote the set of all positive integers. Then **P** is itself an inductive set because (a) it contains 1, and (b) it contains $x+1$ whenever it contains $x$. Since the members of **P** belong to every inductive set, we refer to **P** as the *smallest* inductive set. This property of the set **P** forms the logical basis for a type of reasoning that mathematicians call *proof by induction*, a detailed discussion of which is given in Part 4 of this Introduction.

The negatives of the positive integers are called the negative integers. The positive integers, together with the negative integers and 0 (zero), form a set **Z** which we call simply the *set of integers.*

In a thorough treatment of the real-number system, it would be necessary at this stage to prove certain theorems about integers. For example, the sum, difference, or product of two integers is an integer, but the quotient of two integers need not be an integer. However, we shall not enter into the details of such proofs.

Quotients of integers $a/b$ (where $b\neq 0$) are called *rational numbers.* The set of rational numbers, denoted by **Q**, contains **Z** as a subset. The reader should realize that all the field axioms and the order axioms are satisfied by **Q**. For this reason, we say that the set of rational numbers is an *ordered field*. Real numbers that are not in **Q** are called irrational.

## 课文 4-B Geometric interpretation of real numbers as points on a line

The reader is undoubtedly familiar with the geometric representation of real numbers by means of points on a straight line. A point is selected to represent 0 and another, to the right of 0, to represent 1, as illustrated in Figure 2-4-1. This choice determines the scale. If one adopts an appropriate set of axioms for Euclidean geometry, then each real number corresponds to exactly one point on this line and, conversely, each point on the line corresponds to one and only one real number. For this reason the line is often called the *real line* or the *real axis*, and it is customary to use the words real number and point interchangeably. Thus we often speak of the point $x$ rather than the point corresponding to the real numbers.

The ordering relation among the real numbers has a simple geometric interpretation. If $x<y$, the point $x$ lies to the left of the point $y$ as shown in Figure 2-4-1. Positive numbers lie to the right of 0 and negative numbers to the left of 0. If $a<b$, a point $x$ satisfies the inequalities $a<x<b$ if and only if $x$ is between $a$ and $b$.

This device for representing real numbers geometrically is a very worthwhile aid that helps us to discover and understand better certain properties of real numbers. However, the reader should realize that all properties of real numbers that are to be accepted as theorems must be deducible from the axioms without any reference to geometry[3]. This does not mean that one should not make use of geometry in studying properties of real numbers. On the contrary, the geometry often suggests the method of proof of a particular theorem, and sometimes a geometric argument is more illuminating than a purely *analytic* proof (one depending entirely on the axioms for the real numbers). In this book, geometric arguments are used to a large extent to help motivate or clarify a particular discuss. Nevertheless, the proofs of all the important theorems are presented in analytic form.

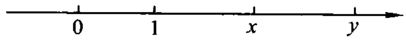


Fig. 2-4-1 Real numbers represented geometrically on a line.

### 生词与词组(一)

adopt[ə'dɔpt] v. 采用

appropriate[ə'prəupriit] adj. 适当的

conversely[ 'kɔnvəːsli ] adv. 反之
correspond[ ˌkɔris'pɔnd ] v. 对应
deducible [ di'djuːsəbl ] adj. 可推导[论]的
difference[ 'difərəns ] n. 差
distinguished[ dis'tiŋgwiʃt ] adj. 著名的，显著的
entirely complete 完整的
Euclid[ 'juːklid ] n. (人名)欧几里得
Euclidean[ juː'klidiən ] adj. 欧几里得的，欧氏的
field[ fiːld ] n. 域，场
  the field axiom 域公理
geometric interpretation 几何意义，几何解释
illuminating[ i'ljuːmiˌneitiŋ ] adj. 明朗的，清晰的
induction [ in'dʌkʃən ] n. (数学)归纳法
  proof by induction (用数学)归纳法证明
inductive set 归纳集
inequality[ ˌini'kwɔliti ] n. 不等式
integer[ 'intidʒə ] n. 整数
interchangeably adv. 可互相交换的
intuitive[ in'tjuːitiv ] adi. 直观的
irrational[ i'ræʃənəl ] adi. 无理的
  irrational number 无理数
negative[ 'negətiv ] adj. 负的，否定的，负面的
  the negative 否定，相反数
  negative number 负数
the order axiom 序公理
ordered[ 'ɔːdəd ] adj. 有序的
product[ 'prɔdəkt ] n. 积
quotient[ 'kwəuʃənt ] n. 商
rational[ 'ræʃənl ] adj. 有理的
  rational number 有理数
real line 实直线
real axis 实轴
reasoning[ riːzniŋ ] n. 推理
scale[ skeil ] n. 尺度，刻度
sum[ sʌm ] n. 和
to the left of 在……的左边
to the right of 在……的右边

**预习要求**

1. 预习生词与词组(一)，浏览课文 A、B 并在疑难处做上记号。
2. 复习或借助词典学习以下单词、词组与短语的含义及用法：

(1) be accepted as sth. /sb.
(2) clarify
(3) convenient method
(4) depend on sth. /sb.
(5) be familiar with sth. /sb.
(6) as illustrated in Fig. 1
(7) mean sth. /sb. by sth. /sb.
(8) by means of sth.
(9) more precise
(10) motivate
(11) be presented in sth.
(12) be shared by sth.
(13) a very worthwhile aid
(14) without any reference to geometry

3. 复习(1)形容词比较级与最高级的用法,(2)形式主语和形式宾语的用法。

**注释与说明**

1. The numbers 1,2,3,⋯, obtained in this way by repeated addition of 1 are all positive, and they are called the *positive integers*. 其中由过去分词 obtained 引起一个短语,说明 The numbers。整句可译成:通过反复加上 1 所得到的数 1,2,3,⋯,都是正的,它们被称为正整数。

2. Although the intuitive meaning of expressions may seem clear, in a careful treatment of the real-number system it is necessary to give a more precise definition of the positive integers. 这里 Although 引起一个让步状语从句,其主句中 it 是形式主语。这里 expressions 代表前面的"and so on"和"repeated addition of 1",可译成"表述"或"说法"。整句可译成:虽然这些说法的直观意思似乎是清楚的,但是在认真处理实数系统时必须给出一个更准确的关于正整数的定义。

3. However, the reader should realize that all properties of real numbers that are to be accepted as theorems must be deducible from the axioms without any reference to geometry. 这句有两处是 that 引起的从句。第一个 that 从句作 realize 的宾语;第二个 that 从句作 properties 的定语,to be accepted 是该从句的表语。整句可译成:然而,读者应该认识到,拟被采用作为定理的所有关于实数的性质都必须不借助于几何就能从公理推出。

**课外作业**

1. 把下列各组的单词、词组与短语译成英语,并按它们的相关性联想记忆:

(1) 整数、有理数、无理数、实数、负数、相反数;实直线、实轴;尺度、在……的左边/右边。

(2) 和、差、积、商、幂,不等式。

(3) 公理、序公理、域公理。

(4) 有序的、完整的、欧氏的、适当的、显著的、清晰的。

(5) 可推导的公式、可互相交换的运算、采用一组公理、对应于某个对象、(用数学)归纳法证明,对检验结论的有效性很有用,把这两个集加以区别。

2. 汉译英:

(1) 严格说,这样描述整数是不完整的,因为我们并没有说明"以此类推"或"反复加 1"的含义是什么。

(2) 两个整数的和、差或积是一个整数,但是两个整数的商未必是一个整数。

(3) 这种用几何来表示实数的办法对于帮助我们更好地发现与理解实数的性质是非常有价值的。

(4) 几何经常为一些特定的定理提供证明思路(建议),而且,有时几何的论证比纯分析的(完全依赖于实数公理的)证明更清晰。

(5) 一个由实数组成的集若满足如下条件则称为开区间(open interval)。

(6) 实数 $a$ 是 $-a$ 的相反数,它们的绝对值相等,且当 $a\neq 0$ 时,其符号不同。

(7) 每个实数刚好对应着实轴上的一点,反之,对实轴上的每一点,有且只有一个实数与之对应。

(8) 在几何上,实数之间的次序关系可以在数轴上清楚地表示出来。

3. 英译汉:

(1) A common mistake is to think that $-x$ is a negative number. But $-x$ can be positive, 0, or negative, depending on the value of $x$.

(2) Each property that we covered in the last section involves only one operation, such as $ab=ba$ and $0+a=a$. We now consider a property that links addition and multiplication. It is called the ***distributive property*** or ***multiplication distributive over addition*** and is illustrated with the following formulas:

$$a(b+c)=ab+ac;\ (b+c)a=ba+ca.$$

(3) Consider the decimal formed by writing the natural numbers in order:

$$.123456789101112131415\cdots$$

Since the natural numbers do not terminate or repeat, this is a nonterminating nonrepeating decimal. The decimal which cannot be converted to the ration of two integers is called irrational number. This set of numbers is denoted by the symbol $\boldsymbol{H}$, and

$$\boldsymbol{H}=\{x: x \text{ is a nonterminating nonrepeating decimal}\}.$$

4. 借助下面的生词与词组(二),将课文 4-C 译成汉语。

## 课文 4-C Fractions and decimal fractions[1]

Let us first turn our attention to fractions. You have surely met the expressions "half" and "quarter", they are used when the denominator of the fraction equals 2 or 4. $\frac{1}{3}$ is read "one third". Other fractions are read in the same way. Thus we read $\frac{1}{5},\frac{1}{6},\frac{1}{7},\frac{1}{10},\frac{1}{25},\frac{1}{100}$ as one fifth, one sixth, one seventh, one tenth, one twenty-fifth and one hundredth. These expressions are regarded as nouns and may therefore have a plural. Thus we read $\frac{2}{3}$ as two thirds; similarly $\frac{5}{6},\frac{9}{10},\frac{5}{100}$ are read as five sixths, nine tenths and five hundredths. However, if the last digit of the denominator is a 1 or a 2, then we do not read the fraction in the above-mentioned way. For example, we pronounce $\frac{5}{21}$ as "five over twenty-one". This method is also used in the other case. If the fraction is not a common one $\left(\text{e. g.}, \frac{1}{1089} \text{ or } \frac{501}{1205}\right)$, then we say "one over a thousand and eighty-nine" or "five hundred and one over twelve hundred and five".

Next, let us examine decimal fractions. They are very simple to pronounce. You just read the integral part of the number in the ordinary way, then say "point" (stands for "decimal point") and then read the decimal place one after the other. Thus 12.65 is read twelve-point-six-five; $\pi$ correct to 6 decimal place, equals three-point-one-four-one-five-nine-two, correct to five significant figures, equals three-point-one-four-one-six. When the decimal fraction is smaller than one, it is not usual in England to write, for instance, 0.56, but only .56. .56 is read "point-five-six", .0007 is read "point-nought-nought-nought-seven" or more usually "point-three 0's-seven".

Now for algebraical expressions, fractions are again read "over". $\frac{2a-1}{ax+b}$ is read $2a-1$ over $ax+b$ and brackets are indicated by the word "into", e. g., $(a+b)(a-b)$ is read "$a$ plus $b$ into $a$ minus $b$". Powers are indicated by indices or exponents. The index 2 is read "squared" and the indices 3 "cubed", or "to the third". Other indices are read "to the fourth, to the fifth, to the minus second, to the $n$th". The identity

$$a^3+b^3=(a+b)(a^2-ab+b^2)$$

reads "$a$ cubed plus $b$ cubed equals $a$ plus $b$ into $a$ squared minus $ab$ plus $b$ squared". Or the equation

$$x^{-\frac{2}{3}}+\sqrt[5]{a^2}=0$$

reads "$x$ to the minus two thirds plus the fifth root of $a$ squared equals zero".

**生词与词组(二)**

be converted to 被转换成

decimal['desiməl] adj. 十进制的，小数的；n. 小数

decimal fraction 十进制小数

nonterminating nonrepeating decimal 无限不循环小数

denominator[di'nɔmineitə] n. 分母

distributive property 分配性[律]

exponent[eks'pəunənt] n. 指数

the fifth root of $x$ squared $x$ 平方的五

次方根

fraction[ 'frækʃən ] n. 分数,分式

index[ 'indeks ] n. 指数,指标

minus[ 'mainəs ] prep. 减;adj. 负的;n. 减号,负号,亏损

$x$ to the minus two thirds $x$ 的负三分之二次幂

multiplication distributive over addition 乘法对加法的分配

numerator[ 'nju:məreitə ] n. 分子

plural[ 'pluərəl ] n. 复数(英语语法用语)

plus[ plʌs ] prep. 加;adj. 正的;n. 正号,加号,盈余

power[ 'pauə ] n. 幂,乘方,势

pronounce[ prə'nauns ] v. 发音,把……读作

## §2.5 笛卡儿几何学的基本概念
## (Basic Concepts of Cartesian Geometry)

### 课文 5-A The coordinate system of Cartesian geometry

As mentioned earlier, one of the applications of the integral is the calculation of area. Ordinarily we do not talk about area by itself, instead, we talk about the area of *something*. This means that we have certain objects (polygonal regions, circular regions, parabolic segments etc.) whose areas we wish to measure. If we hope to arrive at a treatment of area that will enable us to deal with many different kinds of objects, we must first find an effective way to describe these objects[1].

The most primitive way of doing this is by drawing figures, as was done by the ancient Greeks. A much better way was suggested by Rene Descartes (1596–1650), who introduced the subject of analytic geometry (also known as *Cartesian geometry*)[2]. Descartes' idea was to represent geometric points by numbers. The procedure for points in a plane is this:

Two perpendicular reference lines (called coordinate axes) are chosen, one horizontal (called the "$x$-axis"), the other vertical (the "$y$-axis"). Their point of intersection, denoted by $O$, is called the origin. On the $x$-axis a convenient point is chosen to the right of $O$ and its distance from $O$ is called the *unit distance*. Vertical distances along the $y$-axis are usually measured with the same unit distance, although sometimes it is convenient to use a different scale on the $y$-axis[3]. Now each point in the plane (sometimes called the $xy$-plane) is assigned a pair of numbers, called its *coordinates*. These numbers tell us how to locate the point.

Figure 2-5-1 illustrates some examples. The point with coordinates (3,2) lies three units to the right of the $y$-axis and two units above the $x$-axis. The number 3 is called the $x$-coordinate of the point, 2 its $y$-coordinate. Points to the left of the $y$-axis have a negative $x$-coordinate; those below the $x$-axis have a negative $y$-coordinate. The $x$-coordinate of a point is sometimes called its *abscissa* and the $y$-coordinate is called its *ordinate*.

When we write a pair of numbers such as $(a,b)$ to represent a point, we agree that the abscissa or $x$-coordinate, $a$, is written first. For this reason, the pair $(a,b)$ is often referred to as an *ordered pair*. It is clear that two ordered pairs $(a,b)$ and $(c,d)$ represent the same point if and only if we have $a=c$ and $b=d$. Points $(a,b)$ with both $a$ and $b$ positive are said to lie in the *first quadrant*, those with $a<0$ and $b>0$ are in the *second quadrant*; and those with $a<0$ and $b<0$ are in the *third quadrant*; and those with $a>0$ and $b<0$ are in the *fourth quadrant*. Figure 2-5-1 shows one point in each quadrant.

The procedure for points in space is similar. We take three mutually perpendicular lines in space intersecting at a point (the origin). These lines determine three mutually perpendicular planes, and each point in space can be completely described by specifying, with appropriate regard for signs, its distances from these planes. We shall discuss three-dimensional Cartesian geometry in more detail later on; for the present we confine our attention to plane analytic geometry.

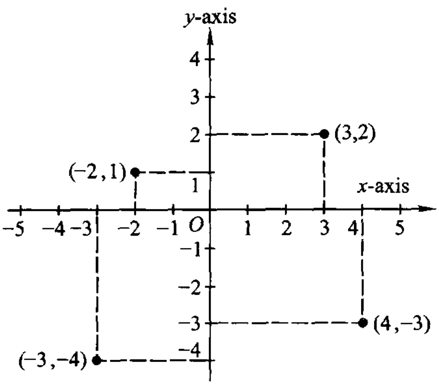


Fig. 2-5-1

## 课文 5-B　Geometric figures

A geometric figure, such as a curve in the plane, is a collection of points satisfying one or more special conditions. By translating these conditions into expressions, involving the coordinates $x$ and $y$, we obtain one or more equations which characterize the figure in question[4]. For example, consider a circle of radius $r$ with its center at the origin, as shown in Figure 2-5-2. Let $P$ be an arbitrary point on this circle, and suppose $P$ has coordinates $(x,y)$. Then the line segment $OP$ is the hypotenuse of a right triangle whose legs have lengths $|x|$ and $|y|$ and hence, by the theorem of Pythagoras,

$$x^2+y^2=r^2.$$

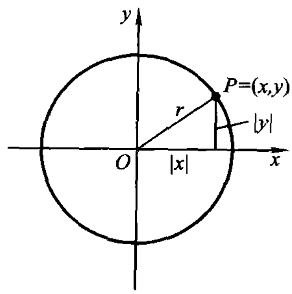

Fig. 2-5-2

This equation, called a *Cartesian equation* of the circle, is satisfied by all points $(x,y)$ on the circle and by no others, so the equation completely characterizes the circle. This example illustrates how analytic geometry is used to reduce geometrical statements about points to analytical statements about real numbers.

Throughout their historical development, calculus and analytic geometry have been intimately intertwined. New discoveries in one subject led to improvements in the other. The development of calculus and analytic geometry in this book is similar to the historical development, in that the two subjects are treated together. However, our primary purpose is to discuss calculus. Concepts from analytic geometry that are required for this purpose will be discussed as needed. Actually, only a few very elementary concepts of plane analytic geometry are required to understand the rudiments of calculus. A deeper study of analytic geometry is needed to extend the scope and applications of calculus, and this study will be carried out in later chapters using vector methods as well as the methods of calculus. Until then, all that is required from analytic geometry is a little familiarity with drawing graph of function.

**生词与词组(一)**

abscissa[æb'sisə]n. 横坐标
analytic geometry 解析几何
arbitrary['ɑːbitrəri]adj. 任意的
area['ɛəriə]n. 面积,区域
calculation[ˌkælkju'leiʃən]n. 计算
Cartesian[kɑː'tiːzjən]adj. 笛卡儿的,卡氏的
  Cartesian geometry 笛卡儿几何,卡氏儿几何
  Rene Descartes(人名)R. 笛卡儿
circular['səːkjulə]adj. 圆的,圆周的
  circular region 圆域
confine[kən'fain]v. 局限于
coordinate[kəu'ɔːdinit]坐标
  coordinate axis 坐标轴
  coordinate system 坐标系
distance['distəns]n. 距离
  distance from sth. 到……的距离

hence[hens]adv. 因此
horizontal[ˌhɔri'zɔntl]adj. 水平的
hypotenuse[hai'pɔtinjuːz]n. 斜边
illustrate['iləstreit]v. 说明
improvement[im'pruːvmənt]n. 改进，改善
integral['intigrəl]adj. 整数的，积分的；n. 积分
intersect[ˌintə'sekt]v. 相交
intertwine[ˌintə'twain]v. 融合，结合
intimately['intimitli]adv. 紧密地，亲密地
leg[leg]n. 侧边，直角边
ordered pair 有序对
ordinate['ɔːdinit]n. 纵坐标
the origin 坐标原点
parabolic[ˌpærə'bɔlik]adj. 抛物线的
  parabolic segment 抛物弓形
perpendicular[ˌpəːpən'dikjulə] adj. [互相]垂直的
polygonal['pɔligənl]adj. 多边形的
  polygonal region 多边形区域
the procedure for points 找点的程序
quadrant['kwɔdrənt]n. 象限
reduce[ri'djuːs]v. 归结[为]，化简
regard[ri'gɑːd]n. 评价，认定；v. 看待，把……视为
region['riːdʒən]n. 区域
rudiments['ruːdimənts]n.（复数）入门，基础
scale[skeil]n. 尺度，刻度
segment['segmənt]n. 线段，线节
specify['spesifai]v. 指定，明确地叙述
subject['sʌbdʒikt]n. 学科，主题
three-dimensional[θriː'dimenʃənəl] adj. 三维的
triangle['traiæŋgl]n. 三角形
  right triangle 直角三角形
the unit distance 单位长度
vector['vektə]n. 向量，矢量
vertical['vəːtikəl]adj. 竖直的

**预习要求**

1. 预习生词与词组(一)，浏览课文 A、B 并在疑难处做上记号。
2. 复习或借助词典学习以下单词、词组与短语的含义及用法：

(1) arrive at sth.
(2) as mentioned earlier
(3) by no other
(4) confine our attention to sth./sb.
(5) a geometric figure
(6) hypotenuse of a right triangle
(7) be intimately intertwined
(8) locate the point
(9) a little familiarity with sth./sb.
(10) a much better way
(11) mutually perpendicular planes
(12) reduce sth. to sth.
(13) the rudiments of calculus
(14) three units to the right of the $y$-axis
(15) two units above the $x$-axis
(16) with appropriate regard for signs

3. 复习状语从句的用法。

**注释与说明**

1. If we hope to arrive at a treatment of area that will enable us to deal with many different kinds of objects, we must first find an effective way to describe these objects. 这里 treatment 和 deal with 的字义都是处理，而实质的意思是计算。因此，翻译时可根据中文的表达习惯选词。全句可译成：如果我们希望获得面积的计算方法以便能用它来处理多种不同类型的图形，我们就必须首先找出描述这些图形的有效办法。

2. A much better way was suggested by Rene Descartes(1596-1650), who introduced the subject of analytic geometry(also known as *Cartesian geometry*). 这种被动句最好译成主动句：R. 笛卡儿(1596—1650)提出了一种好得多的办法，并建立了解析几何(也称为笛卡儿几何)这个学科。

3. Vertical distances along the $y$-axis are usually measured with the same unit distance, although sometimes it is convenient to use a different scale on the $y$-axis. 主句是被动式，但未必要译成主动句；从句中 it 是形式主语，不译出。although 引起的从句通常放在主句之前，全句译成"虽然……但是……"的形式，和中文习惯一样。读者务必注意的是：若 although 引起的从句在后，意思有所不同，这时 although 应译作"但是"、"不过"。全句可译成：沿着 $y$-轴的竖直距离通常用同样的单位长度来测量，不过有时采用不同的尺度(单位长度)较为方便。

4. By translating these conditions into expressions, involving the coordinates $x$ and $y$, we obtain one or more equations which characterize the figure in question. 介词 by 引起一个现在分词短语作方式状语，而 involving 引起的现在分词短语修饰 expressions。the figure in question 意为"在谈论中的图形"、"该图形"。全句可译成：通过把这些条件转化成含有坐标 $x$ 和 $y$ 的表达式，我们就得到了一个或多个能刻画该图形特征的方程。

**课外作业**

1. 把下列各组的单词、词组与短语译成英语，并按它们的相关性联想记忆：

(1) 解析几何、笛卡儿几何、三维的；坐标、坐标系、坐标原点、横坐标、纵坐标、坐标轴、象限；有序对、尺度、单位长度。

(2) 向量、线段，垂直的、水平的、竖直的，相交、交点。

(3) 三角形、直角三角形、斜边、直角边；区域、多边形的、多边形区域；抛物线的、抛物线弓形；圆的、圆域。

(4) 积分的计算、整数的性质、微积分的基本定理。

(5) 对符号做适当认定、把一个问题转化为另一问题、把条件翻译成表达式、紧密融合在一起、刻画了该曲线的特征。

2. 汉译英：

(1) 计算图形的面积是积分的一种重要应用。

(2) 在 $x$-轴上 $O$ 点右边选定一个适当的点，并把它到 $O$ 点的距离称为单位长度。

(3) 对 $xy$-平面上的每一个点都指定了一个数对，称为它的坐标。

(4) 选取两条互相垂直的直线，其中一条是水平的，另一条是竖立的，把它们的交点记作 $O$，称为原点。

(5) 当我们用一对数 $(a,b)$ 来表示平面的点时，商定要把横坐标写在第一个位置上。

(6) 微积分与解析几何在它们的发展史上已经互相融合在一起了。

(7) 如果想拓展微积分的范围与应用，需要进一步研究解析几何，而这种研究需用到向量的方法。

(8) 今后我们要对三维解析几何做详细研究，但目前只限于考虑平面解析几何。

3. 英译汉：

(1) Let $A(x_1,y_1)$ be a fixed point on a plane with slope $m$. Let $P(x,y)$ represent any point on the plane, where $x \neq x_1$. Then the slope determined by $P$ and $A$ is equal to $m$. That is

$$\frac{y-y_1}{x-x_1}=m, \text{ or } y-y_1=m(x-x_1)。$$

The last equation is called the ***point-slope form*** of the equation of a line.

(2) If a moving circle touches a circle $(x+2)^2+y^2=4$ externally and touches a straight line $x=2$, find the equation of the locus of the center of the moving circle.

(3) Given $C: x^2+y^2+Dx+Ey+F=0$. If $A(x_1,y_1)$ is a point outside $C$, then two tangents can be drawn; if $B(x_2,y_2)$ is a point on the circumference of the circle $C$, then the equation of tangent to $C$ at $A$ is

$$x_2x+y_2y+D\left(\frac{x+x_2}{2}\right)+E\left(\frac{y+y_2}{2}\right)+F=0。$$

4. 自学课文 5-C 并把它翻译成汉语。

## 课文 5-C Sets of points in the plane

We have already shown that there is a one-to-one correspondence between points in a plane and pairs of numbers $(x,y)$. Certain sets of points in the plane may be of special interest. For example, we may wish to examine the set of points comprising the circumference of a certain circle, or the set of points constituting the interior of a certain triangle. One may wonder if such sets of points may be succinctly described in a compact mathematical notation.

We may write

$$\{(x,y)\mid y=2x\} \tag{1}$$

to describe the set of ordered pairs $(x,y)$, or corresponding points, such that the ordinate is equal to twice the abscissas. In effect, then, the vertical line in (1) is read "such that". By "the graph of the set of ordered pairs" is meant the set of all points of the plane corresponding to the set of ordered pairs. The student will readily infer that the set of points constituting the graph lies on a straight line.

Consider the set

$$\{(x,y)\mid y=x^2\}.$$

Consistent with our previous interpretation, this symbol represents the set of ordered pairs $(x,y)$ such that the ordinate is equal to the square of the abscissa. Here, the total graph comprises a simple recognizable geometrical figure, a curve known as a parabola.

On the basis of these two examples, one may be tempted to believe that any arbitrarily drawn curve, which of course determines a set of points or ordered pairs, could be described succinctly by a simple equation. Unfortunately, this is not the case. For example, the broken line in figure 2-2-3 is one of such curves.

Consider now the set

$$\{(x,y)\mid y>2x\} \tag{2}$$

to describe the set of points $(x,y)$ whose ordinate is greater than twice its abscissa. In this case, our set of points constitutes not a curve, but a region of the coordinate plane.

### 生词与词组(二)

compact[kəmˈpækt]adj. 简洁的，紧的
comprise[kəmˈpraiz]v. 组成
consistent[kənˈsistənt]adj. 相容的，一致的
constitute[ˈkɔnstitjuːt]v. 组成
determine[diˈtəːmin]v. 确定
infer[inˈfəː]v. 推知，推断
locus[ˈləukəs]n. 轨迹
one-to-one 一对一[的]
parabola[pəˈræbələ]n. 抛物线
recognizable[ˈrekəgnaizəbl]adj. 可识别的
slope[sləup]n. 斜率
　point-slope form 点斜式
succinctly[səkˈsiŋktli]adv. 简明地
tangent[ˈtændʒənt]n. 切线，正切；a. 切线的
touch[tʌtʃ]v. 切，接触
　touch externally 外切

# § 2.6 函数的概念与函数思想
# (Function concept and function idea)

## 课文 6-A Informal description of functions

Various fields of human have to do with relationships that exist between one collection of objects and another[1]. Graphs, charts, curves, tables, formulas, and Gallup polls are familiar to everyone who reads the newspapers. These are merely devices for describing special relations in a quantitative fashion. Mathematicians refer to certain types of these relations as functions. In this section, we give an informal description of the function concept. A formal definition is given in Section 3.

EXAMPLE 1. The force $F$ necessary to stretch a steel spring a distance $x$ beyond its natural length is proportional to $x^2$. That is, $F=cx$, where $c$ is a number independent of $x$ called the spring constant. This formula, discovered by Robert Hooke in the mid-17th century, is called *Hooke's law*, and it is said to express the force as a function of the displacement.

EXAMPLE 2. The volume of a cube is a function of its edge-length. If the edges have length $x$, the volume $V$ is given by the formula $V=x^3$.

EXAMPLE 3. A *prime* is any integer $n>1$ that cannot be expressed in the form $n=ab$, where $a$ and $b$ are positive integers, both less than $n$. The first few primes are 2, 3, 5, 7, 11, 13, 17, 19. For a given real number $x>0$, it is possible to count the number of primes less than or equal to $x$. This number is said to be a function of $x$ even though no simple algebraic formula is known for computing it (without counting) when $x$ is known[3].

The word "function" was introduced into mathematics by Leibniz, who used the term primarily to refer to certain kinds of mathematical formulas. It was later realized that Leibniz's idea of function was much too limited in its scope, and the meaning of the word has since undergone many stages of generalization[4]. Today, the meaning of function is essentially this: Given two sets, say $X$ and $Y$, a *function* is a correspondence which associates with each element of $X$ one and only one element of $Y$. The set $X$ is called the *domain* of the function. Those elements of $Y$ associated with the elements in $X$ form a set called the *range* of the function. (This may be all of $Y$, but it need not be.)

Letters of the English and Greek alphabets are often used to denote functions.

The particular letters $f, g, h, F, G, H$, and $\varphi$ are frequently used for this purpose. If $f$ is a given function and if $x$ is an object of its domain, the notation $f(x)$ is used to designate that object in the range which is associated to $x$ by the function $f$; and it is called the value of $f$ at $x$ or the image of $x$ under $f$. The symbol $f(x)$ is read as "$f$ of $x$."

### 课文 6-B The function idea

The function idea may be illustrated schematically in many ways. For example, in Figure 2-6-1(a) the collections $X$ and $Y$ are thought of as sets of points and an arrow is used to suggest a "pairing" of a typical point $x$ in $X$ with the image point $f(x)$ in $Y$. Another scheme is shown in Figure 2-6-1(b). Here the function $f$ is imagined to be like a machine into which objects of the collection $X$ are fed and objects of $Y$ are produced. When an object $x$ is fed into the machine, the output is the object $f(x)$.

Although the function idea places no restriction on the nature of the objects in the domain $X$ and in the range $Y$, in elementary calculus we are primarily interested in functions whose domain and range are sets of real numbers. Such functions are called *real-valued functions of a real variable*, or, more briefly, *real functions*, and they may be illustrated geometrically by a graph in the $xy$-plane. We plot the domain $X$ on the $x$-axis, and above each point $x$ in $X$ we plot the point $(x, y)$, where $y = f(x)$. The totality of such points $(x, y)$ is called the *graph* of the function.

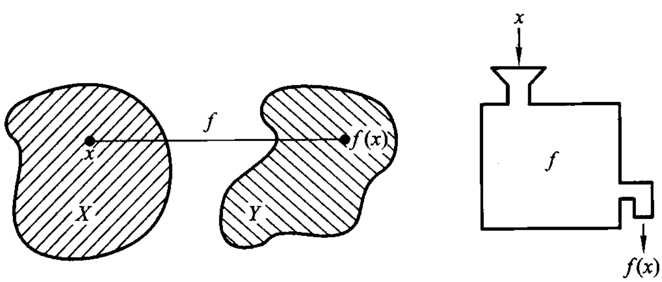


Fig. 2-6-1 Schematic representations of the function idea

Now we consider some more examples of real functions.

EXAMPLE 4. The *identity function*. Suppose that $f(x) = x$ for all real $x$. This function is often called the *identity function*. Its domain is the real line, that is, the set of all real numbers. Here $x = y$ for each point $(x, y)$ on the graph of $f$. The graph is a straight line making equal angles with the coordinates axes (see Figure 2-6-2).

The range of $f$ is the set of all real numbers.

EXAMPLE 5. The *absolute-value function*. Consider the function which assigns to each real number $x$ the nonnegative number $|x|$. A portion of its graph is shown in Figure 2-6-3. Denoting this function by $\varphi$, we have $\varphi(x) = |x|$ for all real $x$. For example, $\varphi(0)=0$, $\varphi(2)=2$, $\varphi(-3)=3$. We list here some properties of absolute values expressed in function notation.

(a) $\varphi(-x)=\varphi(x)$. (b) $\varphi(x^2)=x^2$.

(c) $\varphi(x+y)\leqslant\varphi(x)+\varphi(y)$ (the triangle inequality).

(d) $\varphi[\varphi(x)]=\varphi(x)$. (e) $\varphi(x)=\sqrt{x^2}$.

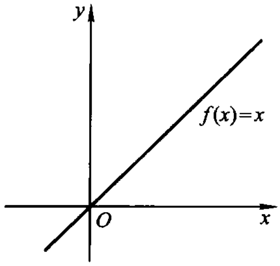


Fig. 2-6-2 Graph of the identity function $f(x)=x$

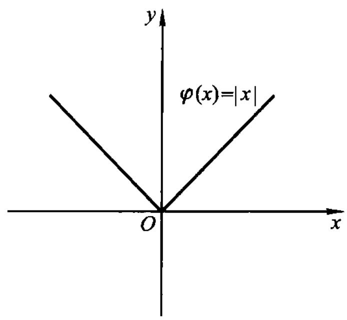


Fig. 2-6-3 Absolute-value function $\varphi(x)=|x|$

**生词与词组(一)**

the absolute - value function 绝对值函数

alphabet[ 'ælfəbit ] n. 字母表

correspondence[ ˌkɔris'pɔndəns ] n. 对应

cube[ kjuːb ] n. 立方体

displacement[ dis'pleismənt ] n. 位移

domain[ dəu'mein ] n. 区域,定义域

edge[ edʒ ] n. 棱,边

function idea 函数思想

Gallup poll[ 'gæləp pəul ] 盖洛普民意测验

generalization[ ˌdʒenərəlai'zeiʃən ] n. 推广,一般化

graph[ grɑːf ] n. 图,图形

Hooke's law 胡克定律

Robert Hooke(人名)R. 胡克

the identity function 恒等函数

image[ 'imidʒ ] v. 想象 n. (映射的)像,图像

Leibniz (人名)莱布尼茨

limit[ 'limit ] v. 限制;n. 极限

nonnegative[ nɔn'negətiv ] adj. 非负的

output[ 'autput ] n. ,v. 输出

plot[ plɔt ] v. 画(草图)

prime[ praim ] n. 素数,质数

proportional [prəˈpɔːʃənəl] adj. 成比例的
range [reindʒ] n. 值域,范围
the real-valued function 实值函数
real variable 实变量
restriction [risˈtrikʃən] n. 限制
scheme [skiːm] n. 方案,计划
schematic representation 图解表示
schematically [skiˈmætikəli] adv. 图解式地
spring constant 弹性系数
stretch [stretʃ] v. 拉伸
totality [təuˈtæliti] n. 全部,全体
the triangle inequality 三角不等式
volume [ˈvɔljuːm] n. 体积,容积,卷

**预习要求**

1. 预习生词与词组(一),浏览课文 A、B 并在疑难处做上记号。
2. 复习或借助词典学习以下单词、词组与短语的含义及用法:
(1) associate with sth./sb.
(2) certain kinds of sth./sb.
(3) do with sth.
(4) be expressed in the form
(5) be familiar to sb.
(6) be fed into the machine
(7) independent of sth./sb.
(8) be much too limited in sth.
(9) pair sth./sb. with sth./sb.
(10) place no restriction on sth.
(11) in a quantitative fashion
(12) refer to sth./sb.
(13) be referred to as sth./sb.
(14) without counting
3. 复习现在分词的用法。

**注释与说明**

1. Various fields of human have to do with relationships that exist between one collection of objects and another. 其中 have to 接动词不定式,意思为“必须”或“不得不”。do with 接名词,意思为“处理”。全句可译成:各行各业的人们都必须处理一类事物与另一类事物之间存在的各种关系。

2. The force $F$ necessary to stretch a steel spring a distance $x$ beyond its natural length is proportional to $x$. 形容词短语 necessary to…作定语修饰 force。全句可译成:把一条钢制的弹簧拉伸到超过其自然长度的距离为 $x$ 时所需要的力 $F$ 与 $x$ 成正比。

3. This number is said to be a function of $x$ even though no simple algebraic formula is known for computing it (without counting) when $x$ is known. 从这句可知 computing 和 counting 含义之差异。全句可译成:这个数称为 $x$ 的函数,尽管还没有一个简单代数式可以由已知的 $x$ 计算(不通过计数求)出它的值。

4. It was later realized that Leibniz's idea of function was much too limited in

its scope, and the meaning of the word has since undergone many stages of generalization. 这里 that 引起的名词从句为被动句，当主语。本句译成主动句较好，可译成：后来人们才认识到，莱布尼茨的函数思想适用的范围太过局限了，这个术语的含义从那时起已经过了多次推广。

**课外作业**

1. 把下列各组的单词、词组与短语译成英语，并按它们的相关性联想记忆：

(1) 函数、定义域、值域；恒等函数、绝对值函数、实值函数、实变量。

(2) 立方体、体积、棱长，素数、全部。

(3) 胡克定律、拉伸、位移、弹性系数、成比例的。

(4) 图解表示、画（草图）、图像、输出、输入。

(5) 不难想象、莱布尼茨的函数定义、过于局限的思想、$x$ 在 $f$ 下的像、在……上做出限制。

2. 汉译英：

(1) 常用英语字母和希腊字母来表示函数。

(2) 若 $f$ 是一个给定的函数，$x$ 是定义域里的一个元素，那么记号 $f(x)$ 用来表示由 $f$ 确定的对应于 $x$ 的值。

(3) 该射线将两个坐标轴的夹角分成两个相等的角。

(4) 可以用许多方式给出函数思想的图解说明。

(5) 容易证明，绝对值函数满足三角不等式。

(6) 对于实数 $x>0$，函数 $g(x)$ 表示不超过 $x$ 的素数的个数。

(7) 函数是一种对应，它未必可以表示成一个简单的代数公式。

(8) 在函数的定义中，关于定义域和值域中的对象，没对其性质做出任何限制。

3. 英译汉：

(1) We can use the concepts of ordered pairs to give a new definition of functions as follows. A function is a set of ordered pairs such that for each first coordinate only one second coordinate exists. The ***domain*** of a function is the set of all first coordinates. The ***range*** of a function is the set of all second coordinates.

(2) Let $X$ and $Y$ be sets and suppose $f: X \to Y$ is a function. If $g: Y \to X$ is another function and has the property that

$$y = f(x) \quad \text{if and only if} \quad x = g(y),$$

then we call $g$ the ***inverse function*** to $f$. Observe that $x = g(y)$ is what we obtained by solving the equation $y = f(x)$ for $x$ in term of $y$. However, in the general case, the equation $y = f(x)$ may have no solutions at all or else may have many solutions. Thus, for $f:X\to Y$ to admit an inverse function, it is necessary that, for each $y$ in the set $Y$, the equation $y=f(x)$ has a unique solution $x$ in the set $X$.

(3) A complex function $f$ on a measurable space $X$ whose range consists of only finitely many points will be called a ***simple function***. Among these are the nonnegative simple functions, whose range is a finite subset of $[0,\infty)$. Note that we explicitly exclude $\infty$ from the values of a simple function. If $\alpha_1,\alpha_2,\cdots,\alpha_n$ are the distinct values of a simple function $f$, and if we set $A_i=\{x: f(x)=\alpha_i\}$, then clearly

$$f=\sum_{i=1}^{n}\alpha_i\chi_{A_i},$$

where $\chi_{A_i}$ is the characteristic function of $A_i$.

4. 自学课文 6-C 并将它译成汉语。

## 课文 6-C The concept of function[1]

Seldom has a single concept played so important a role in mathematics as has the concept of function. It is desirable to know how the concept has developed.

This concept, like many others, originates in physics. The physical quantities were the forerunners of mathematical variables, and relation among them was called a function relation in the late 16th century.

For example, the formula $s=16t^2$ for the number of feet $s$ a body falls in any number of seconds $t$ is a function relation between $s$ and $t$, it describes the way $s$ varies with $t$. The study of such relations led people in the 18th century to think of a function relation as nothing but a formula.

Only after the rise of modern analysis in the early 19th century could the concept of function be extended. In the extended sense, a function may be defined as follows: If a variable $y$ depends on another variable $x$ in such a way that to each value of $x$ corresponds a definite value of $y$, then $y$ is a function of $x$. This definition serves many a practical purpose even today.

Not specified by this definition is the manner of setting up the correspondence. It may be done by a formula as the 18th century mathematics presumed, but it can equally well be done by a tabulation such as a statistical chart, or by some other form of description.

A typical example is the room temperature, which obviously is a function of time. But this function admits of no formula representation, although it can be recorded in a tabular form or traced out graphically by an automatic device.

The modern definition of a function $y$ of $x$ is simply a mapping from a space $X$ to another space $Y$. A mapping is defined when every point $x$ of $X$ has a definite image $y$, a point of $Y$. The mapping concept is close to intuition, and therefore desirable to serve as a basis of the function concept. Moreover, as the space concept is incorporated in this modern definition, its generality contributes much to the generality of the function concept.

**生词与词组(二)**

admit[əd'mit]v. 准许
  admit of 容许
  admit of no 不容许
characteristic function 特征函数
extend[iks'tend]v. 推广,延拓,扩展
  extended[iks'tendid]adj. 广义的
finitely many 有限多个
forerunner['fɔːˌrʌnə]n. 先行者
incorporate[in'kɔːpəreit]v. 并入,结合
in term of $y$ 用 $y$ 来表示
intuition['intju'iʃən]n. 直观
inverse function 反函数
mapping['mæpiŋ]n. 映射
measurable['meʒərəbl]adj. 可测的
obviously['ɔbviəsli]adv. 显然地
ordered pair 有序对
parabola[pə'ræbələ]n. 抛物线
presume[pri'zjuːm]v. 假定
set up 建立
simple function 简单函数
trace[treis]n. 迹,痕迹;v. 追踪

## §2.7 序列及其极限
## (Sequences and Their Limits)

### 课文 7-A The definition of sequences

In everyday usage of the English language, the words "sequence" and "series" are synonyms, and they are used to suggest a succession of things or events arranged in some order[1]. In mathematics these words have special technical meanings. The word "sequence" is employed as in the common use of the term to convey the idea of a set of things arranged in order, but the word "series" is used in a somewhat different sense[2]. The concept of a sequence will be discussed in this section, and series will be defined in Section 11.

If for every positive integer $n$ there is associated a real or complex number $a_n$, then the ordered set

$$a_1, a_2, a_3, \cdots, a_n, \cdots$$

is said to define an infinite sequence. The important thing here is that each member of the set has been labeled with an integer so that we may speak of the *first term* $a_1$, the *second term* $a_2$, and, in general, the $n$th *term* $a_n$. Each term $a_n$ has a successor $a_{n+1}$ and hence there is no "last" term.

The most common examples of sequences can be constructed if we give some rule or formula for describing the $n$th term. Thus, for example, the formula $a_n = 1/n$ defines a sequence whose first five terms are

$$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}.$$

Sometimes two or more formulas may be employed as, for example,

$$a_{2n-1} = 1. \quad a_{2n} = 2n^2,$$

the first few terms in this case being

$$1, 2, 1, 8, 1, 18, 1, 32, 1.$$

Another common way to define a sequence is by a set of instructions which explains how to carry on after a given start[3]. Thus we may have

$$a_1 = a_2 = 1, \quad a_{n+1} = a_n + a_{n-1} \qquad \text{for } n \geqslant 2.$$

This particular rule is known as a recursion formula and it defines a famous sequence whose terms are called the *Fibonacci numbers*. The first few terms are

$$1, 1, 2, 3, 5, 8, 13, 21, 34.$$

In any sequence the essential thing is that there be some function $f$ defined on the positive integers such that $f(n)$ is the $n$th term of the sequence for each $n = 1, 2, 3, \cdots$. In fact, this is probably the most convenient way to state a technical definition of sequence.

DEFINITION. A function $f$ whose domain is the set of all positive integers $1, 2, 3, \cdots$ is called an infinite sequence. The function value $f(n)$ is called the $n$th term of the sequence.

The *range* of the function (that is, the set of function values) is usually displayed by writing the terms in order, thus:

$$f(1), f(2), f(3), \cdots, f(n), \cdots.$$

For brevity, the notation $\{f(n)\}$ is used to denote the sequence whose $n$th term is $f(n)$. Very often the dependence on $n$ is denoted by using subscripts, and we write $a_n, s_n, x_n, u_n$, or something similar instead of $f(n)$. Unless otherwise specified, all sequences in this chapter are assumed to have real or complex terms.

## 课文 7-B The limit of a sequence

The main question we are concerned with here is to decide whether or not the terms $f(n)$ tend to a finite limit as $n$ increases infinitely. To treat this problem, we must extend the limit concept to sequences. This is done as follows.

DEFINITION. *A sequence $\{f(n)\}$ is said to have a limit L if, for every positive number $\varepsilon$, there is another positive number N (which may depend on $\varepsilon$) such that*

$$|f(n)-L|<\varepsilon \quad \text{for all } n \geqslant N.$$

*In this case, we say the sequence $\{f(n)\}$ converges to L and we write*

$$\lim_{n\to\infty} f(n)=L, \quad \text{or} \quad f(n)\to L \quad \text{as} \quad n\to\infty.$$

*A sequence which does not converge is called divergent.*

In this definition the function values $f(n)$ and the limit $L$ may be real or complex numbers. If $f$ and $L$ are complex, we may decompose them into their real and imaginary parts, say $f=u+iv$ and $L=a+ib$. Then we have $f(n)-L=u(n)-a+i[v(n)-b]$. The inequalities

$$|u(n)-a|\leqslant|f(n)-L| \quad \text{and} \quad |v(n)-b|\leqslant|f(n)-L|$$

show that the relation $f(n)\to L$ implies $u(n)\to a$ and $v(n)\to b$ as $n\to\infty$. Conversely, the inequality

$$|f(n)-L|\leqslant|u(n)-a|+|v(n)-b|$$

shows that the two relations $u(n)\to a$ and $v(n)\to b$ imply $f(n)\to L$ as $n\to\infty$. In other words, a complex-valued sequence $f$ converges if and only if both the real part $u$ and the imaginary part $v$ converge separately, in which case we have

$$\lim_{n\to\infty} f(n)=\lim_{n\to\infty} u(n)+i\lim_{n\to\infty} v(n).$$

It is clear that any function defined for all positive real $x$ may be used to construct a sequence by restricting $x$ to take only *integer* values[4]. This explains the strong analogy between the definition just given and the one in Section 6.4 for more general functions. The analogy carries over to *infinite limits* as well, and we leave it for the reader to define the symbols

$$\lim_{n\to\infty} f(n)=+\infty \quad \text{and} \quad \lim_{n\to\infty} f(n)=-\infty$$

as was done in Section 6.5 when $f$ is real-valued. If $f$ is complex, we write $f(n)\to\infty$ as $n\to\infty$ if $|f(n)|\to+\infty$.

The phrase "convergent sequence" is used only for a sequence whose limit is finite. A sequence with an infinite limit is said to diverge. There are, of course, divergent sequences that do not have infinite limits. Examples are defined by the following formulas:

$$f(n)=(-1)^n,\quad f(n)=\sin\frac{n\pi}{2},\quad f(n)=(-1)^n\left(1+\frac{1}{n}\right),\quad f(n)=e^{\frac{n\pi}{2}i}.$$

The basic rules for dealing with limits of sums, products, etc., also hold for limits of convergent sequences. The reader should have no difficulty in formulating these theorems for himself. Their proofs are somewhat similar to those given in Section 3.5.

### 生词与词组(一)

assume[ə'sju:m] v. 假定,取(值)
carry over to 继续做下去
complex-valued sequence 复值序列
converge[kən'və:dʒ] v. 收敛
convergence[kən'və:dʒəns] n. 收敛
convergent[kən'və:dʒənt] adj. 收敛的
conversely ['kɔnvə:sli] adv. 反之,反过来
convey[kən'vei] v. 表达,传递
diverge[dai'və:dʒ] v. 发散
divergence[dai'və:dʒəns] n. 发散
divergent[dai'və:dʒənt] adj. 发散的
event[i'vent] n. 事件
Fibonacci number 菲波那契数
imaginary part 虚部
imply [im'plai] v. 蕴涵,推出
infinite sequence 无穷序列
instruction[in'strʌkʃən] n. 指令
phrase[freiz] n. 短语,惯用语,说法; v. 用语言表示
real part 实部
real valued sequence 实值序列
recursion formula 递推公式
sequence['si:kwəns] n. 序列,数列
series['siəri:z] n. 级数,序列
subscript['sʌbskript] n. 下标
succession[sək'seʃən] n. 连贯性
  a succession of 一连串的
successor[sək'sesə] n. 后继
suggest[sə'dʒest] v. 建议,暗示

### 预习要求

1. 预习生词与词组(一),浏览课文 A、B 并在疑难处做上记号。
2. 复习或借助词典学习以下单词、词组与短语的含义及用法:

(1) carry on
(2) carries over to sth.
(3) converge separately
(4) decompose sth. into sth.
(5) extend sth. to sth.
(6) formulate the theorems
(7) be known as sth./sb.
(8) be labeled with sth.
(9) leave sth. for sb. to do
(10) make the convention
(11) restrict $x$ to take only integer values
(12) a set of instructions
(13) similar to sth./sb.
(14) somewhat
(15) analogy between sth./sb. and sth./sb.
(16) unless otherwise specified

3. 复习介词和数词的用法。

**注释与说明**

1. In everyday usage of the English language, the words "sequence" and "series" are synonyms, and they are used to suggest a succession of things or events arranged in some order. 这句"sequence"与"series"可照抄不译,arranged in some order 是过去分词短语修饰 things or events。采用直译较不好表达,可按原意译成:在日常英语中,单词"sequence"与"series"是同义词,用以表示按某种次序排列的一串东西或事件。

2. The word "sequence" is employed as in the common use of the term to convey the idea of a set of things arranged in order, but the word "series" is used in a somewhat different sense. 其中 employ 和动词 use 是同义词,但本句出现的 use 是名词,表示"用法"。全句可译成:像通常用法一样,术语"sequence"用以表达按次序排列的一串东西的意思,但是,"series"一词则用于某种别的意思。(注:series 表示的数学术语是"级数"。)

3. Another common way to define a sequence is by a set of instructions which explains how to carry on after a given start. 这句用 by 引起的短语作表语。全句可译成:另一种常用的定义序列的方法是,通过一串指令说明在给定首项后如何给出后面的各项。

4. It is clear that any function defined for all positive real $x$ may be used to construct a sequence by restricting $x$ to take only *integer* values. 这句可译成一个长句,也可以译成两个短句,例如可译成:显然,每一个对所有正实数 $x$ 有定义的函数都可以用来构造一个序列,其办法是限制 $x$ 只取整数值。

**课外作业**

1. 把下列各组的单词、词组与短语译成英语,并按它们的相关性联想记忆:

(1) 序列、实值序列、复值序列、无穷序列、级数、菲波那契数;后继、连贯性、下标、递推公式。

(2) 实部、虚部、分解。

(3) 收敛、收敛的、发散、发散的。

(4) 假定结论成立,$x$ 取值为 3、表达思想、蕴涵着另一等式、提示证明方法、反之亦然、除非另加说明。

2. 汉译英:

(1) 序列各项对 $n$ 的相关性常利用下标来表示,写成如下形式:$a_n$,$x_n$等。

(2) 以正整数集为定义域的函数称为序列。

(3) 一个复值序列收敛当且仅当它的实部和虚部分别收敛。

(4) 一个序列$\{a_n\}$若满足:对任意正数$\varepsilon$,存在另一个正数$N$($N$可能与$\varepsilon$有关)使得$|a_n-L|<\varepsilon$对所有$n\geqslant N$成立,就称$\{a_n\}$收敛于$L$。

(5) 重要的是,该集的每一个成员都用一个正整数标上记号。这样一来,就可以谈论第一项、第二项和一般项,即第$n$项。

(6) 若无另加申明,本章研究的序列都假定具有实的项或复的项。

(7) 作为日常用语,sequence 和 series 是同义词;但作为数学术语,它们表示不同的概念。

(8) 术语"收敛序列"指的是具有有限极限的序列,因此,极限为无限的序列不是收敛的,而是发散的。

3. 英译汉:

(1) A quantity that can take on successively different numerical values is called a variable. It should be remembered that, although it is common for the successive values of the variable to be related to each other in accordance with some law, these values need not have any definite relations one to another. As stated before, a variable is commonly represented by a letter such as $x, y$, or $z$.

(2) If a variable $V$ takes on successively a series of values which approach closer and closer, to a fixed number $L$ in such a manner that the absolute value of $V-L$ becomes and remains less than any finite number however small, then $V$ is said to approach the limit $L$.

This may be written limit of $V=L$. The symbol→gives us the equivalent notation $V\to L$, which is read $V$ approaches $L$ as a limit.

(3) This is the most important function in mathematics. It is defined, for every complex number $z$, by the formula

$$\exp(z)=\sum_{n=0}^{\infty}\frac{z^n}{n!}. \qquad (*)$$

The series $(*)$ converges absolutely for every $z$ and converges uniformly on every bounded subset of the complex plane. Thus exp $(z)$ is a ***continuous function***. By the absolute convergence of $(*)$ one can prove the important addition formula

$$\exp(a)+\exp(b)=\exp(a+b)$$

is valid for all complex number $a$ and $b$.

**注** 本题(1)、(2)中的 successively 都可译成"接连不断地"。作为数学术语的连续函数不译成"successive function",而应如(3)那样,译成"continuous function"。

4. 自学课文 7-C 并将它译成汉语。

## 课文 7-C Monotonic sequences of real numbers

A sequence $\{f(n)\}$ is said to be increasing if

$$f(n) \leqslant f(n+1) \qquad \text{for all } n \geqslant 1.$$

We indicate this briefly by writing $f(n)\nearrow$. If, on the other hand, we have

$$f(n) \geqslant f(n+1) \qquad \text{for all } n \geqslant 1,$$

we call the sequence decreasing and write $f(n)\searrow$. A sequence is called monotonic if it is increasing or if it is decreasing.

Monotonic sequences are pleasant to work with because their convergence or divergence is particularly easy to determine. In fact, we have the following simple criterion.

THEOREM 7.1. *A monotonic sequence converges if and only if it is bounded.*

*Note*: A sequence $\{f(n)\}$ is called *bounded* if there exists a positive number $M$ such that

$|f(n)| \leqslant M$ for all $n$. A sequence that is not bounded is called *unbounded.*

*Proof.* It is clear that an unbounded sequence cannot converge. Therefore, all we need to prove is that a bounded monotonic sequence must converge.

Assume $f(n)\nearrow$ and let $L$ denote the least upper bound of the set of function values. (Since the sequence is bounded, it has a least upper bound by Axiom 10 of the real-number system.) Then $f(n) \leqslant L$ for all $n$, and we shall prove that the sequence converges to $L$.

Choose any positive number $\varepsilon$. Since $L-\varepsilon$ cannot be an upper bound for all numbers $f(n)$, we must have $L-\varepsilon<f(N)$ for some $N$. (This $N$ may depend on $\varepsilon$.) If $n \geqslant N$, we have $f(N) \leqslant f(n)$ since $f(n)\nearrow$. Hence, we have $L-\varepsilon<f(n) \leqslant L$ for all $n \geqslant N$, as illustrated in Figure 2-7-1. From these inequalities we find that

$$0 \leqslant L-f(n)<\varepsilon \qquad \text{for all } n \geqslant N$$

and this means that the sequence converges to $L$, as asserted.

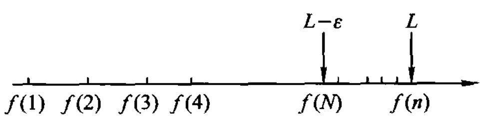


Fig. 2-7-1 A bounded increasing sequence converges to its least upper bound.

If $f(n)\searrow$, the proof is similar, the limit in this case being the greatest lower bound of the set of function values.

**生词与词组(二)**

absolute convergence 绝对收敛性
bound[baund]n. 界,限
lower bound 下界
upper bound 上界
bounded['baundid]adj. 有界的
complex plane 复平面
continuous function 连续函数
converge absolutely 绝对收敛
converge uniformly 一致收敛
criterion[krai'tiəriən]n. 判别法,准则
decreasing[di'kriːsiŋ]adj. 递减的
definite['definit]adj. 定的,确定的
however small 无论多么小
in accordance with… 根据……
increasing[in'kriːsiŋ]adj. 递增的
monotonic[mɔnəu'tɔnik]adj. 单调的
remain[ri'mein]v. 仍然,剩下,尚待
series['siəriːz]n. 级数
successive[sək'sesiv]adj. 逐次的,相继的
unbounded['ʌn'baundid]adj. 无界的

# §2.8 函数的导数和它的几何意义 (The Derivative of a Function and Its Geometric interpretation)

## 课文 8-A The derivative of a function

The example described in the foregoing section points the way to the introduction of the concept of derivative. We begin with a function $f$ defined at least on some open interval $(a,b)$ on the $x$-axis. Then we choose a fixed point $x$ in this interval and introduce the difference quotient

$$\frac{f(x+h)-f(x)}{h}, \tag{8.1}$$

where the number $h$, which may be positive or negative (but not zero), is such that $x+h$ also lie in $(a,b)$. The numerator of this quotient measures the change in the function when $x$ changes from $x$ to $x+h$. The quotient itself is referred to as the *average rate of the change* of $f$ in the interval joining $x$ to $x+h$.

Now we let $h$ approach zero and see what happens to this quotient. If the quotient approaches some definite value as a limit (which implies that the limit is the same whether $h$ approaches zero through positive values or through negative values), then this limit is called the *derivative* of $f$ at $x$ and is denoted by the symbol $f'(x)$ (read as "$f$ prime of $x$")[1]. Thus, the formal definition of $f'(x)$ may be stated as follows:

DEFINITION OF DERIVATIVE. *The derivative $f'(x)$ is defined by the equation*

(8.2) $$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h},$$

*provided the limit exists. The number $f'(x)$ is also called the rate of change of $f$ at $x$.*

By comparing (8.2) with (7.3) in the foregoing section, we see that the concept of instantaneous velocity is merely an example of the concept of derivative. The velocity $v(t)$ is equal to the derivative $f'(t)$, where $f$ is the function which measures position. This is often described by saying that velocity is the rate of change of position with respect to time. In the example worked out in Section 7.2, the position function $f$ is described by the equation

$$f(t)=144t-16t^2,$$

and its derivative $f'$ is a new function (velocity) given by

(8.3) $$f'(t)=144-32t.$$

In general, the limit process which produces $f'(x)$ from $f(x)$ gives us a way of obtaining a new function $f'$ from a given function $f$.[2] The process is called *differentiation*, and $f'$ is called the first derivative of $f$. If $f'$, in turn, is defined on an open interval, we can try to compute its first derivative, denoted by $f''$ and called the second derivative of $f$. Similarly, the $n$th derivative of $f$, denoted by $f^{(n)}$, is defined to be the first derivative of $f^{(n-1)}$. We make the convention that $f^{(0)}=f$, that is, the zeroth derivative is the function itself.

For rectilinear motion, the first derivative of velocity (second derivative of position) is called *acceleration*. For example, to compute the acceleration in the example of Section 7.2, we can use Equation (7.2) to form the difference quotient

$$\frac{v(t+h)-v(t)}{h}=\frac{[144-32(t+h)]-[144-32t]}{h}=\frac{-32h}{h}=-32.$$

Since this quotient has the constant value $-32$ for each $h\neq 0$, its limit as $h\to 0$ is also $-32$. Thus, the acceleration in this problem is constant and equal to $-32$. This result tells us that the velocity is decreasing at the rate of 32 feet per second every second. In 9 seconds the total decrease in velocity is $9\cdot 32=288$ feet per second. This agrees with the fact that during the 9 seconds of motion the velocity changes from $v(0)=144$ to $v(9)=-144$.

## 课文 8-B Geometric interpretation of the derivative as a slope

The procedure used to define the derivative has a geometric interpretation which leads in a natural way to the idea of a tangent line to a curve. A portion of the graph of a function $f$ is shown in Figure 2-8-1. Two of its points $P$ and $Q$ are shown with respective coordinates $(x, f(x))$ and $(x+h, f(x+h))$. Consider the right triangle with hypotenuse $PQ$; its altitude, $f(x+h)-f(x)$, represents the difference of the ordinates of the two points $Q$ and $P$. Therefore, the difference quotient

$$\frac{f(x+h)-f(x)}{h} \tag{8.4}$$

represents the trigonometric tangent of the angle $\alpha$ that $PQ$ makes with the horizontal. The real number $\tan\alpha$ is called the *slope* of the line through $P$ and $Q$ and it provides a way of measuring the "steepness" of this line. For example, if $f$ is a linear function, say $f(x)=mx+b$, the difference quotient (8.4) has the value $m$, so $m$ is the slope of the line.

Some examples of lines of various slopes are shown in Figure 2-8-2. For a horizontal line, $\alpha=0$ and the slope, $\tan\alpha$, is also 0. If $\alpha$ lies between 0 and $\pi/2$, the line is rising as we move from left to right and the slope is positive. If $\alpha$ lies between $\pi/2$ and $\pi$, the line is falling as we move from left to right and the slope is negative. A line for which $\alpha=\pi/4$ has slope 1. As $\alpha$ increases from 0 to $\pi/2$, $\tan\alpha$ increases without bound, and the corresponding lines of slope $\tan\alpha$ approach a vertical position. Since $\tan\pi/2$ is not defined, we say that *vertical lines have no slope*.

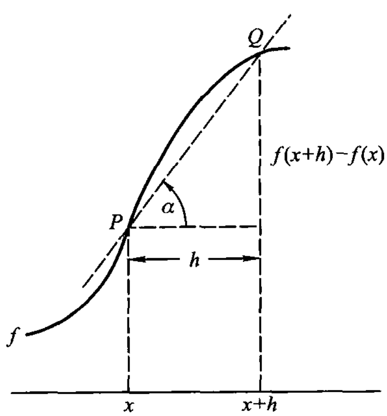


Fig. 2-8-1 Geometric interpretation of the difference quotient as the tangent of an angle

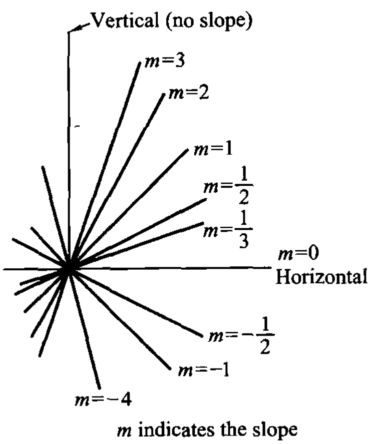


Fig. 2-8-2 Lines of various slopes

Suppose now that $f$ has a derivative at $x$. This means that the difference quotient approaches a certain limit $f'(x)$ as $h$ approaches 0. When this is interpreted geometrically it tells us that, as $h$ gets nearer to 0, the point $P$ remains fixed, $Q$ moves along the curve toward $P$, and the line through $PQ$ changes its direction in such a way that its slope approaches the number $f'(x)$ as a limit[3]. For this reason it seems natural to define the *slope of the curve* at $P$ to be the number $f'(x)$. The line through $P$ having this slope is called the *tangent line* at $P$.

**生词与词组(一)**

acceleration[ækˌseləˈreiʃən] n. 加速度
altitude[ˈæltitjuːd] n. 高度
approach[əˈprəutʃ] v. 趋于,逼近
  approach zero 趋于0
average[ˈævəridʒ] n. 平均
  average rate 平均变化率
  average value 平均值
bound[baund] n. 界,限
  without bound 无界,无限
constant[ˈkɔnstənt] n. 常数
derivative[diˈrivətiv] n. 导数
  the first derivative 一阶导数
  the second derivative 二阶导数
difference equation 差商
differentiation[ˌdifəˌrenʃiˈeiʃən] n. 微分法
fraction[ˈfrækʃən] n. 分数,分式
interval[ˈintəvəl] n. 区间,线节
  open interval 开区间
linear function 线性函数
numerator[ˈnjuːməreitə] n. 分子
position function 位置函数
rectilinear motion 直线运动
respective coordinates 各自的坐标
steepness[ˈstiːpnis] n. 陡峭
slope[sləup] n. 坡度,斜率
tangent[ˈtændʒənt] n. 正切,切线
velocity[viˈlɔsiti] n. 速度
  instantaneous velocity 瞬时速度

**预习要求**

1. 预习生词与词组(一),浏览课文 A、B 并在疑难处做上记号。
2. 复习或借助词典学习以下单词、词组与短语的含义及用法:
(1) approach sth. through sth.
(2) begin with sth.
(3) compare sth./sb. with sth./sb.
(4) happen to sth./sb.
(5) be interpreted geometrically
(6) joint sth. to sth.
(7) make the convention
(8) move along sth. toward sth.
(9) the total decrease in velocity
(10) with respect to sth.
3. 复习动名词、同位语与插入语的用法。

**注释与说明**

1. If the quotient approaches some definite value as a limit (which implies that the limit is the same whether $h$ approaches zero through positive values or through negative values), then this limit is called the derivative of $f$ at $x$ and is denoted by the symbol $f'(x)$ (read as "$f$ prime of $x$"). 此句宜用意译，译成：如果差商以某个确定的值为极限（这蕴涵着不论 $h$ 取正的值趋于 0 还是取负的值趋于 0，其极限一样），那么这个极限称为 $f$ 在 $x$ 的导数，记作 $f'(x)$（读成"$f$ 一撇 $x$"）。

2. In general, the limit process which produces $f'(x)$ from $f(x)$ gives us a way of obtaining a new function $f'$ from a given function $f$. 这句 a way 后的定语，可按原来次序翻译，译成长句。本句可译成：一般地，由 $f(x)$ 产生 $f'(x)$ 的极限过程向我们提供了一种方法，从一个给定的函数 $f$ 得到一个新的函数 $f'$。

3. When this is interpreted geometrically it tells us that, as $h$ gets nearer to 0, the point $P$ remains fixed, $Q$ moves along the curve toward $P$, and the line through $PQ$ changes its direction in such a way that its slope approaches the number $f'(x)$ as a limit. 此句若用意译则较为简练且符合中文的表达方式，可译成：其几何意义为，当 $h$ 趋于 0 时，点 $P$ 保持不动，而点 $Q$ 沿曲线趋近 $P$；同时，经过 $PQ$ 的直线不断地改变方向，结果其斜率趋于数值 $f'(x)$，并以它为极限。

**课外作业**

1. 把下列各组的单词、词组与短语译成英语，并按它们的相关性联想记忆：

(1) 导数、一阶导数、二阶导数、微分法。

(2) 直线运动、位置函数、高度、位移、速度、即时速度、加速度。

(3) 切线、斜率、陡峭、区间、界/限。

(4) 分数、分子、差商、平均、平均值、变化率。

(5) 趋于 0、连接 $x$ 与 $x+h$ 的区间、作出约定、无限增加、切线斜率上升/下降、相对于时间的变化率，提供计算斜率的方法。

2. 汉译英：

(1) 差商表示函数 $f$ 在连接 $x$ 与 $x+h$ 的区间上的平均变化率。

(2) 速度等于位置函数的导数。

(3) 由定义导数的过程所提供的几何解释以一种自然的方式导出了关于曲线的切线的思想。

(4) 差商表示直线 $PQ$ 与水平线的夹角的正切。

(5) 在直线运动中，速度的一阶导数称为加速度。

(6) 我们约定 $f^{(0)}=f$，即函数 $f$ 的零阶导数就等于它本身。

(7) 在运动的 9 秒钟内，物体的速度由 $v(0)=-144$ 变成了 $v(9)=144$，也就

是说,速度总共增加了每秒 288 英尺。

(8) 当 $\alpha$ 从 0 增加到 $\frac{\pi}{2}$ 时, $\tan\alpha$ 无限增加,而 $\tan\alpha$ 所对应的直线趋于竖直位置。

3. 英译汉:

(1) Examples of derivatives

EXAMPLE 1. *Derivative of a constant function.* Suppose $f$ is a constant function, say $f(x)=c$ for all $x$. The difference quotient is

$$\frac{f(x+h)-f(x)}{h}=\frac{c-c}{h}=0.$$

Since the quotient is 0 for all $h\neq 0$, its limit, $f'(x)$ is also 0 for every $x$. In the other words, a constant function has a zero derivative everywhere.

EXAMPLE 2. *Derivative of a linear function.* Suppose $f$ is a linear function, say $f(x)=mx+b$ for all real $x$. If $h\neq 0$, we have

$$\frac{f(x+h)-f(x)}{h}=\frac{m(x+h)+b-(mx+b)}{h}=\frac{mh}{h}=m.$$

Since the difference quotient does not change when $h$ approaches 0, we conclude that

$$f'(x)=m \qquad \text{for every } x.$$

Thus, the derivative of a linear function is a constant function.

(2) These three numbers, i.e., $f'(a)$, $f'_-(a)$ and $f'_+(a)$ are called, respectively, the derivative on the left, the derivative, the derivative on the right of $f(x)$ at $x=a$. For example, if $f(x)=|x|$, then $f'(0)$ does not exist, but $f'_-(a)=-1$ and $f'_+(a)=1$.

(3) We now introduce a natural generalization of partial derivatives. In the definition of $f_1(x_0,y_0)$, the numerator of the difference quotient used involves the value of $f(x,y)$ at two points $(x_0+\Delta x, y_0)$ and $(x_0,y_0)$. As $\Delta x$ approaches zero, the first point approaches the latter along the line $y=y_0$. For $f_2(x_0,y_0)$ a point $(x_0,y_0+\Delta y)$ approaches $(x_0,y_0)$ along the line $x=x_0$. We now replace these two special lines by an arbitrary line through $(x_0,y_0)$.

(4) A direction $\xi_\alpha$ is defined as the direction of any directed line which makes the angle $\alpha$ with the positive $x$-axis (positive angles measured in the counterclockwise sense as usual). Thus the line segment directed from the point $(0,0)$ to the point $(-1,-1)$ has the direction $\xi_{5\pi/4}$ or $\xi_{-3\pi/4}$.

The directional derivative of $f(x,y)$ in the direction $\xi_\alpha$ at $(a,b)$ is

$$\left.\frac{\partial f}{\partial \xi_\alpha}\right|_{(a,b)}=\frac{f(a+\Delta s\cos\alpha,b+\Delta s\sin\alpha)-f(a,b)}{\Delta s}。$$

4. 自学课文 8-C 并将它译成汉语。

## 课文 8-C Logarithms[1]

Logarithms were invented to shorten the work of extended numerical computations, which involve one or more of the operations of multiplication, division, involution, and evolution. Their use has decreased the labour of computing to such an extent that many calculations, which would require hours without the use of logarithms, can be performed with their aid in a small fraction of that time.

If we write the equation

$$n=b^a, \tag{1}$$

we express then the essential relation between a number, $n$, and its logarithms, $a$, for a given base, $b$. In the notation of logarithms this is written

$$\log_b n=a, \tag{2}$$

and it is read "the logarithm of $n$ to the base $b$ equals $a$". We can define verbally in one statement both logarithm and base as follows:

The logarithm of a given number is the power to which another number, called the base, must be raised in order to equal the given number.

It is important to realize that equations (1) and (2) are merely two different ways of expressing precisely the same relations, one the exponential way, the other the logarithmic. Above all it is necessary to keep in mind the fact that a logarithm is an exponent.

Thus in $81=3^4$, the given number is 81, the base is 3, and the logarithm is 4; that is, $\log_3 81=4$.

The base of the common system of logarithm is 10. Hence a table of common logarithms is really a table of exponents of the number 10. Since the greater portion of these exponents is approximate values of irrational numbers, it follows that computations by means of logarithms give only approximate results. Tables exist, however in which each logarithm is given to twenty or more decimals; hence practically any desired degree of accuracy can be obtained by using the proper table. The common system is used in numerical work almost exclusively.

The only other system of logarithms used in computations is called the natural system. It has for its base the irrational number $2.7182^+$, which is usually denoted by the letter e and is used mainly for theoretical purposes.

**生词与词组(二)**

counterclockwise[ˌkauntəˈklɔkwaiz] adj./adv. 逆时针的/地
in the counterclockwise sense 按逆时针方向
directed line 有向直线
directional derivative 方向导数
the derivative on the left 左导数
the derivative on the right 右导数
evolution[ˌiːvəˈluːʃən] n. 开方
exponential [ˌekspəuˈnenʃəl] adj. 指数的
involution[ˌinvəˈluːʃən] n. 乘方
labour[ˈleibə] n. 劳动,(辛苦的)工作
logarithm[ˈlɔgəriðəm] n. 对数
table of common logarithms 常用对数表
logarithmic[ˌlɔgəˈriðmik] adj. 对数的
the natural system 自然(对数)系
numerator[ˈnjuːməreitə] n. 分子
partial derivative 偏导数
portion[ˈpɔːʃən] n. 部分
the greater portion of … 大部分的
raise[reiz] v. 增加,种植
be raised to the power 按该幂次自乘
verbally[ˈvəːbəli] adv. 在语言上

# §2.9 微分方程简介
# (Introduction to Differential Equations)

## 课文 9-A Introduction

A large variety of scientific problems arise in which one tries to determine something from its rate of change.[1] For example, we could try to compute the position of a moving particle from a knowledge of its velocity or acceleration. Or a radioactive substance may be disintegrating at a known rate and we may be required to determine the amount of material present after a given time.[2] In examples like these, we are trying to determine an unknown function from prescribed information expressed in the form of an equation involving at least one of the derivatives of the unknown function[3]. These equations are called differential equations, and their study forms one of the most challenging branches of mathematics.

Differential equations are classified under two main headings: ordinary and partial, depending on whether the unknown is a function of just one variable or of two or more variables[4]. A simple example of an ordinary differential equation is the relation

$$f'(x)=f(x) \tag{9.1}$$

which is satisfied, in particular by the exponential function, $f(x)=e^x$. We shall see presently that every solution of (9.1) must be of the form $f(x)=Ce^x$, where $C$ may be any constant.

On the other hand, an equation like

$$\frac{\partial^2 f(x,y)}{\partial x^2}+\frac{\partial^2 f(x,y)}{\partial y^2}=0$$

is an example of a partial differential equation. This particular one, called *Laplace's equation*, appears in the theory of electricity and magnetism, fluid mechanics, and elsewhere. It has many different kinds of solutions, among which are $f(x,y)=x+2y$, $f(x,y)=e^x\cos y$, and $f(x,y)=\log(x^2+y^2)$.

The study of differential equations is one part of mathematics that, perhaps more than any other, has been directly inspired by mechanics, astronomy, and mathematical physics. Its history began in the 17th century when Newton, Leibniz, and the Bernoullis solved some simple differential equations arising from problems in geometry and mechanics. These early discoveries, beginning about 1690, gradually led to the development of a lot of "special tricks" for solving certain special kinds of differential equations. Although these special tricks are applicable in relatively few cases, they do enable us to solve many differential equations that arise in mechanics and geometry, so their study is of practical importance. Some of these special methods and some of the problems which they help us solve are discussed near the end of this chapter.

Experience has shown that it is difficult to obtain mathematical theories of much generality about solutions of differential equations, except for a few types. Among these are the so-called *linear* differential equations which occur in a great variety of scientific problems. The simplest types of linear differential equations and some of their applications are also discussed in this introductory chapter. A more thorough study of linear equations is carried out in Volume Ⅱ.

## 课文 9-B Terminology and notation

When we work with a differential equation such as (9.1), it is customary to write $y$ in place of $f(x)$ and $y'$ in place of $f'(x)$, the higher derivatives being denoted by $y''$, $y'''$, etc. Of course, other letters such as $u,v,z$, etc. are also used instead of $y$. By the order of an equation is meant the order of the highest derivative which appears[5]. For example, (9.1) is a first-order equation which may be written as $y'=y$. The differential equation $y'=x^3y+\sin(xy'')$ is one of second order.

In this chapter we shall begin our study with first-order equations which can be solved for $y'$ and written as follows:

(9.2) $$y' = f(x, y),$$

where the expression $f(x, y)$ on the right has various special forms. A differentiable function $y = Y(x)$ will be called a *solution* of (9.2) on an interval $I$ if the function $Y$ and its derivative $Y'$ satisfy the relation

$$Y'(x) = f[x, Y(x)]$$

for every $x$ in $I$. The simplest case occurs when $f(x, y)$ is independent of $y$. In this case, (9.2) becomes

(9.3) $$y' = Q(x),$$

say, where $Q$ is assumed to be a given function defined on some interval $I$. To solve the differential equation (9.3) means to find a primitive of $Q$. The Second fundamental theorem of calculus tells us how to do it when $Q$ is continuous on an open interval $I$. We simply integrate $Q$ and add any constant[6]. Thus, every solution of (9.3) is included in the formula

(9.4) $$y = \int Q(x)\,dx + C,$$

where $C$ is any constant (usually called an arbitrary constant of integration). The differential equation (9.3) has infinitely many solutions, one for each value of $C$.

If it is not possible to evaluate the integral in (9.4) in terms of familiar functions, such as polynomials, rational functions, trigonometric and inverse trigonometric functions, logarithms, and exponentials①, still we consider the differential equation as having been solved if the solution can be expressed in terms of integrals of known functions. In actual practice, there are various methods for obtaining approximate evaluations of integrals which lead to useful information about the solution. Automatic high-speed computing machines are often designed with this kind of problem in mind[7].

EXAMPLE. Linear motion determined from the velocity. Suppose a particle moves along a straight line in such a way that its velocity at time $t$ is $2\sin t$. Determine its position at time $t$.

Solution. If $Y(t)$ denotes the position at time $t$ measured from some starting point, then the derivative $Y'(t)$ represents the velocity at time $t$. We are given that

$$Y'(t) = 2\sin t.$$

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① 由上下文知,这里 logarithms 和 exponentials 分别是 logarithm functions 和 exponential functions 的简写形式。注意,若无上下文可参照时,一般不宜采用这种简写。

Integrating, we find that

$$Y(t)=2\int \sin t dt+C=-2\cos t+C.$$

This is all we can deduce about $Y(t)$ from a knowledge of the velocity alone; some other piece of information is needed to fix the position function. We can determine $C$ if we know the value of $Y$ at some particular instant. For example, if $Y(0)=0$, then $C=2$ and the position function is $Y(t)=2-2\cos t$. But if $Y(0)=2$, then $C=4$ and the position function is $Y(t)=4-2\cos t$.

In some respects the example just solved is typical of what happens in general. Some-where in the process of solving a first-order differential equation, an integration is required to remove the derivative $y'$ and in this step an arbitrary constant $C$ appears[8]. The way in which the arbitrary constant $C$ enters into the solution will depend on the nature of the given differential equation. It may appear as an additive constant, as in Equation (9.4), but it is more likely to appear in some other way. For example, when we solve the equation $y'=y$ in Section 9.3, we shall find that every solution has the form $y=Ce^x$.

In many problems it is necessary to select from the collection of all solutions one having a prescribed value at some point. The prescribed value is called an initial condition, and the problem of determining such a solution is called an initial-value problem. This terminology originated in mechanics where, as in the above example, the prescribed value represents the displacement at some initial time.

## 生词与词组(一)

approximate evaluation 近似估计
astronomy[əˈstrɔnəmi]n. 天文学
differential equation 微分方程
Bernoulli (人名) 伯努利
  the Bernoullis 伯努利家族[的数学家们]
disintegrate [disˈintigreit] v. 解体,衰变
differentiable [ˌdifəˈrenʃiəbl] adj. 可微的
displacement[disˈpleismənt]n. 位移
exponential [ˌekspəuˈnenʃəl] adj. 指数的
  exponential function 指数函数
initial[iˈniʃəl]adj. 初始的
  *initial condition* 初始条件
  initial-value problem 初值问题
integrate[ˈintigreit]v. 对……积分
integration[ˌintiˈgreiʃən]n. 积分
logarithm[ˈlɔgəriðəm]n. 对数
  logarithm function 对数函数(=logarithmic function)
mathematical physics 数学物理
mechanics[miˈkæniks]n. 力学

nature[ˈneitʃə]n. 性质,自然

ordinary differential equation 常微分方程

partial differential equation 偏微分方程

polynomial[ˌpɔliˈnəumjəl]n. 多项式

rational function 有理函数

terminology[ˌtəːmiˈnɔlədʒi]n. 术语

trigonometric function 三角函数

inverse trigonometric function 反三角函数

**预习要求**

1. 预习生词与词组(一),浏览课文 A、B 并在疑难处做上记号。
2. 复习或借助词典学习以下单词、词组与短语的含义及用法:

(1) arise from (in) sth.
(2) be classified under sth.
(3) by sth. is meant
(4) deduce sth. from sth.
(5) do enable sb. to do sth.
(6) enter into sth.
(7) infinitely many
(8) be inspired by sth./sb.
(9) in term of sth.
(10) a knowledge of sth./sb.
(11) a large variety of sth.
(12) must be of the form of sth.
(13) a prescribed value
(14) radioactive substance
(15) be required to do sth.
(16) at some particular instant

3. 复习"it is (was)…"句型,倒装句与省略句的用法.

**注释与说明**

1. A large variety of scientific problems arise in which one tries to determine something from its rate of change. 这里 which 引起的从句修饰 problems,直译可能不够简洁明白。据上下文的意思,可采用意译:大量的科学问题需要人们根据事物的变化率去确定该事物(的量)。

2. Or a radioactive substance may be disintegrating at a known rate and we may be required to determine the amount of material present after a given time. 这里 present 是形容词,意义为"留存的"。结合上一句开头为"For example",这句的"or"可译成"又如"。全句可译成:又如,某种放射性物质可能正在以已知的速度进行衰变,需要我们确定在给定的时间后遗留物质的总量。

3. In examples like these, we are trying to determine an unknown function from prescribed information expressed in the form of an equation involving at least one of the derivatives of the unknown function. 全句可译成两个短句:在类似的例子中,我们力求由以方程的形式表述的信息来确定未知函数,而这种方程至少包含了未知函数的一个导数。

4. Differential equations are classified under two main headings: ordinary and partial, depending on whether the unknown is a function of just one variable or of two or more variables. 这句翻译时需据数学知识做些补充和修改,使得数学术语完整出现。全句可译成:微分方程根据未知量是单变量函数或多变量函数分成两个主题:常微分方程和偏微分方程。

5. By the order of an equation is meant the order of the highest derivative which appears. 这是被动式语句。句型"By A is meant B"是动词 mean 的习惯用法,表示"A 的含义为 B"或"A 即 B"。句中"the order of the highest derivative which appears"可译为"出现在其中的最高阶导数的阶"或"出现在其中的导数的最高阶数"。

6. We simply integrate $Q$ and add any constant. 其中 integrate 是及物动词, simply 意思为"直接地"。全句可译成:我们直接对 $Q$ 积分并加上任意常数。注意:被积函数 $Q$ 是动词 integrate 的宾语。若要同时指明"关于变量 $x$ 进行积分",应采用"with respect to $x$"这一词组,放在句中"integrate $Q$"之后。

7. Automatic high-speed computing machines are often designed with this kind of problem in mind. 其中 with this kind of problem in mind 是行为方式状语,说明设计时就考虑到这类问题的处理。

8. Some-where in the process of solving a first-order differential equation, an integration is required to remove the derivative $y'$ and in this step an arbitrary constant $C$ appears. 其中 to remove the derivative $y'$是目的状语。被动式可译成主动式或不出现主语。全句可译成:在解一阶微分方程的过程中,为了消去导数 $y'$,需要在某一步进行积分,这时候就出现了一个任意常数。

**课外作业**

1. 把下列各组的单词、词组与短语译成英语,并按它们的相关性联想记忆:

(1) 微分方程、常微分方程、偏微分方程;初值问题、初始条件;积分、原函数。

(2) 对数函数、指数函数、有理函数、三角函数、反三角函数、多项式、可微的函数。

(3) 力学、天文学、数学物理。

(4) 对该函数积分、做近似估计、可以关于 $y$ 解出、有无限多个解、为了去掉导数、加上一个任意常数。

2. 汉译英:

(1) 此时,微分方程就有无穷多个解,$C$ 的每个值对应一个解。

(2) 微分方程的阶指的是方程中最高阶导数的阶。

（3）我们可以由已知的粒子运动速度或者加速度计算出粒子的位置。

（4）如果一个微分方程的未知函数是多元函数，则称为偏微分方程。

（5）微分方程的研究直接受到力学、天文学和数学物理的推动。

（6）许多应用问题要求我们从方程的解集中选出一个在某个点具有指定值的解。

（7）确定满足边界条件的解的问题称为边值问题。

（8）人们设计许多高速运行的计算机来对各种积分做出近似估计。

3. 英译汉：

（1）Boundary conditions

The rate if increase with time $t$ of a population $P$ of a colony of bacteria is proportional to the population $P$ at time $t$, i. e.

$$\frac{dP}{dt}=kP,$$

where $k$ is a constant. It is easy to check that the general solution of this differential equation is

$$P(t)=ce^{kt},$$

where $c$ is an arbitrary constant. Suppose that one needs to calculate the proportional at time $t=100$ in the case $k=2$. Then $P(t)=ce^{200}$.

To proceed any further, it is necessary to assign a particular value to the arbitrary constant $c$. For this, further information is necessary. The conditions which supply this information are usually called the *boundary conditions* for the problem.

（2）Histor ical introduction to complex numbers

The quadratic equation $x^2+1=0$ has no solution in the real-number system because there is no real number whose square is $-1$. New types of numbers, called *complex numbers*, have been introduced to provide to solutions to such equation. In this brief chapter we discuss complex number and show that they are important in solving algebraic equations and that they have an impact on differential and integral calculus.

As early as the 16th century, a symbol $\sqrt{-1}$ was introduced to provide solutions of the quadratic equation $x^2+1=0$. This symbol, later denoted by the letter i, was regarded as a fictitious or imaginary number which could be manipulated algebraically like an ordinary real number, except that its square was $-1$. Thus, for example, the quadratic solutions $x^2+1=0$ was factored by writing $x^2+1=x^2-i^2=(x-i)(x+i)$, and the solutions of $x^2+1=0$ were exhibited as $x=\pm i$, without any concern regarding the meaning or validity of such formulas. Expressions such as 2+3i were called complex numbers, and they were used in purely formal way for nearly 300 years before they were described in a manner that would be considered satisfactory by present-day standards.

Early in the 19th century, Karl Friedrich Gauss (1777－1855) and William Rowan Hamilton (1805－1865) independently and almost simultaneously proposed the idea of defining complex numbers as ordered pairs $(a, b)$ of real numbers endowed with certain special properties. This idea is widely accepted today and is described in the next section.

4. 自学课文 9-C 并将它译成汉语。

## 课文 9-C The basic properties of the integral

From the definition of the integral, it is possible to deduce the following properties. Proofs are given in Section 1.27.

THEOREM 1. LINEARITY WITH RESPECT TO THE INTEGRAND. *If both* $f$ *and* $g$ *are integrable on* $[a, b]$, *so is* $c_1 f + c_2 g$ *for every pair of constants* $c_1$ *and* $c_2$. *Furthermore, we have*

$$\int_a^b [c_1 f(x) + c_2 g(x)]\,dx = c_1 \int_a^b f(x)\,dx + c_2 \int_a^b g(x)\,dx.$$

Note: By use of mathematical induction, the linearity property can be generalized as follows: If $f_1, f_2, \cdots, f_n$ are integrable on $[a, b]$, then so is $c_1 f_1 + c_2 f_2 + \cdots + c_n f_n$ for all real *constants* $c_1, c_2, \cdots, c_n$, and

$$\int_a^b \left[\sum_{k=1}^{n} c_k f_k(x)\right] dx = \sum_{k=1}^{n} \left[c_k \int_a^b f_k(x)\,dx\right].$$

THEOREM 2. ADDITIVITY WITH RESPECT TO THE INTERVAL OF INTEGRATION. *If two of the following three integrals exist, the third also exists, and we have*

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx.$$

Note: In particular, if $f$ is monotonic on $[a, c]$ and also on $[c, b]$, then both integrals $\int_a^c f(x)\,dx$ and $\int_c^b f(x)\,dx$ exist, so $\int_a^b f(x)\,dx$ also exists and is equal to the sum of the other two integrals.

THEOREM 3. INVARIANCE UNDER TRANSLATION. *If* $f$ *is integrable on* $[a, b]$, *then for every real* $c$ *we have*

$$\int_{a}^{b} f(x)\,\mathrm{d}x = \int_{a+c}^{b+c} f(x-c)\,\mathrm{d}x.$$

THEOREM 4. EXPANSION OR CONTRACTION OF THE INTERVAL OF INTEGRATION. *If $f$ is inlegrable on $[a,b]$, then for every real $k \neq 0$ we have*

$$\int_{a}^{b} f(x)\,\mathrm{d}x = \frac{1}{k}\int_{ka}^{kb} f\left(\frac{x}{k}\right)\mathrm{d}x.$$

Note: In both Theorems 3 and 4, the existence of one of the integrals implies the existence of the other. When $k=-1$, Theorem 4 is called the *reflection property*.

THEOREM 5. COMPARISON THEOREM. *If both $f$ and $g$ are integrable on $[a,b]$ and if $g(x) \leqslant f(x)$ for every $x$ in $[a,b]$, then we have*

$$\int_{a}^{b} g(x)\,\mathrm{d}x \leqslant \int_{a}^{b} f(x)\,\mathrm{d}x.$$

An important special case of Theorem 5 occurs when $g(x)=0$ for every $x$. In this case, the theorem states that if $g(x) \geqslant 0$ everywhere on $[a,b]$, then $\int_{a}^{b} g(x)\,\mathrm{d}x \geqslant 0$. In other words, a nonnegative function has a nonnegative integral. It can also be shown that if we have the *strict* inequality $g(x) < f(x)$ for all $x$ in $[a,b]$, then the same strict inequality holds for the integrals, but the proof is not easy to give at this stage.

In Chapter 5 we shall discuss various methods for calculating the value of an integral without the necessity of using the definition in each case. These methods, however, are applicable to only a relatively small number of functions, and for most integrable functions the actual numerical value of the integral can only be estimated. This is usually done by approximating the integrand above and below by step functions or by other simple functions whose integrals can be evaluated exactly. Then the comparison theorem is used to obtain corresponding approximations for the integral of the function in question.

**生词与词组(二)**

algebraically[ˌældʒiˈbreiikəli] adv. 在代数上,用代数方法

approximate[əˈprɔksimeit] v. 逼近,约等于

arbitrary constant 任意常数

boundary condition 边界条件

a colony of bacteria 菌落

complex number 复数(相对于实数而言)

comparison theorem 比较定理

contraction[ kən'trækʃən ] n. 收缩
evaluate[ i'væljueit ] v. 估计,求值
expansion[ iks'pænʃən ] n. 扩张
factor[ 'fæktə ] n. 因式,因子; v. 因式分解
fictitious[ fik'tiʃəs ] adj. 虚构的,假的
impact[ 'impækt ] n. 冲击,影响
imaginary[ i'mædʒinəri ] adj. 想象的
imaginary number 虚数
integrable[ 'intigrəbl ] adj. 可积的
integrand[ 'intigrænd ] n. 被积函数
integration[ ˌinti'greiʃən ] n. 积分
linearity[ ˌlini'æriti ] n. 线性
manipulate[ mə'nipjuleit ] v. 操作
population [ ˌpɔpju'leiʃən ] n. 总体,总量
simultaneously[ siməl'teiniəsli ] adv. 同时地
step function 阶梯函数
strict inequality 严格不等式

# §2.10 线性空间中的相关与无关集
# (Dependent and Independent Sets in a Linear Space)

## 课文 10-A Dependent and independent sets in a linear space

DEFINITION. *A set $S$ of elements in a linear space $V$ is called dependent if there is a finite set of distinct elements in $S$, say $\boldsymbol{x}_1, \cdots, \boldsymbol{x}_k$, and corresponding set of scalars $c_1, \cdots, c_k$, not all zero, such that*[1]

$$\sum_{i=1}^{k} c_i \boldsymbol{x}_i = \boldsymbol{O}.^{2}$$

*The set $S$ is called independent if it is not dependent. In this case, for all choices of distinct elements $\boldsymbol{x}_1, \cdots, \boldsymbol{x}_k$ in $S$ and scalars $c_1, \cdots, c_k$,*

$$\sum_{i=1}^{k} c_i \boldsymbol{x}_i = \boldsymbol{O} \qquad \textit{implies } c_1 = c_2 = \cdots = c_k = 0.$$

Although dependence and independence are properties of sets of elements, we also apply these terms to the elements themselves. For example, the elements in an independent set are called independent elements.

If $S$ is a finite set, the foregoing definition agrees with that given in Chapter 8 for the space $V_n$. However, the present definition is not restricted to finite sets.

EXAMPLE 1. If a subset $T$ of a set $S$ is dependent, then $S$ itself is dependent. This is logically equivalent to the statement that every subset of an independent set is independent.

EXAMPLE 2. If one element in $S$ is a scalar multiple of another, then $S$ is dependent.

EXAMPLE 3. If $\boldsymbol{O} \in S$, then $S$ is dependent.

EXAMPLE 4. The empty set is independent.

Many examples of dependent and independent sets of vectors in $V_n$ were discussed in Chapter 8. The following examples illustrate these concepts in function spaces. In each case the underlying linear space $V$ is the set of all real-valued functions defined on the real line.

EXAMPLE 5. Let $u_1(t) = \cos^2 t, u_2(t) = \sin^2 t, u_3(t) = 1$ for all real $t$. The Pythagorean identity shows that $u_1 + u_2 - u_3 = \boldsymbol{O}$, so the three functions $u_1, u_2, u_3$ are dependent.

EXAMPLE 6. Let $u_k(t) = t^k$ for $k = 0, 1, 2, \cdots$, and $t$ real. The set $S = \{u_0, u_1, u_2, \cdots\}$ is independent. To prove this, it suffices to show that for each $n$ the $n+1$ polynomials $u_0, u_1, \cdots, u_n$ are independent. A relation of the form $\sum c_k u_k = 0$ means that

$$\sum c_k t^k = 0 \tag{10.1}$$

for all real $t$. When $t = 0$, this gives $c_0 = 0$. Differentiating (10.1) and setting $t = 0$, we find that $c_1 = 0$. Repeating the process, we find that each coefficient $c_k$ is zero.

EXAMPLE 7. If $a_1, \cdots, a_n$ are distinct real numbers, the $n$ exponential functions

$$u_1(x) = e^{a_1 x}, \cdots, u_n(x) = e^{a_n x}$$

are independent. We can prove this by induction on $n$. The result holds trivially when $n = 1$. Therefore, assume it is true for $n-1$ exponential functions and consider scalars $c_1, \cdots, c_n$ such that

$$\sum_{k=1}^{n} c_k e^{a_k x} = 0. \tag{10.2}$$

Let $a_M$ be the largest of the $n$ numbers $a_1, \cdots, a_n$. Multiplying both members of (10.2) by $e^{-a_M x}$, we obtain

$$\sum_{k=1}^{n} c_k e^{(a_k - a_M)x} = 0. \tag{10.3}$$

If $k \neq M$, the number $a_k - a_M$ is negative. Therefore, when $x \to +\infty$ in Equation (10.3), each term with $k \neq M$ tends to zero and we find that $c_M = 0$. Deleting the $M$th term from (10.2) and applying the induction hypothesis, we find that each of the remaining $n-1$ coefficients $c_k$ is zero.

THEOREM 10.5. Let $S$ be an independent set consisting of $k$ elements in a linear space $V$ and let $L(S)$ be the subspace spanned by $S$. Then every set of $k+1$ elements in $L(S)$ is dependent.

*Proof.* When $V = V_n$, Theorem 10.5 reduces to Theorem 8.8. If we examine the proof of Theorem 8.8, we find that it is based only on the fact that $V_n$ is a linear space and not on any other special property of $V_n$. Therefore the proof given for Theorem 8.8 is valid for any linear space $V$.

## 课文 10-B Basis and dimension

DEFINITION. *A finite set $S$ of elements in a linear space $V$ is called a finite basis for $V$ if $S$ is independent and spans $V$. The space $V$ is called finite dimensional if it has a finite basis, or if $V$ consists of $\mathbf{O}$ alone. Otherwise $V$ is called infinite dimensional.*

THEOREM 10.6. *Let $V$ be a finite-dimensional linear space. Then every finite basis for $V$ has the same number of elements.*

*Proof.* Let $S$ and $T$ be two finite bases for $V$. Suppose $S$ consists of $k$ elements and $T$ consists of $m$ elements. Since $S$ is independent and spans $V$, Theorem 10.5 tells us that every set of $k+1$ elements in $V$ is dependent. Therefore, every set of more than $k$ elements in $V$ is dependent. Since $T$ is an independent set, we must have $m \leqslant k$. The same argument with $S$ and $T$ interchanged shows that $k \leqslant m$[3]. Therefore $k = m$.

DEFINITION. *If a linear space $V$ has a basis of $n$ elements, the integer $n$ is called the dimension of $V$. We write $n = \dim V$. if $V = \{\mathbf{O}\}$, we say $V$ has dimension 0.*

EXAMPLE 1. The space $V_n$ has dimension $n$. One basis is the set of $n$ unit coordinate vectors.

EXAMPLE 2. The space of all polynomials $p(t)$ of degree $\leqslant n$ has dimension $n+1$.[4] One basis is the set of $n+1$ polynomials $\{1, t, t^2, \cdots, t^n\}$. Every polynomial of degree $\leqslant n$ is a linear combination of these $n+1$ polynomials.

EXAMPLE 3. The space of solutions of the differential equation $y'' - 2y' - 3y = 0$ has dimension 2. One basis consists of the two functions $u_1(x) = e^{-x}$, $u_2(x) = e^{3x}$. Every solution is a linear combination of these two.

EXAMPLE 4. The space of all polynomials $p(t)$ is infinite-dimensional. Although the infinite set $\{1, t, t^2, \cdots\}$ spans this space, no finite set of polynomials spans the space.

THEOREM 10.7. *Let $V$ be a finite-dimensional linear space with $\dim V = n$. Then we have the following.*

(a) *Any set of independent elements in $V$ is a subset of some basis for $V$.*

(b) *Any set of $n$ independent elements is a basis for $V$.*

*Proof.* The proof of (a) is identical to that of part (b) of Theorem 8.10. The proof of (b) is identical to that of part (c) of Theorem 8.10.

Let $V$ be a linear space of dimension $n$ and consider a basis whose elements $\boldsymbol{e}_1, \cdots, \boldsymbol{e}_n$ are taken in a given order. We denote such an ordered basis as an $n$-tuple $(\boldsymbol{e}_1, \cdots, \boldsymbol{e}_n)$. If $\boldsymbol{x} \in V$, we can express $\boldsymbol{x}$ as a linear combination of these basis elements:

$$\boldsymbol{x} = \sum_{i=1}^{n} c_i \boldsymbol{e}_i. \tag{10.4}$$

The coefficients in this equation determine an $n$-tuple of numbers $(c_1, \cdots, c_n)$ that is uniquely determined by $\boldsymbol{x}$. In fact, if we have another representation of $\boldsymbol{x}$ as a linear combination of $\boldsymbol{e}_1, \cdots, \boldsymbol{e}_n$, say $\boldsymbol{x} = \sum_{i=1}^{n} d_i \boldsymbol{e}_i$, then by subtraction from (10.4), we find that $\sum_{i=1}^{n} (c_i - d_i)\boldsymbol{e}_i = \boldsymbol{0}$. But since the basis elements are independent, this implies $c_i = d_i$ for each $i$, so we have $(c_1, \cdots, c_n) = (d_1, \cdots, d_n)$.

The components of the ordered $n$-tuple $(c_1, \cdots, c_n)$ determined by Equation (10.4) are called the components of $\boldsymbol{x}$ relative to the ordered basis $(\boldsymbol{e}_1, \cdots, \boldsymbol{e}_n)$.

**生词与词组(一)**

basis[ ˈbeisis ] n. 基,基底
　finite basis 有限基
　ordered basis 有序基
coefficient[ ˌkəuiˈfiʃənt ] n. 系数
component [ kəmˈpəunənt ] n. 成分,分量
dependent[ diˈpendənt ] adj. 相关的
differentiate[ ˌdifəˈrenʃieit ] v. 求导数,求微分
distinct[ disˈtiŋkt ] adj. 互不相同的
finite dimensional 有限维的
hold[ həuld ] v. 成立,有效,适用
　hold trivially 显然成立
hypothesis[ haiˈpɔθisis ] n. 假设
　the induction hypothesis 归纳法的假设
independent [ ˌindiˈpendənt ] adj. 无关的
infinite dimensional 无限维的
linear combination 线性组合
linear space 线性空间
logically equivalent 逻辑等价的
multiple[ ˈmʌltipl ] n. 倍数;adj. 多样的
multiply[ ˈmʌltiplai ] v. 乘,倍增;adv. 多样地,以多种形式地
The Pythagorean identity 毕达哥拉斯等式(西方国家对勾股定理的称呼)
remain[ riˈmein ] v. 剩下,余留
scalar[ ˈskeilə ] n. 标量,数量
span[ spæn ] v. 张成,支撑
　the subspace spanned by $S$ 由 $S$ 张成的子空间
$n$-tuple[ ˌenˈtju:pl ] n. $n$-元组
　the ordered $n$-tuple 有序 $n$-元组
the underlying linear space 基础线性空间

uniquely[ juː'niːkli ] adv. 唯一地

vector[ 'vektə ] n. 向量,矢量

unit coordinate vector 单位坐标向量

valid[ 'vælid ] adj. 成立的,有效的

be valid for 对……成立

**预习要求**

1. 预习生词与词组(一),浏览课文 A、B 并在疑难处做上记号。
2. 复习或借助词典学习以下单词、词组与短语的含义及用法:

(1) agree with sth./sb.
(2) base on sth.
(3) be identical to sth.
(4) by induction on $n$
(5) it suffices to do sth.
(6) reduce sth. to sth.
(7) relative to sth./sb.
(8) be restricted to sth.
(9) the same argument with sth.
(10) span the space
(11) by subtraction from sth.
(12) take in a given order
(13) uniquely determine

3. 复习(1)介词和数词的用法;(2)否定句的表达方式。

**注释与说明**

1. *A set S of elements in a linear space V is called dependent if there is a finite set of distinct elements in S, say* $\boldsymbol{x}_1,\cdots,\boldsymbol{x}_k$, *and corresponding set of scalars* $c_1,\cdots,c_k$, *not all zero, such that*…其中 A set $S$ of elements in a linear space $V$ 可直译为"线性空间 $V$ 中的元素组成的一个集 $S$";若根据数学知识,意译为"线性空间 $V$ 的一个子集 $S$",则更为简单明了。又,英文数学命题(特别是定义)常用主句表示结论,放在前面,同时把条件从句放在后面。当条件从句较长时,中文文献也常按这种次序翻译。这样一来,这句可译成:线性空间 $V$ 的一个子集 $S$ 称为相关的,如果在 $S$ 中存在有限个不同的元素,例如 $\boldsymbol{x}_1,\cdots,\boldsymbol{x}_k$,和相应的一组不全为 0 的数 $c_1,\cdots,c_k$使得……。

2. 本文用 $\mathbf{0}$ 表示零向量。在连续函数空间中,零向量可以被定义为恒等于零的函数。

3. The same argument with $S$ and $T$ interchanged shows that $k\leqslant m$. 这句可译成:把 $S$ 与 $T$ 交换后,作同样的讨论可证明 $k\leqslant m$。

4. The space of all polynomials $p(t)$ of degree $\leqslant n$ has dimension $n+1$. 整句可译成:阶数小于或等于 $n$ 的所有多项式组成的空间的维数是 $n+1$。通常 has dimension $n+1$ 不译成"有维数 $n+1$"。

**课外作业**

1. 把下列各组的单词、词组与短语译成英语,并按它们的相关性联想记忆:

(1) 线性空间、基础线性空间、子空间;基、有限基、有序基。

(2) 相关的、无关的;维数、有限维、无限维。

(3) 标量、向量、分量/成分、单位坐标向量、$n$ 元组;乘法、倍数、系数。

(4) 该等式对所有的 $x$ 成立、对函数 $f(t,x)$ 关于变量 $t$ 求导、对自然数 $n$ 进行归纳;唯一地确定极限、$A$ 所张成的线性空间、与前面给出的定义一致;只要证明这个结论就够了。

2. 汉译英:

(1) 该式的两边同时关于 $t$ 积分,我们就得到一个所需要的结论。

(2) 不难看出,这个命题仅仅建立在该空间是线性的这一事实上,与空间的其他性质无关。

(3) 如果空间不存在有限基,就称该空间是无限维的。

(4) 假定这个结论对 $n-1$ 个指数函数成立,我们将证明此结论对 $n$ 个指数函数也成立。

(5) 这两个定义在逻辑上是互相等价的。

(6) 设 $X$ 是线性空间 $V$ 中 $k$ 个元素组成的一个线性无关集合,$L(X)$ 是由 $X$ 张成的子空间。那么,$L(X)$ 的每一个元素都可以表示成 $X$ 的元素的线性组合。

(7) 设 $V$ 是一个 $n$ 维线性空间,考虑它的一个基,其元素按给定的次序排列为 $\boldsymbol{e}_1,\boldsymbol{e}_2,\cdots,\boldsymbol{e}_n$。

(8) 该线性表示的系数构成一个 $n$ 元组,它由向量 $\boldsymbol{x}$ 唯一确定。

3. 英译汉:

(1) let $\boldsymbol{x}_1,\boldsymbol{x}_2,\cdots,\boldsymbol{x}_n$ denote vectors. Any vector expressible in the form

$$\alpha_1\boldsymbol{x}_1+\alpha_2\boldsymbol{x}_2+\cdots+\alpha_n\boldsymbol{x}_n\text{( with }\alpha_1,\alpha_2,\cdots,\alpha_n\text{ scalar)}$$

is said to be (expressible as) a linear combination of $\boldsymbol{x}_1,\boldsymbol{x}_2,\cdots,\boldsymbol{x}_n$. (In the trivial case $n=1$, where we would be talking about "a linear combination of $\boldsymbol{x}_1$", that would simpy mean a scalar multiple of $\boldsymbol{x}_1$.

(2) We shall now define (for any $\boldsymbol{x},\boldsymbol{y}\in E_3$) something called ***scalar product*** of $\boldsymbol{x}$ and $\boldsymbol{y}$. As the name suggests, this is a scalar (depending on $\boldsymbol{x}$ and $\boldsymbol{y}$). It is denoted by $\boldsymbol{x}\cdot\boldsymbol{y}$, and it is defined as follow:

(a) If $\boldsymbol{x}$ and $\boldsymbol{y}$ are nonzero, then $\boldsymbol{x}\cdot\boldsymbol{y}=|\boldsymbol{x}||\boldsymbol{y}|\cos\theta$, where $\theta$ is the angle between $\boldsymbol{x}$ and $\boldsymbol{y}$; (b) If $\boldsymbol{x}$ or $\boldsymbol{y}$ is $\boldsymbol{0}$ (or if both are $\boldsymbol{0}$), then $\boldsymbol{x}\cdot\boldsymbol{y}=0$.

(3) A polynomial in $x$ is an algebraic expression of the $a_0x^n+a_1x^{n-1}+a_2x^{n-2}+\cdots+a_n$, where $x$ is a variable whose value is not fixed and $a_0,a_1,a_2,\cdots,a_n$ are all constants with $a_0\neq0$, and $n$ is a non-negative integer and is known as the ***degree*** of the polynomial, $a_0$ is called the ***leading coefficient*** of the polynomial.

(4) A matrix with the same number of rows and columns is called a **square**

**matrix**. And an $n$ th-order square matrix, which is denoted by $\boldsymbol{A}_{n\times n}$, consists of $n^2$ numbers with $n$ rows and $n$ columns. For any matrix $\boldsymbol{A}_{n\times n}$ with real numbers entries, there is associated a unique number called ***determinant***, designated by $|\boldsymbol{A}|$. Also, is said to be a determinant of order $n$.

4. 自学课文 10-C,并把它译成汉语。

## 课文 10-C Applications of matrices

In recent years the applications of matrices in mathematics and in many diverse fields have increased with remarkable speed. Matrix theory plays a central role in modern physics in the study of quantum mechanics. Matrix methods are used to solve problems in applied differential equations, specifically, in the area of aerodynamics, stress and structure analysis. One of the most powerful mathematical methods for psychological studies is factor analysis, a subject that makes wide use of matrix methods. Recent developments in mathematical economics and in problems of business administration have led to extensive use of matrix methods. The biological sciences, and in particular genetics, use matrix techniques to good advantage. No matter what the student's field of major interest is, knowledge of the rudiments of matrices is likely to broaden the range of literature that he can read with understanding.

In this section we will give some elementary examples of how matrices are utilized.

The solution of $n$ simultaneous linear equations in $n$ unknowns is one of the important problems of applied mathematics. Descartes, the inventor of analytic geometry and one of the founders of modern algebraic notation, believed that all problems could ultimately be reduced to the solution of a set of simultaneous linear equations. Although this belief is now known to be untenable, we know that a large group of significant applied problems from many different disciplines are reducible to such equations. Many of the applications, require the solution of a large number of simultaneous linear equations, sometimes in the hundreds. The advent of computers has made the matrix methods effective in the solution of these formidable problems. Example 1. Solve the simultaneous equations for $x_1$, $x_2$ and $x_3$.

$$2x_1+3x_2+4x_3=4,$$
$$2x_1+x_2+x_3=-2,$$
$$-x_1+x_2+2x_3=2.$$

Solution. We may rewrite these equations in matrix form

$$\begin{pmatrix} 2 & 3 & 4 \\ 2 & 1 & 1 \\ -1 & 1 & 2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}=\begin{pmatrix} 4 \\ -2 \\ 2 \end{pmatrix} \tag{1}$$

and call the matrix of coefficients $\boldsymbol{A}$, the $3\times1$ matrix of unknowns $\boldsymbol{x}$, and the $3\times1$ matrix on the right $\boldsymbol{k}$, we may then write Equation (1) in the form

$$\boldsymbol{Ax}=\boldsymbol{k}. \tag{2}$$

If it were possible to find a $3\times3$ matrix, which is designated by $\boldsymbol{A}^{-1}$ and is called the inverse of matrix, such that

$$\boldsymbol{A}^{-1}\boldsymbol{A}=\boldsymbol{I}, \tag{3}$$

where $\boldsymbol{I}$ is the identity matrix, then we would multiply both members of Equation (2) by $\boldsymbol{A}^{-1}$. Equation (2) would then become

$$\boldsymbol{A}^{-1}\boldsymbol{Ax}=\boldsymbol{A}^{-1}\boldsymbol{k}. \tag{4}$$

Using Equation (3), we could rewrite Equation (4) as

$$\boldsymbol{Ix}=\boldsymbol{A}^{-1}\boldsymbol{k},$$

$$\boldsymbol{x}=\boldsymbol{A}^{-1}\boldsymbol{k}. \tag{5}$$

Specifically for this case, without telling you how we get it,

$$\boldsymbol{A}^{-1}=\begin{pmatrix} -1 & 2 & 1 \\ 5 & -8 & -6 \\ -3 & 5 & 4 \end{pmatrix}.$$

Using this in Equation (5), we get

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}=\begin{pmatrix} -1 & 2 & 1 \\ 5 & -8 & -6 \\ -3 & 5 & 4 \end{pmatrix}\begin{pmatrix} 4 \\ -2 \\ 2 \end{pmatrix}=\begin{pmatrix} -6 \\ 24 \\ -14 \end{pmatrix}.$$

Thus $x_1=-6$, $x_2=24$, and $x_3=-14$. From the above discussion, we see that the problem of solving $n$ simultaneous linear equation in $n$ unknowns is reduced to the problem of finding the inverse of the matrix of coefficients. It is therefore not surprising that in books on the theory of matrices the techniques of finding inverse matrices occupy considerable space. Of course, we will not in our limited treatment discuss such techniques. Not only are matrix methods useful in solving simultaneous equations, but they are also useful in discovering whether or not the set of equations are consistent, in the sense that they lead to solutions, and in discovering whether or not the set of equations are determinate, in the sense that they lead to unique solutions.

**生词与词组(二)**

consistent[ kən'sistənt ] adj. 相容的

column[ 'kɔləm ] n. 列

determinate[ di'təːminit ] n. 行列式

discipline[ 'disiplin ] n. 领域,学科

diverse[ dai'vəːs ] adj. 不同的

entry[ 'entri ] n. 进入,通路,表中值

entry of matrix 矩阵的元

inverse[ 'in'vəːs ] n. 逆

matrix[ 'meitriks ] n. 矩阵

matrix of coefficients 系数矩阵

square matrix 方阵

$n$ th-order square matrix $n$ 阶方阵

nonzero[ 'nɔnziərəu ] n. 非零元

polynomial[ ˌpɔli'nəumjəl ] n. 多项式

polynomial in $x$ [关于] $x$ 的多项式

psychological[ ˌsaikə'lɔdʒikəl ] adj. 心理学的

reducible[ ri'djuːsəbl ] adj. 可简化的,可归约的

row[ rəu ] n. 行

simultaneous linear equations 联立方程[组]

ultimately[ 'ʌltimitli ] adv. 最终,最后

untenable [ 'ʌn'tenəbl ] adj. 不可达到的

**插页:名家谈数学①**

**What is mathematics? ——什么是数学?**

● 数学王子高斯(C. F. Gauss)把数学尊崇到"君临天下"的位置,他说:

Mathematics is the Queen of the Sciences, and Arithmetic the Queen of Mathematics.

(数学是科学的女王,而数论是数学的女王。)

● 德国哲学家康德(I. Kant, 1724 ~ 1804)曾很感慨地说过:

The science of mathematics presents the most brilliant example of how pure reason may successfully enlarge its domain without the aid of experience.

(数学科学呈现出一个最辉煌的例子,表明不用借助实验,纯粹的推理就能成功地扩大其自身的领域。)

● 杰出的物理学家爱因斯坦(A. Einstein)就他的研究工作体验说道:

But there is another reason for the high repute of mathematics: it is mathematics that offers the exact natural sciences a certain measure of security which, without mathematics, they could not attain.

(数学受到高度尊崇的另一个原因在于:恰恰是数学,给精密的自然科学提供了无可置疑的可靠保证,没有数学,它们无法达到这样的可靠程度。)

① 本书第二章起的插页均摘自文献[4]。

# § 2.11 数理逻辑入门
# (Elementary Mathematical Logic)

## 课文 11-A Predicates①

Statements involving variables, such as

"$x>3$", "$x=y+3$", and "$x+y=z$",

are often found in mathematical assertions and in computer programs. These statements are neither true nor false when the values of the variables are not specified[1]. In this section we will discuss the ways that propositions can be produced from such statements.

The statement "$x$ is greater than 3" has two parts. The first part, the variables, is the subject of the statement. The second part — the *predicate*, "is greater than 3"— refers to a property that the subject of the statement can have. We can denote the statement "$x$ is greater than 3" by $P(x)$, where $P$ denotes the predicate "is greater than 3" and $x$ is the variable. The statement $P(x)$ is also said to be the value of the *propositional function* $P$ at $x$. Once a value has been assigned to the variable $x$, the statement $P(x)$ becomes a proposition and has a truth value. Consider the following example.

EXAMPLE 1. Let $P(x)$ denote the statement "$x>3$". What are the truth values of $P(4)$ and $P(2)$?

*Solution*: The statement $P(4)$ is obtained by setting $x=4$ in the statement "$x>3$". Hence, $P(4)$, which is the statement "$4>3$", is true. However, $P(2)$, which is the statement "$2>3$", is false.

We can also have statements that involve more than one variable. For instance, consider the statement "$x=y+3$". We can denote this statement by $Q(x,y)$, where $x$ and $y$ are variables and $Q$ is the predicate. When values are assigned to the variables $x$ and $y$, the statement $Q(x,y)$ has a truth value.

EXAMPLE 2. Let $Q(x,y)$ denote the statement "$x=y+3$". What are the truth values of the propositions $Q(1,2)$ and $Q(3,0)$?

*Solution*: To obtain $Q(1,2)$, set $x=1$ and $y=2$ in the statement $Q(x,y)$.

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① 本节三篇短文均摘自:K. H. Rosen. Discrete Mathematics and Its Application. 4th Edition. New York: McGraw-Hill, 1999.

Hence, $Q(1,2)$ is the statement "$1=2+3$", which is false. The statement $Q(3,0)$ is the proposition "$3=0+3$", which is true.

Similarly, we can let $R(x,y,z)$ denote the statement "$x+y=z$". When values are assigned to the variables $x,y$, and $z$, this statement has a truth value.

EXAMPLE 3. What are the truth values of the propositions $R(1,2,3)$ and $R(0,0,1)$?

*Solution*: The proposition $R(1,2,3)$ is obtained by setting $x=1, y=2$, and $z=3$ in the statement $R(x,y,z)$. We see that $R(1,2,3)$ is the statement "$1+2=3$", which is true. Also note that $R(0,0,1)$, which is the statement "$0+0=1$", is false.

In general, a statement involving the $n$ variables $x_1, x_2, \cdots, x_n$ can be denoted by $P(x_1, x_2, \cdots, x_n)$.

A statement of the form $P(x_1, x_2, \cdots, x_n)$ is the value of the *propositional function P* at the $n$-tuple $(x_1, x_2, \cdots, x_n)$, and $P$ is also called a *predicate.*

Propositional functions occur in computer programs, as the following example demonstrates.

EXAMPLE 4. Consider the statement

$$\text{if } x>0 \text{ then } \quad x := x+1.$$

When this statement is encountered in a program, the value of the variable $x$ at that point in the execution of the program is inserted into $P(x)$, which is "$x>0$". If $P(x)$ is true for this value of $x$, the assignment statement $x:=x+1$ is executed, so the value of $x$ is increased by 1. If $P(x)$ is false for this value of $x$, the assignment statement is not executed, so the value of $x$ is not changed.

## 课文 11-B Quantifiers ①

When all the variables in a propositional function are assigned values, the resulting statement has a truth value. However, there is another important way, called quantification, to create a proposition from a propositional function. Two types of quantification will be discussed here, namely, universal quantification and existential quantification.

Many mathematical statements assert that a property is true for all values of a

① Recall the following 4 symbels: $\neg$, $\wedge$, $\vee$, $\rightarrow$ which we read respectively as: 'not', 'and', 'or' and 'implies' and which we call *symbols for propositional connectives.* The symbols $\neg$, $\wedge$, $\vee$, $\rightarrow$ are respectively called: the symbol for *negation*, the symbol for *conjunction*, the symbol for *disjunction* and the symbol for *implication*[2].

variable in a particular domain, called the universe of discourse. Such a statement is expressed using a universal quantification. The universal quantification of a propositional function is the proposition that asserts that $P(x)$ is true for all values of $x$ in the universe of discourse. The universe of discourse specifies the possible values of the variable $x$.

DEFINITION 1. The *universal quantification* of $P(x)$ is the proposition "$P(x)$ is true for all values of $x$ in the universe of discourse."

The notation

$$\forall x P(x)$$

denotes the universal quantification of $P(x)$. Here $\forall$ is called the *universal quantifier.*

The proposition $\forall x P(x)$ is also expressed as "for all $xP(x)$" or "for every $xP(x)$".

*Remark*[3]: it is best to avoid the word "any" since it is often ambiguous as to whether it means "every" or "some". In some cases, "any" is unambiguous, such as when it is used in negatives, for example, "there is not any reason not to study hard".

EXAMPLE 5. Express the statement "Every student in this class has studied calculus" as a universal quantification.

*Solution*: Let $P(x)$ denote the statement "$x$ has studied calculus".

Then the statement "Every student in this class has studied calculus" can be written as $\forall x P(x)$, where the universe of discourse consists of the students in this class.

This statement can also be expressed as

$$\forall x(S(x) \to P(x)),$$

where $S(x)$ is the statement

"$x$ is in this class."

$P(x)$ is as before, and the universe of discourse is the set of all students.

Example 5 illustrates that there is often more than one good way to express a quantification.

Many mathematical statements assert that there is an element with a certain property. Such statements are expressed using existential quantification. With existential quantification, we form a proposition that is true if and only if $P(x)$ is true for at least one value of $x$ in the universe of discourse.

DEFINITION 2. The *existential quantification* of $P(x)$ is the proposition "There exists an element $x$ in the universe of discourse such that $P(x)$ is true." We use the notation

$$\exists xP(x)$$

for the existential quantification of $P(x)$. Here $\exists$ is called the *existential quantifier*.

The existential quantification $\exists xP(x)$ is also expressed as

"There is an $x$ such that $P(x)$",

"There is at least one $x$ such that $P(x)$", or "For some $xP(x)$".

EXAMPLE 6. Let $P(x)$ denote the statement "$x>3$". What is the truth value of the quantification $\exists xP(x)$, where the universe of discourse is the set of real numbers?

*Solution*: Since "$x>3$" is true — for instance, when $x=4$ — the existential quantification of $P(x)$, which is $\exists xP(x)$, is true.

**生词与词组(一)**

assert[ə'səːt]v. 断言,主张
assertion[ə'səːʃən]n. 论断,断言
conjunction[kən'dʒʌŋkʃən]n. 合取
connective[kə'nektiv]n. 连词
disjunction[dis'dʒʌŋkʃən]n. 析取
exclusive[iks'kluːsiv]adj. 不可兼的
execute['eksikjuːt]v. 执行
false[fɔːls]adj. 假的
inclusive[in'kluːsiv]adj. 可兼的
phrase[freiz]v. 用语言表示
predicate['predikit]n. 谓词
preferable['prefərəbl]adj. 更可取的,更好的
proposition[ˌprɔpə'ziʃən]n. 命题
prepositional [ˌprɔpə'ziʃənəl] adj. 命题的
  prepositional connective 命题连词
  propositional function 命题函数
quantifier['kwɔntifaiə]n. 量词
  existential quantifier 存在量词
  universal quantifier 全称量词
quantification[ˌkwɔntifi'keiʃən] n. 量词化
  universal quantification 全称量词化
  existential quantification 存在量词化
statement['steitmənt]n. 语句
  subject of the statement 语句的主语
  assignment statement 赋值语句
truth value 真假值
universe['juːnivəːs]n. 通集,底集
  universe of discourse 论域,通集

**预习要求**

1. 预习生词与词组(一),浏览课文 A、B 并在疑难处做上记号。
2. 复习或借助词典学习以下单词、词组与短语的含义及用法:

(1) assert  (2) be assigned to sth.

(3) be encourtered in some place  (4) execute

(5) be phrased in term of sth.　　(6) produce sth. from sth.

(7) specify　　(8) universe of discourse

3. 复习分词短语的用法。

## 注释与说明

1. These statements are neither true nor false when the values of the variables are not specified. 注意在此句中 variables 前面有冠词 the 和无冠词时意义的差别. specify values 意为“指定取值”或“确定值”,简称“赋值”。这句可改变词序和语态译成:若未给语句中的所有变量赋值,则不能判断该语句是真是假。

2. 在逻辑运算中,主要的运算符号为“¬ ”,“ ∨ ”,“ ∧ ”和“→”,它们分别读作“not(译成‘非’)”,“or(译成‘或’)”,“and(译成‘与’)”和“implies(译成‘蕴涵’)”,它们构成四种基本运算。这四个符号分别称为否定符号,析取符号,合取符号与蕴涵符号,其含义可参阅第三章 §3.2.2。

3. Remark 之后的语句说明不用 any 这个词的原因。翻译时,其中例句最好先照抄,再附译文放在其后的括号中。即可译成:最好不用 any 这个词,因为它可能意指“每一个”或“某一个”,经常导致意义含糊不清。而在某些情况下,例如用于否定句时,如在句子“there is not any reason not to study hard”(没有任何不努力学习的理由)中,any 的含义却是清楚的。

## 课外作业

1. 把下列各组的单词、词组与短语译成英语,并按它们的相关性联想记忆:

(1) 语句、主语、谓词、连接词;量词、全称量词、存在量词。

(2) 命题、命题函数、命题连词;论断、论域、通集。

(3) 真的、假的、真假值、真值表;可兼的、不可兼的、明确的、含混不清的。

(4) 合取、析取、否定、蕴涵;量词化、全称量词化、存在量词化;推演规则。

(5) 包含变量的语句、由这样的命题产生、该语句被执行、断言某结论成立、由已知的命题构造新命题、任何不工作的理由。

2. 汉译英:

(1) 本节要讨论由这种语句生成命题的办法。

(2) 当一个命题函数的所有变量都赋上了值,所得到的语句就有了真假值。

(3) $P(x)$的全称量词化是这样一个命题:对论域中的所有 $x$,$P(x)$都是真的。把 $P(x)$的全称量词化记作 $\forall xP(x)$。

(4) 设 $P(x)$ 为语句“$x^2-2x+3>0$”。若论域为实数集,则全称量词化 $\forall xP(x)$的值是真的。

(5) $P(x)$的存在量词化是这样一个命题:在论域中存在一个元素 $x$ 使得

$P(x)$都是真的。把 $P(x)$的存在量词化记作 $\exists xP(x)$。

(6) 利用逻辑符号和量词,可以用逻辑表达式来表示英语语句,这在数学命题、逻辑编程和人工智能中特别有用。

(7) 如果对 $x$ 的这个值 $P(x)$是真的,则程序的语句 $x:=x-1$ 被执行,结果 $x$ 的值减少了 1.

(8) 这些例子说明,对于一个定理,可采用不止一种的好办法来证明。

3. 英译汉:

(1) Thus $p\leftrightarrow q$ is true if both $p$ and $q$ are true or if both $p$ and $q$ are false. The following are English-language equivalents to $p\leftrightarrow q$: "$p$ if and only if $q$," "$p$ is a necessary and sufficient condition for $q$," and "$p$ precisely if $q$."

It is worth emphasizing that the compound proposition $p\rightarrow q$ and its converse $q\rightarrow p$ are quire different; they have different truth tables.

(2) Two compound propositions $P$ and $Q$ are regarded as ***logically equivalent*** if they have the same truth values for all choices of truth values of the variables $p, q$, etc. In other words, the final columns of their truth tables are the same. When this occurs, we write $P\Leftrightarrow Q$. Since the table for $P\leftrightarrow Q$ has truth values true precisely where the truth values of $P$ and $Q$ agree, we see that

$$P\Leftrightarrow Q \text{ if and only if } P\leftrightarrow Q \text{ is a tautology.}$$

(3) We are typically faced with a set of hypotheses $H_1, \cdots, H_n$ from which we want to infer a conclusion $C$. One of the most natural sorts of proofs is the ***direct proof*** in which we show

$$H_1 \wedge H_2 \wedge \cdots \wedge H_n \Rightarrow C.$$

Many of the proofs that we gave in Chapter 1 — for example, proof about greatest common divisors and common multiples — were of this sort.

(4) Proofs that are not directive are called ***indirect***. The first type of indirect proof is ***proof of contrapositive***:

$$\neg C \Rightarrow \neg(H_1 \wedge H_2 \wedge \cdots \wedge H_n).$$

Another type of indirect proof is a ***proof by contradiction***:

$$(H_1 \wedge H_2 \wedge \cdots \wedge H_n) \wedge \neg C \Rightarrow \text{a contradiction.}$$

4. 自学课文 11-C 并将它译成汉语:

## 课文 11-C Translation sentences into logical expression

In Section 1.1 we illustrated the process of translating English sentences into logical expressions involving propositions and logical connectives. Now that we have discussed quantifiers, we can express a wider variety of English sentences using logical expressions. Doing so eliminates ambiguity and makes it possible to reason with these sentences. (Section 3.1 covers rules of inference for reasoning with logical expressions.)

The following examples show how to use logical operators and quantifiers to express English sentences, similar to the kind that occur frequently in mathematical statements, in logic programming, and in artificial intelligence.

EXAMPLE 7. Express the statements "Some student in this class has visited Mexico" and "Every student in this class has visited either Canada or Mexico" using quantifiers.

*Solution.* Let the universe of discourse for the variable $x$ be the set of students in your class. Let $M(x)$ be the statement "$x$ has visited Mexico" and $C(x)$ the statement "$x$ has visited Canada". The statement "Some student in this class has visited Mexico" can be written as $\exists x M(x)$. The statement "Every student in this class has visited either Canada or Mexico" can be written as $\forall x(C(x) \vee M(x))$ (assuming that the inclusive, rather than the exclusive, or is what is meant here).

EXAMPLE 8. Express the statement "Everyone has exactly one best friend" as a logical expression.

*Solution.* Let $B(x,y)$ be the statement "$y$ is the best friend of $x$". To translate the sentence in the example, note that it says that for every person $x$ there is another person $y$ such that $y$ is the best friend of $x$ and that if $z$ is a person other than $y$, then $z$ is not the best friend of $x$. Consequently, we can translate the sentence as

$$\forall x \exists y \forall z(B(x,y) \wedge ((z \neq y) \rightarrow \neg B(x,z))).$$

EXAMPLE 9. Express the statement "If somebody is female and is a parent, then this person is someone's mother" as a logical expression.

*Solution.* Let $F(x)$ be the statement "$x$ is female", let $P(x)$ be the statement "$x$ is a parent", and let $M(x,y)$ be the statement "$x$ is the mother of $y$". Since the statement in the example pertains to all people, we can write it symbolically as

$$\forall x((F(x) \wedge P(x) \rightarrow \neg \exists y M(x,y)).$$

EXAMPLE 10. Use quantifiers to express the statement "There is a woman who has taken a flight on every airline in the world".

*Solution.* Let $P(w,f)$ be "$w$ has taken $f$" and $Q(f,a)$ be "$f$ is a flight on $a$." We can express the statement as

$$\exists w \forall a \exists f(P(w,f) \wedge Q(f,a)),$$

where the universes of discourse for $w$, $f$, and $a$ consist of all the women in the world, all airplane flights, and all airlines, respectively.

The statement could also be expressed as

$$\exists w \forall a \exists f \quad R(w,f,a),$$

where $R(w,f,a)$ is "$w$ has taken $f$ on $a$". Although this is more compact, it somewhat obscures the relationships between the variables. Consequently, the first solution is usually preferable.

As mentioned earlier, quantifiers are often used in the definition of mathematical concepts. One example that you may be familiar with is the concept of limit, which is important in calculus.

EXAMPLE 11. (Calculus required) Express the definition of a limit using quantifiers.

*Solution.* Recall that the definition of the statement

$$\lim_{x \to a} f(x) = L$$

is: For every real number $\varepsilon > 0$ there exists a real number $\delta > 0$ such that $|f(x) - L| < \varepsilon$ whenever $0 < |x - a| < \delta$. This definition of a limit can be phrased in terms of quantifiers by

$$\forall \varepsilon \exists \delta \forall x (0 < |x-a| < \delta \rightarrow |f(x) - L| < \varepsilon),$$

where the universe of discourse for the variables $\varepsilon$ and $\delta$ is the set of positive real numbers and for $x$ is the set of real numbers.

This definition can also be expressed as

$$\forall \varepsilon > 0 \exists \delta > 0 \forall x (0 < |x-a| < \delta \rightarrow |f(x) - L| < \varepsilon),$$

where the universe of discourse for the variables $\varepsilon$ and $\delta$ is the set of all real numbers, rather than the set of positive real numbers.

## 生词与词组(二)

ambiguous[æm'bigjuəs] adj. 二义的，不清楚的

unambiguous[ˌʌnæm'bigjuəs] adj. 明确的，清楚的

artificial intelligence 人工智能

compound proposition 复合命题

eliminate[i'limineit] v. 消除

emphasize['emfəsaiz] v. 强调

obscure[əb'skjuə] v. 使得难理解

pertain to 与……有关，属于……

preferable['prefərəbl] adj. 更可取的

infer[in'fəː] v. 推断、推导

inference['infərəns] n. 推断、推导

rule of inference 推导的规则

proof by contradiction 矛盾证明，反证法

proof of contrapositive 反向证明

tautology[tɔː'tɔlədʒi] n. 重言式

## 插页:数学珍言

**数学家是怎样的一种人?**

近代名人,包括哲学家、科学家、数学家本身以至于皇帝,都从不同的角度提出自己的看法。

吉恩斯(J. H. Jeans)说:

The great Architect of the Universe now begins to appear as a pure mathematician.

(宇宙的伟大建筑师现在开始以纯粹数学家的身份出现。)

以提出爱尔朗根纲领(Erlanger Program)而闻名的数学家克莱因(F. Klein, 1849~1925)认为:

The greatest mathematicians, as Archimedes, Newton and Gauss, always united theory and application in equal measure.

(如阿基米得、牛顿与高斯这样的最伟大的数学家,总是不偏不倚地把理论与应用结合起来。)

凯瑟(C. J. Keyser)在回忆连续变换群理论的奠基人索福斯·李(Sophus Lie,1842~1899)时说:

When the late Sophus Lie was asked to name the characteristic endowment of the mathematician, his answer was the following quaternion: fancy, energy, self-confident, self-criticism.

(已故的索福斯·李曾被问道,什么是数学家所特有的禀赋。他回答说是下面这个四元组:想象力、干劲、自信心与自我批评。)

"现代分析学之父"魏尔斯特拉斯关于数学家的秉赋有如下一段言简意赅、常被人引用的名言:

A mathematician, who is not also something of a poet, will never be a complete mathematician.

(没有一些诗人气质的数学家绝对不是一位完全的数学家。)

法国皇帝拿破仑对当时的数学家拉格朗日的评价是:

Lagrange is the lofty pyramid of mathematical sciences.

(拉格朗日是数学科学中高耸的金字塔。)

# §2.12 概率论与数理统计
# (Probability Theory and Mathematical Statistics)

## 课文 12-A Special terminology peculiar to probability theory ①

In discussions involving probability, one often sees phrases from everyday language such as "two events are equally likely," "an event is impossible," or "an event is certain to occur." Expressions of this sort have intuitive appeal and it is both pleasant and helpful to be able to employ such colorful language in mathematical discussions. Before we can do so, however, it is necessary to explain the meaning of this language in terms of the fundamental concepts of our theory.

Because of the way probability is used in practice, it is convenient to imagine that each probability space $(S, \boldsymbol{B}, P)$ is associated with a real or conceptual experiment. The universal set $S$ can then be thought of as the collection of all conceivable outcomes of the experiment, as in the example of coin tossing discussed in the foregoing section. Each element of $S$ is called an *outcome* or a *sample* and the subsets of $S$ that occur in the Boolean algebra $\boldsymbol{B}$ are called *events*. The reasons for this terminology will become more apparent when we treat some examples.

Assume we have a probability space $(S, \boldsymbol{B}, P)$ associated with an experiment. Let $A$ be an event, and suppose the experiment is performed and that its outcome is $x$. (In other words, let $x$ be a point of $S$.) This outcome $x$ may or may not belong to the set $A$. If it does, we say that the *event $A$ has occurred*. Otherwise, we say that the event *$A$ has not occurred*, in which case $x \in A'$, so the complementary event $A'$ has occurred. An event $A$ is called *impossible* if $A = \varnothing$, because in this case no outcome of the experiment can be an element of $A$. The event $A$ is said to be *certain* if $A = S$, because then every outcome is automatically an element of $A$.

Each event $A$ has a probability $P(A)$ assigned to it by the probability function $P$. [The actual value of $P(A)$ or the manner in which $P(A)$ is assigned does not concern us at present.] The number $P(A)$ is also called the *probability that an outcome of the experiment is one of the elements of $A$*[1]. We also say that $P(A)$ is *the probability that the event $A$ occurs* when the experiment is performed.

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① 课文 12-A 摘自:T. M. Apostol. Calculus, Vol. 2. New York: John Wiley & Sohn Inc. 1969.

The impossible event $\varnothing$ must be assigned probability zero because $P$ is a finitely additive measure. However, there may be events with probability zero that are not impossible. In other words, some of the nonempty subsets of $S$ may be assigned probability zero. The certain event $S$ must be assigned probability 1 by the very definition of probability, but there may be other subsets as well that are assigned probability 1. In Example 1 of Section 6.8 there are nonempty subsets with probability zero and proper subsets of $S$ that have probability 1.

Two events $A$ and $B$ are said to be *equally likely* if $P(A)=P(B)$. The event $A$ is called *more likely* than $B$ if $P(A)>P(B)$, and *at least as likely as* $B$ if $P(A) \geqslant P(B)$. Table 2-12-1 provides a glossary of further everyday language that is often used in probability discussions. The letters $A$ and $B$ represent events, and $x$ represents an outcome of an experiment associated with the sample space $S$. Each entry in the left-hand column is a statement about the events $A$ and $B$, and the corresponding entry in the right-hand column defines the statement in terms of set theory.

**TABLE 2-12-1. Glossary of Probability Terms**

| Statement | Meaning in set theory |
|---|---|
| At least one of $A$ or $B$ occurs | $x \in A \cup B$ |
| Both events $A$ and $B$ occur | $x \in A \cap B$ |
| Neither $A$ nor $B$ occurs | $x \in A' \cap B'$ |
| $A$ occurs and $B$ does not occur | $x \in A \cap B'$ |
| Exactly one of $A$ or $B$ occurs | $x \in (A \cap B') \cup (A' \cap B)$ |
| Not more than one of $A$ or $B$ occurs[2] | $x \in (A \cap B)'$ |
| If $A$ occurs, so does $B$ ($A$ implies $B$) | $A \subseteq B$ |
| $A$ and $B$ are mutually exclusive | $A \cap B = \varnothing$ |
| Event $A$ or event $B$ | $A \cup B$ |
| Event $A$ and event $B$ | $A \cap B$ |

## 课文 12-B Two basic statistics concepts — population and sample ①

In the preceding sections, we cited a few examples of situations where evaluation of factual information is essential for acquiring new knowledge. Although these exam-

① 课文 12-B 取自：R. A. Johnson & G. K. Bhattacharyya. Statistics — Principles and Methods, 3rd Edition, New York: John Wiley & Sons, 1996.

ples are drawn from widely differing fields and only sketchy descriptions of the scope and objectives of the studies are provided, a few common characteristics are readily discernible.

First, in order to acquire new knowledge, relevant data must be collected. Second, some amount of variability in the data is unavoidable even though observations are made under the same or closely similar conditions. For instance, the treatment for an allergy may provide long-lasting relief for some individuals whereas it may bring only transient relief or even none at all to others[3]. Likewise, it is unrealistic to expect that college freshmen whose high school records were alike would perform equally well in college. Nature does not follow such a rigid law[4].

A third notable feature is that access to a complete set of data is either physically impossible or from a practical standpoint not feasible. When data are obtained from laboratory experiments or field trials, no matter how much experimentation has been performed, more can always be done. In public opinion or consumer expenditure studies, a complete body of information would emerge only if data were gathered from every individual in the nation — undoubtedly a monumental if not an impossible task[5]. To collect an exhaustive set of data related to the damage sustained by all cars of a particular model under collision at a specified speed, every car of that model coming off the production lines would have to be subjected to a collision![6] Thus, the limitations of time, resources, and facilities, and sometimes the destructive nature of the testing, mean that we must work with incomplete information — the data that are actually collected in the course of an experimental study.

The preceding discussions highlight a distinction between the data set that is actually acquired through the process of observation and the vast collection of all potential observations that can be conceived in a given context. The statistical name for the former is sample; for the latter, it is population, or statistical population. To further elucidate these concepts, we observe that each measurement in a data set originates from a distinct source which may be a patient, tree, farm, household, or some other entity depending on the object of a study. The source of each measurement is called a *sampling unit*, or simply, a *unit*. A *sample* or *sample data set* then consists of measurements recorded for those units that are actually observed. The observed units constitute a part of a far larger collection about which we wish to make inferences. The set of measurements that would result if all the units in the larger collection could be observed is defined as the population.

**Definition 1** A statistical *population* is the set of measurements (or record of some qualitative trait) corresponding to the entire collection of units about which information is sought[7].

The population represents the target of an investigation. We learn about the population by taking a sample from the population.

**Definition 2** A *sample* from a statistical population is the set of measurements that are actually collected in the course of an investigation.

## 生词与词组(一)

allergy['ælədʒi]n. 过敏,厌恶
Boolean algebra 布尔代数
destructive nature 破坏性,有害的性质
discernible[di'səːnəbl]adj. 可识别的
elucidate[i'luːsideit]v. 阐明,解释
equally likely 同等可能的
event[i'vent]n. 事件
  certain event 必然事件
  complementary event 余事件
  impossible event 不可能事件
exhaustive[ig'zɔːstiv]adj. 穷竭的,穷尽的
expenditure [iks'penditʃə] n. 消费,支出
feasible['fiːzəbl]adi. 可行的
finitely additive measure 有限可加测度
highlight['hailait]v. 强调,凸显;n. 最显著部分
inference['infərəns]n. 推测,推断
monumental[ˌmɔnju'mentl] adj. 非常大的
mutually exclusive 互斥的
outcome['autkʌm]n. 结果,结局
potential [pə'tenʃəl] adj. 潜在的;n. 位势
population[ˌpɔpju'leiʃən]n. 总体(个体总数),人口
probability[ˌprɔbə'biliti]n. 概率
  probability space 概率空间
  probability theory 概率论
  probability zero 零概率
relief[ri'liːf]n. (痛苦等)缓和,减轻
rigid law 刚性定律
sample['sæmpl]n. 样本
  sampling unit 样本单位,样本个体
statistics[stə'tistiks]n. 统计学
  mathematical statistics 数理统计
sustain[sə'stein]v. 持续,支持
transient['trænziənt] adj. 瞬时的,过渡的
trait[treit]n. 特征,特性
trial['traiəl]n. 试验,试用
universal set 通集,全集
unrealistic['ʌnriə'listik]adj. 非现实的
variability [ˌvɛəriə'biliti] n. 可变性,差异

## 预习要求

1. 预习生词与词组(一),浏览课文 A、B 并在疑难处做上记号。
2. 复习或借助词典学习以下单词、词组与短语的含义及用法:

(1) be acquired through sth.
(2) closely similar
(3) coin tossing
(4) complete body of sth.
(5) conceive
(6) draw from sth.
(7) field trials
(8) learn about sth.
(9) make inference
(10) originate from sth.
(11) particular to sth.
(12) perform an experiment
(13) be subjected to sth./sb.
(14) sustain
(15) the very definition

3. 复习介词和数词的用法。

**注释与说明**

1. The number $P(A)$ is also called the probability that an outcome of the experiment is one of the elements of $A$. 本句由 that 引起一个定语从句,用于修饰 probability。整句可译成:数 $P(A)$ 又称为试验的结果,是 $A$ 的一个元素的概率。

2. "Not more than one of $A$ or $B$ occurs"的含义是"事件 $A$ 与 $B$ 至多只有一件发生"。

3. For instance, the treatment for an allergy may provide long-lasting relief for some individuals whereas it may bring only transient relief or even none at all to others. 本句由 whereas 引导一个起对比作用的从句,其中 none 是 bring 的宾语的一部分, at all 修饰 none。整句可译成:例如,对一种过敏的治疗可以使某些人得到长时间的缓解,但对于其他人可能只是短时间的缓解或者完全无效。

4. Likewise, it is unrealistic to expect that college freshmen whose high school records were alike would perform equally well in college. Nature does not follow such a rigid law. 这两句可译成:类似地,希望那些在高中时成绩一样的大学一年级新生在大学中表现也一样好是不现实的。自然界不会遵循这样一个固定不变的规律运行。

5. In public opinion or consumer expenditure studies, a complete body of information would emerge only if data were gathered from every individual in the nation — undoubtedly a monumental if not an impossible task. 这个虚拟语句结构较复杂,直译不易。可改变句子结构,译成:在公众意见或顾客消费的统计研究中,要想得到一个完全的信息体,必须收集到该国家的每一个体的数据。无疑地,这个任务即使可能完成,也是极其庞大的。

6. To collect an exhaustive set of data related to the damage sustained by all cars of a particular model under collision at a specified speed, every car of that model coming off the production lines would have to be subjected to a collision! 这句也是

虚拟语句，句子较长，可译成：如果要完全地收集某种特殊型号的汽车在一定速度之下碰撞时的耐损能力的有关数据，那么来自该生产线的该型号的每一辆汽车都要接受一场碰撞。

7. A statistical *population* is the set of measurements (or record of some qualitative trait) corresponding to the entire collection of units about which information is sought. 这个定义很长，corresponding 引起的分词短语做定语，短语中又含有一个从句。整句可译成：统计的总体是与有待于收集信息的个体组成的整体相对应的，由观测数据（或关于某种特性的记录）组成的集。

**课外作业**

1. 把下列各组的单词、词组与短语译成英语，并按它们的相关性联想记忆：

(1) 概率、概率论、概率空间、零概率、有限可加测度、全集；事件、必然事件、不可能事件、同等可能、互斥的；试验、观测、结局；布尔代数。

(2) 统计、数理统计、样本、样本单位、总体、个体；推断、差异、可识别的。

(3) 特性、破坏性、刚性定律。

(4) 可以想象的、非现实的、非常大的、可行的、穷尽的、瞬时的。

(5) 抛硬币、减轻过敏症状、凸显特性，从总体中取出样本，研究的目标，从每个个体汇总，非常类似的条件，与一个试验相关联的概率空间。

2. 汉译英：

(1) 根据概率论实际应用的方式，把概率空间对应于一个实际的或概念上的试验。

(2) 在试验中事件 $A$ 出现的概率记作 $P(A)$。

(3) 若 $P(A)>P(B)$，则称事件 $A$ 比事件 $B$ 有更大的可能性；如果 $P(A)\geq P(B)$，则称事件 $A$ 至少和事件 $B$ 的可能性一样大。

(4) 仅根据概率的定义就得把必然事件的概率指定为 1，但是可能有别的子集其概率也为 1。

(5) 上述的讨论凸显了两种数据集之间的差异。

(6) 统计的总体既可以看成是由研究对象全体组成的集，也可以看成是这个集所对应的观测数据集。

(7) 统计总体的样本是在一个研究过程中由实际收集的测量数据组成的集合。

(8) 总体代表研究的目标。我们从总体中抽取样本来了解总体。

3. 英译汉：

(1) The general rule for the addition of probabilities is

$$P(A\cup B)=P(A)+P(B)-P(A\cap B),$$

where $P(A \cup B)$ is the probability that either event $A$ or $B$ occurs (at list one of $A$ and $B$ occurs), and $P(A \cap B)$ is the probability that event $A$ and $B$ both occur at the same time.

(2) Two events are said to be mutually exclusive when both cannot happen at the same time. If $E$ denotes an event, then the event that $E$ does not happen is called the ***complementary event*** of $E$, and is denoted by $E'$. Obviously, $E$ and $E'$ are mutually exclusive. Since an event either happens or does not happen, the sum of their probabilities is 1.

(3) Given a set of ungrouped data $x_1, x_2, \cdots, x_n$, and let $\bar{x}$ denote the mean of the data, the ***mean deviation*** of the data is defined by:

$$\text{mean deviation} = \frac{\sum |x - \bar{x}|}{n},$$

where $n$ is the number of data.

(4) We shall use the Greek letter $\mu$ to denote the arithmetic mean of a population and the Greek letter $\sigma$ to denote the standard deviation of a population. In a normal distribution almost all of the population lies between the values of variable that are four standard deviation ($4\sigma$) either side of the arithmetic mean ($\mu$) of the distribution. In fact, the range $\mu-4\sigma$ to $\mu+4\sigma$ spans 0.9999 or 99.99% of the population.

4. 阅读并翻译课文 12-C:

## 课文 12-C Some basic principles of combinatorial analysis ①

Many problems in probability theory and in other branches of mathematics can be reduced to problems on counting the number of elements in a finite set. Systematic methods for studying such problems form part of a mathematical discipline known as *combinatorial analysis*. In this section we digress briefly to discuss some basic ideas in combinatorial analysis that are useful in analyzing some of the more complicated problems of probability theory.

If all the elements of a finite set are displayed before us, there is usually no difficulty in counting their total number. More often than not, however, a set is described in a way that makes it impossible or undesirable to display all its elements. For example, we might ask for the total number of distinct bridge hands that can be dealt. Each player is dealt 13 cards from a 52-card deck. The number of possible

① 课文 12-C 摘自:T. M. Apostol. Calculus, Vol. 2. New York: John Wiley & Sohn Inc. 1969.

distinct hands is the same as the number of different subsets of 13 elements that can be formed from a set of 52 elements. Since this number exceeds 635 billion, a direct enumeration of all the possibilities is clearly not the best way to attack this problem; however, it can readily be solved by combinatorial analysis.

This problem is a special case of the more general problem of counting the number of distinct subsets of $k$ elements that may be formed from a set of $n$ elements (When we say that a set has $n$ elements, we mean that it has $n$ distinct elements. Such a set is sometimes called an $n$-element set.), where $n \geqslant k$. Let us denote this number by $f(n,k)$. It has long been known that

$$f(n,k)=\binom{n}{k}, \tag{12.1}$$

where, as usual $\binom{n}{k}$ denotes the binomial coefficient,

$$\binom{n}{k}=\frac{n!}{k!\,(n-k)!}.$$

In the problem of bridge hands we have $f(52,13)=\binom{52}{13}=635{,}013{,}559{,}600$ different hands that a player can be dealt.

There are many methods known for proving (12.1). A straightforward approach is to form each subset of $k$ elements by choosing the elements one at a time. There are $n$ possibilities for the first choice, $n-1$ possibilities for the second choice, and $n-(k-1)$ possibilities for the $k$th choice. If we make all possible choices in this manner we obtain a total of

$$n(n-1)\cdots(n-k+1)=\frac{n!}{(n-k)!}$$

subsets of $k$ elements. Of course, these subsets arc not all distinct. For example, if $k=3$ the six subsets

$$\{a,b,c\},\{b,c,a\},\{c,a,b\},\{a,c,b\},\{c,b,a\},\{b,a,c\}$$

are all equal. In general, this method of enumeration counts each $k$-element subset exactly $k!$ times. Therefore we must divide the number $n!/(n-k)!$ by $k!$ to obtain $f(n,k)$. This gives us $f(n,k)=\binom{n}{k}$, as asserted.

## 生词与词组(二)

binomial coefficient 二项式系数　　　　bridge[bridʒ]n. 桥,桥牌

combinatorial analysis 组合分析

commensurability[kəˌmenʃərəˈbiliti] n. 均匀

complementary event 补事件

deck[dek] n. 一副(纸牌);甲板,底层

deviation[ˌdiːviˈeiʃən] n. 偏差

mean deviation 平均偏差

standard deviation 标准差

digress[daiˈgres] v. 偏离(主题)

distribution[ˌdistriˈbjuːʃən] n. 分布

enumeration[iˌnjuːməˈreiʃən] n. 枚举,计数

mean[miːn] n. 平均,平均值

normal distribution 正态分布

precision[priˈsiʒən] n. 明确,精密

span[spæn] v. 张成,支撑

straightforward approach 直接的方法

ungrouped data 未分类数据

**插页:名家谈数学之美**

公元前4世纪,被誉为"百科全书"的科学家阿里士多德(Aristotle)说:

Those who assert that the mathematical sciences say nothing of the beautiful are in error. The chief forms of beauty are order, commensurability and precision.

(译文:认为数学科学无美可言的人是错误的。美的主要形式是次序、均匀与明确。)

当代公认的"计算机之父"冯.诺依曼(Von. Neumann)说:

I think it is correct to say (the mathematician's) criteria of selection, and also those of success, are mainly aesthetical.

(译文:我认为,说数学家选择课题的准则以及判断他是否成功的准则,主要是美的准则,这是正确的。)

近代法国杰出的数学家庞加莱(J. H. Poincaré)说:

The feeling of mathematical beauty, of the harmony of numbers and forms, of geometric elegance. This is a true aesthetic feeling that all real mathematicians know.

(译文:感觉到数学的美,感觉到数与形的协调,感觉到几何的优雅,这是所有真正的数学家都清楚的美的感觉。)

# 附录 1　基本运算符号与算式的读法

首先指出：表示数或量的字母，特别是出现在算式中字母（字母变量通常以斜体字出现，已知其值的字母常数通常以正体形式出现）均按该字母的名称读。例如，英文字母 $A$（或 $a$）和希腊字母 $\pi$ 分别按国际音标[ei]和[pai]读。

下面只列出部分的基本运算符号与算式的读法。更详细的资料可参看专门文献，如书末列出的参考文献[17]；本书的 §2.4 课文 4-C 等一些相关章节也有部分介绍。

## 一、四则运算

1. 加法（Addition）

加号"+"读作 plus。例如，1+2 读作"one plus two"；$a+3$ 读作"$a$ plus three"。

**注**：当"+"看作正号时，可读作"plus 或 positive"，如+5 读作"positive five"或"plus five"。

2. 减法（Subtraction）

减号"-"读作 minus。例如，3-2 读作"three minus two"；$5-b$ 读作"five minus $b$"。

**注**：当"-"看作负号时，可读作"minus"或"negative"，如 -5 读作"negative five"或"minus five"。

3. 乘法（Multiplication）

乘号"×"、"·"或省略均读作"times"或"multiplied by"。例如，3×2 读作"three times two"或"three multiplied by two"；$a \cdot b$ 或 $ab$ 都读作："$a$ times $b$"或"$a$ multiplied by $b$"。

4. 除法（Divition）

除号"÷"读作"divided by"；用"/"或"—"表示除法时读作"over"或"divided by"。例如，10÷4 读作"ten over four"或"ten divided by four"；$a/b$ 或 $\frac{a}{b}$ 读作"$a$ over $b$"或"$a$ divided by $b$"。

**注**：分数（式）的读法比较复杂，把 $\frac{a}{b}$ 看作分数（式）时通常读作"$a$ over $b$"，但有许多具体情形采用其他读法，如：$\frac{1}{4}$ 读作"a quarter"或"fourth"；$\frac{3}{4}$ 读作"three quarters"等。更详细的介绍见 §2.4 课文 4-C。

5. 比例（Proportion）

比例符号“:”读作“is to”。例如，$a:b$ 读作“$a$ is to $b$”；有时读作“the ratio of $a$ to $b$”。

6．乘方(Powers)

$x$ 的 $n$ 次方 $x^n$ 一般读作“$x$ to the $n$th power”。例如，$ab^n$ 读作“$a$ times $b$ to the $n$th power”；$b^{n-1}$ 读作“$b$ to the $n$ minus one power”；$b^{m+n}$ 读作“$b$ to the $m$ plus $n$ power”。但是，当 $n=2,3$ 时读法不同：$x^2$ 读作“$x$ squared”或“$x$ square”；$x^3$ 读作“$x$ cubed 或 $x$ cube”。

7．开方(Root-extracting)

$x$ 的 $n$ 次方根 $\sqrt[n]{x}$（或 $x^{\frac{1}{n}}$）一般读作“the $n$th root of $x$”，例如，$\sqrt[5]{a^2}$ 读作“the fifth root of $a$ squared”。当 $n=2,3$ 时读法不同：$\sqrt{x}$（或 $x^{\frac{1}{2}}$）读作“the square root of $x$”，$\sqrt[3]{x}$（或 $x^{\frac{1}{3}}$）读作“the cube root of $x$”。

## 二、大小关系(Comparison of quantities)

等于：　$a=b$ 读作“$a$ is equal to $b$”或“$a$ equals $b$”，也可读作“$a$ is $b$”；
$x+2=7$ 读作“$x$ plus 2 is equal to 7”或“$x$ plus 2 equals 7”，也可读作“$x$ plus 2 is 7”。

不等于：　$a\neq b$ 读作“$a$ is not equal to $b$”或“$a$ does not equal $b$”，也可读作“$a$ is not $b$”。

恒等于：　$a\equiv b$ 读作“$a$ is identical to $b$”或“$a$ is identical with $b$”。

近似等于：$a\approx b$ 读作“$a$ is approximately equal to $b$”。
$\pi\approx3.14$ 读作“$\pi$ is approximately equal to three point one four”。

小于：　$a<b$ 读作“$a$ is less than $b$”；$3+a<m-3$ 读作“3 plus $a$ is less than $m$ minus 3”。

大于：　$a>b$ 读作“$a$ is greater than $b$”。

小于或等于：　$a\leqslant b$ 读作“$a$ is less than or equal to $b$”。

大于或等于：　$a\geqslant b$ 读作“$a$ is greater than or equal to $b$”。

## 三、括号(Signs of grouping)

圆括号(　　)、方括号[　　]、大括号{　　}分别读作“parentheses”(或“round brackets”)、“square brackets”、“braces”。

括号的完整读法比较麻烦，如$(a+b)$，先读左半括号(open parenthesis)，再读 $a+b$，最后读右半括号(close parenthesis)，整个式子读作“open parenthesis $a$ plus $b$ close parenthesis”。为了简便，$(a+b)$ 可改读成“the quantity $a$ plus $b$”。圆括号换成方括号、大括号时的情形类似。

更进一步的简化是省去 the quantity，如：

$(a+b)(a-b)$读作"$a$ plus $b$ into $a$ minus $b$"。(into 代表"乘")(参见 §2.4 课文 4-C)。

## 四、特殊求值(Special evaluation)

绝对值(Absolute value)：$|x|$读作"the absolute value of $x$"。

最大值(Maximun value)：$\max f(x)$读作"the maximun value of $f(x)$"，$\max\{x_1,\cdots x_n\}$读作"the maximun value of the series $x$ sub one to sub $n$"。

最小值(Minimum value)：$\min f(x)$读作"the minimun value of $f(x)$"，$\mathrm{mix}\{x_1,\cdots,x_n\}$读作"the minimun value of the series $x$ sub one to sub $n$"。

求累加和(the sum of the terms indicated)：$\sum\limits_{k=1}^{n} a_k$读作"Capital sigma $a_n$ from $k$ equals one to $k$ equals $n$"或"Sum of all $a_n$ from $k$ equals one to $n$"；

$\sum\limits_{k=1}^{\infty} a_k$读作"Capital sigma $a_n$ from $k$ equals one to $k$ equals infinity"或"Sum of all $a_n$ from $k$ equals one to infinity"。

求连乘积(the product of the terms indicated)：$n!$ 读作"Factorial $n$"；$\prod\limits_{k=1}^{n} b_k$读作"Product of all $b_n$ from $k$ equals one to $n$"；$\prod\limits_{k=1}^{\infty} b_k$读作"Product of all $b_n$ from $k$ equals one to infinity".

**注：小数点的读法可参见 §2.4 课文 4-C。**

# 第三章　专业文选——进阶需读

本章共六节，每一节含有若干小节。各节介绍的数学内容不同，写法风格也不尽相同。第一节是科技图书的序言与教学指导；第二节是数学基础与数学方法，介绍集合、关系、映射等，并用一个小节介绍数学教与学的方法；第三节是高等数学的内容，包括高等代数、抽象代数、解析几何及函数论的部分基础知识；第四节是数学的应用与应用数学的部分知识，包括微分方程、图论和线性规划及数学模型的构建等方面；第五节是计算数学与计算机科学的基本知识，包括数值计算、微分方程数值解、人工智能和计算理论简介、P与NP问题等方面；第六节介绍数学新的学科分支——分形几何、模糊数学与小波分析的有关知识。

本章可作为泛读的材料供教学选用。为了帮助初学者，每一小节的末尾给出"学习要求"，前两节还给出"注释与说明"，供读者阅读时参考。

与第二章相比较，本章各篇短文所介绍的数学内容专业水平较高，英语的难度也加大了。在此有两点意见供读者参考：

(1) 作为泛读的材料，学习要求可深可浅。读者可首先大体了解全文的中心内容和主要论点，然后根据需要逐步加细。比如，按层次先搞清楚每一个部分(段)的中心意思是什么，再进一步掌握更细更精确的内容。必要时可参照每小节末的"学习要求"，进行自我测试。

(2) 初学者不要急于求成，希望自己在几天之内就全部掌握，而应根据自己的实际水平有所选择地分批分期学习(若有良师指导则更佳)，从自己比较熟悉的内容开始。例如从§3.2和§3.3中那些具体数学内容较为熟悉的短文开始，然后逐步扩展，循序渐进。当然，不论从何开始，都应该把英语和数学结合起来学习。在许多情况下，读者要正确理解一句或一段英语，必须先理解其中的数学含义以及它们与上下文的逻辑关系，并作必要的逻辑推理。

# §3.1 科技图书的序言

一本科技图书通常在开头都有序言(preface,或称为前言),它像一出戏的开场白。通常作者在序言中要介绍本书的写作目的、主要内容、显著特色和读者对象。为了有针对性地获取自己需要的信息、资料和知识,读者应该学会阅读序言。

由于序言也是书的门面,所以许多作者不仅在这里介绍有关的内容,而且想趁机展示一下自己的文采,发表一下自己的意见。因此序言常被写得近似文学作品中的短评,用词丰富灵活,语句生动活泼,表达方式多样,这难免会使许多中国读者感到吃力。与它相比,不少人感到阅读正文的专业知识是一件较容易的事。其实,由于专业内容具有连贯性,阅读时可借助于上、下文来推测、识别和理解,并可从公式、图表获取重要信息,再加上专业用词相对集中,叙述的方式和句型相对固定等因素,故阅读起来相对轻松。

据此,我们应了解:序言的大致内容是什么?是否也有相对固定的用语和表达方式?如果我们能抓住序言的特点和表达规律,就能达到事半功倍的效果。应该说明,要很好地抓住这种特点和规律,只能靠读者在不断练习中摸索。这里仅给出一些供参考的信息。

## 一、序言的基本内容

序言的写作风格因人而异,内容也没有统一要求。但总体上可把内容概括为三大部分:

第一部分:作者的开场白,可包含三个内容:

1. 该书的写作目的与读者范围(指从事什么专业,具备哪些基础知识)。如下面的 Preface 1 中,作者称"The main purpose of our text, then, is to make learning about harmonic functions easier."接下去就写了读者应具备的基础知识。

2. 该书的特色(即与其他书不同之处,主要是优点以及作者的创造性工作)。

3. 该学科近年来的发展概况或编写该书的背景及过程等。

第二部分:该书各章内容简介。通常介绍各不同章节时采用的表达方式(语句)不同,以避免太过单一枯燥,但这就增加了中国初学者阅读英文的困难。

第三部分:作者对支持、帮助该书写作的有关人员致谢。

以上各部分在不同的序言中所占篇幅可多可少,也可省略,或另加作者感兴趣的其他内容。读者不难从阅读本节的两篇序言和一篇读书指导中得到一些基本印象。

## 二、若干常见用语和句型举例

读者在阅读序言时，一方面要注意用词的灵活性和词义的广泛性，摆脱专业用词和词义的框框，例如，不要一见到 current，就以为是电流，一见到 integrate，就以为是要对什么积分。另一方面要努力掌握序言中常见词汇和表达形式，不断积累总结。这样，就能较快提高阅读水平。下面是一些例子：

1. “读者应具有的基础知识”的英语句型

(a) The only prerequisite for this book is ….

这里“prerequisite”意指“应具有的准备条件”。

(b) It is assumed that the reader is familiar with ….

这里 be assumed 意为：本书把读者的基础“假定为”(已掌握……方面的知识)。

(c) A grounding in calculus is assumed. 意为：假定读者已具有微积分的基础知识。

(d) A prior exposure to advanced mathematics is not required. 意为：先前没有学过高等数学的读者也能阅读本书(不要求读者此前学过高等数学)。

2. 作者打算或不打算达到什么目的常用动词

作者打算或不打算达到什么目的或做什么事常用带有“attempt to”的句型。见下面给出的两篇前言。

3. 重复出现的同一个意思或概念的多种表达

同一篇序言中，同一个意思或概念若出现多次，其用词和表达方式常不同。例如，对“论述”，“叙述”，可分别选用 describe，treat，concern，discuss，deal with，present，cover 等，并采用不同句型表达。又如，可能先后出现的“this book”，“this text”，“the material”等都表示“本书”。

4. 表示“归功于”或感谢的单词和语句甚多

序言常出现 be due to…的句型。be due to 有两种含义，一是表示“由于”，“归功于”(见后面的 Preface 2)；二是表示感谢。它出现在序言中时多数属于后者。

下面几个例子都是表示感谢的句子：

(1) A special debt of gratitude is due to Prof. Wang.

(2) The author would like to express his deep gratitude to Mr. Brown.

(3) The author takes great pleasure in thanking Doctor Li for his helpful suggestions.

### 3.1.1 *Preface 1* ①

**编者按:**这是一本书名为 *The theory of harmonic functions* 的教材的序言。

Harmonic functions—the solutions of Laplace's equation—play a crucial role in many areas of mathematics, physics, and engineering. But learning about them is not always easy. At times each of the authors has agreed with Lord Kelvin and Peter Tail, who wrote ([12], Preface[1])

> There can be but one opinion as to the beauty and utility of this analysis of Laplace; but the manner in which it has been hitherto presented has seemed repulsive to the ablest mathematicians, and difficult to ordinary mathematical students[2].

The quotation has been included mostly for the sake of amusement, but it does convey a sense of the difficulties the uninitiated sometimes encounter.

The main purpose of our text, then, is to make learning about harmonic functions easier. The only prerequisite for the book is a solid foundation in real and complex analysis, together with some basic results from functional analysis. The first fifteen chapters of Rudin's Real and Complex Analysis, for example, provide sufficient preparation.

In several cases we simplify standard proofs. For example, we replace the usual tedious calculations showing that the Kelvin transform of a harmonic function is harmonic with some straightforward observations that we believe are more revealing. Another example is our proof of Bôcher's Theorem, which is more elementary than the classical proofs.

We also present material not usually covered in standard treatments of harmonic functions. The section on the Schwarz Lemma and the chapter on Bergman spaces are examples. For completeness, we include some topics in analysis that frequently slip through the cracks in a beginning graduate student's corriculum, such as real-analytic functions.

We rarely attempt to trace the history of the ideas presented in this book. Thus the absence of a reference does not imply originality on our part.

In addition to writing the text, the authors have developed a software package to

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① 本节摘自:S. Axler, P. Bourdon, W. Ramey. The Theory of Harmonic Functions, Berlin: Springer, 2001。

manipulate many of the expressions that arise in harmonic function theory. Our software package, which uses many results from this book, can perform symbolic calculations that would take a prohibitive amount of time if done without a computer. For example, the Poisson integral of any polynomial can be computed exactly. Appendix B explains how readers can obtain our software package free of charge.

This book has its roots in a graduate course at Michigan State University taught by one of the authors and attended by the other authors along with a number of graduate students[3]. The topic of harmonic functions was presented with the intention of moving on to different material after introducing the basic concepts. We did not move on to different material. Instead, we began to ask natural questions about harmonic functions. Lively and illuminating discussions ensued. A freewheeling approach to the course developed; answers to questions someone had raised in class or in the hallway were worked out and then presented in class (or in the hallway). Discovering mathematics in this way was a thoroughly enjoyable experience. We will consider this book a success if some of that enjoyment shines through in these pages.

**注释与说明**

1. ([12],Preface)表示“见文献[12]的序言”,[12]表示该书所列的第 12 个文献。

2. 这段话可看成由两个并列句组成,但后面一句也可看成非限定性定语从句,似乎和常见的句型不一致,是个典型的难句。in which it has been hitherto presented 中的 which 代表 analysis of Laplace, it 代表 the manner;but one opinion as to…,意为“对……只有一种意见(见解)”。整段可译成:人们都一致认为这种 Laplace 分析很漂亮且有用,但是迄今给出的有关阐述方式似乎连最有能力的数学家也感到厌烦,更让普通的数学专业学生感到困难。

3. taught by …和 attended by …是两个并列的过去分词短语,同时修饰 course。句子较长,可译成两小句:本书内容源自密歇根州立大学的一门研究生课程,该课程由一名作者担任主讲,其他作者和一些研究生参与了该课程的建设。(编者注:该书有三个作者。)

**学习要求**

1. 掌握如下内容:

(1) 全文的主题和中心意思、各段的主要内容。

(2) 10 个新旧数学术语,包括 harmonic function(调和函数), Laplace's equation(拉普拉斯方程), polynomial(多项式), software package(软件包),

Kelvin transform(开尔文变换)等。

2. 回答如下问题:

(1) 传统的《调和函数论》教材存在什么缺点?

(2) 作者自称本书有何优点和特色?

(3) 本书是如何编写出来的?

### 3.1.2 *Preface 2* ①

**编者按:** 以下是一本专著 *Counterexamples in Topological Vector Spaces* 的前言.

During the last three decades much progress has been made in the field of topological vector spaces. Many generalizations have been introduced: this was, to a certain extent, due to the curiosity of studying topological vector spaces for which a known theorem of Functional analysis can be proved. To justify that a class $C_1$ of topological vector spaces is a proper generalization of another class $C_2$ of topological vector spaces, it is necessary to construct an example of a topological vector space belonging to $C_1$ but not to $C_2$; such an example is called a counterexample. In this book the author has attempted to present such counterexamples in topological vector spaces, ordered topological vector spaces, topological bases and topological algebras.

The author makes no claim to completeness, obviously because of the vastness of the subject. He makes no attempt to give due recognition to the authorship of most of the counterexamples presented in this book[1].

It is assumed that the reader is familiar with general topology. The reader may refer to B[18] for information about general topology.[2]

To facilitate the reading of this book, some fundamental concepts in vector spaces and ordered vector spaces have been collected in the Chapter called "Prerequisites". Thereafter each Chapter begins with an introduction which presents the relevant definitions and statements of theorems and propositions with references where their proofs can be found. For some counterexamples which require long and complicated proofs, only reference has been made to the literature where they are available.

The books and papers are listed separately in the bibliography at the end of the book. Any reference to a book is indicated by writing B [ ] and to a paper by P [ ][3].

The author would like to express his deep gratitude to Professor T. Husain, Mc-

① 本节课文摘自:S. M. Khaleelulla. Counterexamples in Topological Vector Spaces. Berlin: Springer, 2001。

Master University, Hamilton, Canada, and Dr. I. Tweddle, University of Stirling, Stirling, Scotland, who have given him both moral and material support during the preparation of this book. The author wishes to thank Mr. Mohammed Yousufuddin for typing the manuscript.

The author takes great pleasure in thanking the editors and the staff of Springer's "Lecture Notes in Mathematics" series for their keen interest in the publication of this book.

**注释与说明**

1. He makes no attempt to give due recognition to the authorship of most of the counterexamples presented in this book. 其中 the authorshin 意为"原作者",give due recognition 意为"给出归属认可",makes no attempt to 意为"不打算"。整句可译成:作者不打算对出现在本书中的大部分反例给出其原作者的归属认可。

2. 该书引用文献时把书与论文分别编号,采用不同的记号,例如 B[3]和 B[18]分别表示在书末参考文献中编号为 3 和 18 的书,而 P[3]表示编号为 3 的论文。

3. 见第 2 条注释

**学习要求**

1. 掌握如下内容:

(1) 全文的主题和中心意思、各段的主要内容。

(2) 12 个新旧数学单词与术语,包括 counterexample(反例);topological space(拓扑空间);topological vector space(拓扑向量空间);ordered topological vector space(序拓扑向量空间);topological base(拓扑基);topological algebra(拓扑代数);functional analysis(泛函分析)等。

2. 回答如下问题:

(1) 作者自称本书有何优点和特色?学习本书需要什么预备知识?

(2) 作者如何解释反例的含义?

(3) 本书关于参考文献的排列与标号做了哪些说明?

### 3.1.3 Instructor's guide ①

**编者按:**这是一本《微分方程》(Differential Equations)教材的前言中给出的

① 本节课文摘自:J. Goldberg & M. C. Potter. Differential Equations. New York: John Wiley & Sons, 1982。

"教学指导"。

Accompanying this text is a *Guided Tour of Differential Equations using Computer Technology* by Alexandra Skidmore and Margie Hale. This supplementary book offers computer projects in differential equations with necessary keystroke instructions for Maple, Mathematica and Derive. This work can be combined with selected examples and exercises to constitute either a one or two semester course.

In what follow, we outline the possible of ones semester course following either the traditional study of the $n$th-order equation with systems being relegated to the background, or a study of systems of liner differential equations through the medium of matrix algebra. There is sufficient material here so that the instructor can pick and choose among the latter four chapters. Indeed, one can easily see how to use this text in a two semesters survey of the initial and boundary value problems. Our suggestions follow.

Chapter 0: Complex Numbers, Roots and Matrices

Section 1-3 provides a review of the theory of equations. All of this material is needed in the study of the constant coefficient equation, but particular emphasis should be given to complex roots and the exponential form of complex numbers. Sections 4-6 are best left until needed in Chapter 2.

Chapter 1: First-order Differential Equations

Here we acquaint the student with first-order linear equations. This is a good place to explain what it means to solve a differential equation, to explain the distinction between specific and general solutions and to describe and illustrate initial-value problems. Towards this end, we present a number of simple applied problems. The instructor is encouraged to select two or three examples.

This chapter also provides an introduction to change of variables in a linear equation by offering the method of variation of parameters for the first-order linear equation.

Section 5 offers a brief excursion into the most common integrable nonlinear equations, variables separable and exact equations. Some users may wish to skip this section; this can be safely done because no future work depends explicitly on this material.

Chapter 2: Linear Systems

Instructors who wish to use this text for a more traditional course without relying on systems material may skip this chapter and go immediately to Chapter 3. For those using a systems approach, we recommend first returning to Chapter 0, Sections 4 to review notation, row-reduction, determinants and linear independence. In this regard, the relationship between det $\boldsymbol{A}=\mathbf{0}$ and $\boldsymbol{Ax}=\mathbf{0}$ having nontrivial solutions usually needs special emphasis.

We recommend doing all sections in this chapter although one may skip Sections 3.5 and 2.8 to save time.

Chapter 3: Second-Order Equations

This is the traditional treatment of the second-order constant coefficient linear equation. One might choose to skip Section 3.2 because it is a bit more theoretical than the remaining sections and skip Section 3.3 since sectionally continuous functions are used only in Chapter 5, The Laplace Transform (its natural home!) and Chapter 8.

As in Chapter 1, we offer the instructor a choice of the classical applications to electrical circuits and spring-mass systems.

We have made Section 3.11, The Cauchy-Euler equation of second-order optional. Section 3.12 is the standard, and in our opinion, rather difficult approach to variation of parameters. We believe that Section 4.6, variation of parameters for systems, is a far more natural method of presentation; first, because it is so similar to variation of parameters for the scalar first-order equation and second, because there is no need to puzzle over the hard-to-motivate equations which comprise the classical approach. (One never sees the classical variation of parameters for the third-order equation, and not just because it is tedious!)

Chapter 4: Higher-Order Equations

Sections 1-3 handle the $n$th-order equation without reference to systems. These sections, then, provide a natural extension of the material of Chapter 3 to higher order equations. We recommend these sections because they are good practice on the ideas introduced earlier. Sections 4-6 discuss the central idea of converting an $n$th-order equation to a (companion) system. For instructors using a systems approach we recommend doing the entire chapter, albeit some of the more theoretical aspects can be done lightly or even skipped.

Chapter 5: The Laplace Transform

As mentioned in the preface, Chapter 5 is one of optional chapters. We begin with the traditional approach to the Laplace transform with applications to sectionally continuous forcing functions (square-wave, saw-tooth, and the like). Section 5.8 is optional and Section 5.12 treats the Laplace transform of systems.

Chapter 6: Series Methods

This optional chapter presents a fairly complete discussion of power series methods in the solution of linear differential equations with variable coefficients. Besides the expansion of solutions about an ordinary point, we present solutions about the regular singular point (the Frobenius series) and a method we call the Wronskian method. The classical Bessel, Legendre, Laguerree, Hermite and Bessel-Clifford equations are used as illustrations.

Chapter 7: Numerical Methods

This chapter introduces the methods of Euler, Heun and Runge-Kutta. We use the Euler method to explain the main ideas in numerical approximations. The Heun and Runge-Kutta methods are presented for comparison purposes and to illustrate how improved accuracy is obtained. The last section is devoted to an explanation of these methods applied to systems.

Chapter 8: Boundary-value Problems

This chapter introduces the boundary-value problem as a consequence of the separation of variables in the wave, diffusion and Laplace equations. This leads directly to the expansion of functions in Fourier series. Motivation for this material is provided by physical applications.

**注释与说明**

这篇教学指导写得较通俗,语法难点较少,关键是需了解有关的数学知识和数学专业词汇,才能对本文有较好的理解。数学专业词汇大体可分为四类:

1. 一般数学术语:如 sectionally continuous functions (分段连续函数), boundary-value problem(边值问题), exact equation(恰当方程), variables separable equation(可分离变量的方程), variation of parameter(参数变易法)等。

2. 数学家的名字和用数学家的名字命名的数学术语:如 Euler(欧拉), Laplace transform(拉普拉斯变换), Bessel-Clifford equations(贝塞尔-克利福德方程),等等。

3. 数学符号: det $\boldsymbol{A}$ 表示矩阵 $\boldsymbol{A}$ 的行列式。

4. 数学软件的名称:Maple, Mathematica, Derive。

**学习要求**

1. 掌握如下内容:

(1) 全文的主题和中心意思、各段的主要内容。

(2) 15 个新旧数学单词与术语。

2. 回答如下问题:

（1）该书各章分别介绍什么内容?
（2）第三章内容是如何安排的? 为什么要这样安排?
（3）该书教学内容如何与计算机辅助软件相结合?

## §3.2 数学基础与数学方法

本节介绍数学基础,分为五小节。第一小节介绍集合的概念,对第二章§2.3介绍的内容作了进一步的补充;第二小节是数理逻辑简介;第三小节介绍映射;第四小节介绍等价关系;第五小节介绍数学方法(教与学)。其中一、三和四小节都给出许多练习题,第五小节课文本身就提出许多思考题。这里特别强调,读者应该学会读练习题和思考题,准确理解它们的含义。

### 3.2.1 The basical concept of sets ①

Many of the objects we shall study are themselves collections of other objects. These collections or sets may be finite or infinite; later we shall meet sets with additional structure, but for the moment we shall look at abstract sets and the ways in which they can be combined to form new sets.

By a set we understand, then, any collection of objects. For example, the following are sets: (ⅰ) all the stars visible from my house at 9 pm tonight, (ⅱ) all one-legged magicians, (ⅲ) all odd numbers. We see that in some cases it may be difficult to check which objects belong to the set (e. g. (*i*) above) or whether the set has any members at all ((ⅱ) above). All that matters is that the definition is sufficiently clear for us to be able to tell (in principle at least) whether any given object is or is not a member of the set.

**Remark**: According to G. Cantor (1845—1918), the mathematician who developed set theory, "a set is a grouping together of single objects into a whole." Note that no uniform property of the objects in the set is implied other than that they are grouped together to form the set. The totality of students taking college algebra forms a set. The collection consisting of a desk, a chair, and a book constitutes a set.

We usually denote sets by capitals and their members, also called their elements, by lower case letters. However, it will not always be possible to keep to this convention, especially when we are dealing with sets whose members are themselves

① 本节课文摘自:P. M. Cohn. Algebra. New York: John Wiley & Sons, 1982。

sets. If $S$ is a set, we write $x \in S$ to indicate that $x$ is a member of $S$; in the contrary case we write $x \notin S$.

A set, in this sense, is no more and no less than the totality of its members; no considerations of order or multiplicity enter. Thus: Adam and Eve; Eve and Adam; Adam, Eve and the mother of Cain, all describe the same set. In a more precise form this can be stated by saying that $S$ and $T$ denote the same set: $S = T$, if and only if, for all $x$, $x \in S \Leftrightarrow x \in T$.

Every set encountered in the real world is finite, by this we mean that its members can (at least in principle) be counted, using the natural numbers, and this process stops at a certain number. Otherwise the set is infinite, e. g., the set of all odd numbers is infinite. It is this occurrence of infinite sets in mathematics that requires rather careful analysis. Although no critical situations will arise in these pages, it should be kept in mind that too free a use of set-theoretic notions can easily lead to contradictions. The best known of these is Russell's paradox[2]: A set may be a member of itself, e. g., the Union of all Registered Charities may be a Registered Charity. Now consider the set $M$ of those sets that are not members of themselves. Is $M$ a member of itself, i. e. is $M \in M$? If $M \in M$, then $M \notin M$ (by the definition of $M$), while if $M \notin M$, then $M \in M$. Thus we reach a contradiction in either case. The paradox is resolved by restricting the ways in which sets can be formed, so that it becomes inadmissible to consider "the set of all those sets that are not members of themselves"[3]. There are several ways of doing this, but they need not concern us here; they will not play a role in the rather simple set-theoretical arguments we shall meet.

Let $S$ and $T$ be sets. If every member of $T$ also belongs to $S$, we say that $T$ is a subset of $S$ and write $T \subseteq S$ or also $S \supseteq T$. E. g., the odd numbers form a subset of the set of all numbers. Any set $S$ is a subset of itself; this is called an improper subset of $S$ in contrast to the proper subsets of $S$ which are different from $S$ itself. We write $T \subset S$ or $S \supset T$ to indicate that $T$ is a proper subset of $S$.

Frequently we describe a subset of $S$ by means of a propositional function, thus $\{x \in S \mid P(x)\}$ denotes the subset of $S$ consisting of those (and only those) elements $x$ for which $P(x)$ holds. E. g., if the set of all natural numbers is denoted by $\mathbf{N}$, then the subset of odd numbers may be denoted by $\{x \in \mathbf{N} \mid x \text{ is odd}\}$. On the other hand, forms like $\{x \mid P(x)\}$, in which the domain over which $x$ ranges is left unspecified, are best avoided.

Let $S$ and $T$ be any sets, then the elements which belong to both $S$ and $T$ form a set which is called the intersection of $S$ and $T$ and is denoted by $S \cap T$. E. g. , if $S$ is the set of all one-legged creatures and $T$ the set of magicians, then $S \cap T$ is the set of all one-legged magicians. It may happen that $S \cap T$ has no members at all; this means that $S \cap T$ is the empty set. By definition this is the set with no members; it is generally denoted by $\varnothing$. Two sets whose intersection is the empty set are said to be disjoint.

From two sets $S$, $T$ we can form another set, the union, written $S \cup T$, which consists of all the members of $S$ or $T$. E. g. , a public library may admit as borrower anyone who is either (ⅰ) a householder in the district or (ⅱ) a resident of at least 3 years' standing. Denoting the sets of persons named in (ⅰ), (ⅱ) by $A$, $B$ respectively, we see that the set of people eligible as borrowers is $A \cup B$.

In most cases, the sets under consideration in any given case will all be subsets of some given set $U$, the "universe of discourse". Thus $U$ might be the set of natural numbers, or of triangles in the plane etc. In this situation we can, for any set $S$, form its complement, i. e. , the set of all members of $U$ that are not in $S$; it is denoted by $S'$. Thus, if $S$ is the set of all odd numbers, then its complement (in the set of all natural numbers) is the set of all even numbers. This example makes it clear why the complement, to be useful, has to be taken within a given set as universe. We also note the following brief way of describing intersection, union and complement, which brings out a certain analogy with the rules for combining propositions.

$$S \cap T = \{x \in U \mid x \in S \wedge x \in T\}, \quad \cap S_i = \{x \in U \mid x \in S_i, \text{ for all } i\},$$

$$S \cup T = \{x \in U \mid x \in S \vee x \in T\}, \quad \cup S_i = \{x \in U \mid x \in S_i, \text{ for some } i\},$$

$$S' = \{x \in U \mid x \notin S\}.$$

Here $U$ is the universe containing all the objects under discussion. We also define the relative complement $S \backslash T = \{x \in S \mid x \notin T\}$.

Although many of our sets are infinite, we shall also be dealing with finite sets. In particular, with any object $x$ we can associate the set $\{x\}$ whose only member is $x$. It is important to distinguish between the set $\{x\}$ and the object $x$ (which may itself be a set). E. g. , the set $\mathbf{N}$ of natural numbers is infinite, but the set $\{\mathbf{N}\}$, whose only member is $\mathbf{N}$, is finite. If $S$ is any finite set, its members can (by the definition of finite set) be labelled or indexed by the integers from 1 to $n$, for some integer $n$. Thus if the elements of $S$ are $x_1, \cdots, x_n$, then $S = \{x_1, \cdots, x_n\}$; if distinct elements have received distinct labels, i. e. if $x_i \neq x_j$ for $i \neq j$, then $S$ consists of exactly $n$ elements. But it is usually more convenient not to impose this restriction, so that there may be repetitions among $x_1, \cdots, x_n$. If we wish to consider the objects $x_1, \cdots, x_n$ in the order given, we use parentheses: $(x_1, \cdots, x_n)$, and call the result a *sequence* or, more particularly, an *n-tuple*, e. g., a roster selecting pilots for flying duties is of this form. By contrast, the set $\{x_1, \cdots, x_n\}$ where the order is immaterial, is written with curly brackets (braces). A set which has been indexed in some way by the numbers from 1 to $n$ is also called a *family*; more generally even infinite sets can be indexed if we use an infinite indexing set. E. g. if $\triangle_{ABC}$ denotes the plane triangle with vertices $A$, $B$, $C$, this provides an indexing of all triangles in the plane by triples of points and we may speak of the family $\{\triangle_{ABC}\}$ of triangles obtained in this way.

From any two objects $x$ and $y$ we can form the sequence $(x, y)$; it is called an ordered pair, and of course is different from $(y, x)$, unless $x = y$. If $S$ and $T$ are any sets, we denote by $S \times T$ the set of all ordered pairs $(x, y)$ with $x \in S$ and $y \in T$. When $T = S$, we also write $S^2$ in place of $S \times S$. The set $S \times T$ is called the *Cartesian product* of $S$ and $T$, after R. Descartes who showed how to describe points of the plane by the Cartesian product of the real line with itself.

Examples, (ⅰ) At a dance, let $S$ be the set of gentlemen and $T$ the set of ladies, then $S \times T$ is the set of possible couples, (ⅱ) If $S = \{0, 4, 6\}$, $T = \{1, 4\}$, then $S \times T = \{(0,1), (0,4), (4,1), (4,4), (6,1), (6,4)\}$. (ⅲ) If $S = T = \mathbf{R}$, the set of real numbers, then $\mathbf{R}^2$ is the set of pairs of real numbers and these pairs may be used to represent points in the plane.

More generally, from $n$ sets $S_1, \cdots, S_n$ we can form the product $S_1 \times S_2 \times \cdots \times S_n$ whose elements are all the sequences $(x_1, \cdots, x_n)$, in which $x_i \in S(i = 1, \cdots, n)$; when $S_1 = \cdots = S_n = S$, say, one also writes $S^n$ in place of $S \times S \times \cdots \times S$ and $S^n$ is called the $n$th *Cartesian power* of $S$ or the *Cartesian square* when $n = 2$.

**Exercises**

(1) Prove the following formulae for subsets of a set $U$: (ⅰ) $A \cap A = A$, (ⅱ) $A \cup A = A$, (ⅲ) $A \cap B = B \cap A$, (ⅳ) $A \cup B = B \cup A$, (ⅴ) $(A \cap B) \cap C = A \cap (B \cap C)$, (ⅵ) $(A \cup B) \cup C = A \cup (B \cup C)$, (ⅶ) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$, (ⅷ) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$. (Compare with Ex. (1), 1.1.)

(2) Illustrate the formulae of Ex. (1) by taking $A$, $B$, $C$ to be the set of all quadrilaterals, all regular polygons and all polygons large enough to cover a penny (not necessarily respectively).

(3) If $A$ has $\alpha$ elements and $B$ has $\beta$ elements, find the number of elements in $A\times B$. If, moreover, $A$ and $B$ are disjoint, find the number of elements in $A\cup B$. What is this number when $A\cap B$ has $\delta$ elements?

(4) How many subsets are there in a set of $n$ elements? (Do not forget to include $\varnothing$ and the set itself.)

(5) Give examples of sets such that (ⅰ) all and (ⅱ) none of their members are also subsets.[4]

(6) (De Morgan's laws). Show that $(A\cup B)'=A'\cap B'$, $(A\cap B)'=A'\cup B'$.

## 注释与说明

1. 与第二章第三节相比,本小节有些深度,但学过集合基本概念的同学都可通过具体例子帮助理解全文。

2. Russell's paradox 译成"罗素悖论"或"Russell 悖论",是著名的英国数学家 Russell 提出的一个用于指出早期集合论存在逻辑缺陷的重要例子。悖论的内容由接下来的两行文字表达,意思大体是:若假定一个集合 $M$ 是 $M$ 的成员(即 $M\in M$),就会导出 $M\notin M$,与假定矛盾;同时,若假定 $M\notin M$,也要导出 $M\in M$,仍与假定矛盾。

3. 'the set of all those sets that are not members of themselves.' 这里 the set of …意思为"由……组成的集合",后一个 of 像通常的情况一样表示"……的……";that 引起一个定语从句修饰 sets。整个短语可译成:由那些自己不是自己的成员的集合组成的集合。

4. Give examples of sets such that (ⅰ) all and (ⅱ) none of their members are also subsets. 注意这句省略的成分以及 all 和 none 的用法。全句可译成:给出如下集合的例子:(1) 它的所有成员都是它的子集合;(2) 它的任何成员都不是它的子集。

## 学习要求

1. 掌握如下内容:

(1) 全文的主题和中心意思、各段的主要内容。

(2) 20 个新旧数学术语,包括 improper subset(非真子集),universe(全域,底集,通集),universe of discourse(论域,即一个指定的集,使得在一定范围内讨论的集都是它的子集,常是 universe 的同义词),relative complement(相对余集),$n$-tupe($n$ 元组,$n$ 重的),Cartesian product(笛卡儿积,卡氏积),Cartesian power(笛卡儿幂,卡氏幂),Cartesian square(笛卡儿方形),vertices(顶点),curly brack-

ets(braces)(花括号),indexing of sth.(给……加号标的方法),quadrilaterals(四边形), De Morgan's laws(德·摩根律)。

(3) 理解各条习题的条件、结论及读者的任务。

2. 回答如下问题:

(1) 研究集合有什么重要性?

(2) 一个集合中的事物是否必须具有共同的性质?

(3) 集合是如何表示的? 属于、包含、相等概念是如何定义的?

(4) 如何理解全集(即通集或底集)的概念? 举例说明。

(5) 笛卡儿乘积集的概念是如何推广到 $n$ 个集上去的?

## 3.2.2 Logic ①

Mathematics is the study of relations between certain ideal objects such as numbers, functions and geometrical figures. These objects are not to be regarded as real, but rather as abstract models of physical situations. As examples of the relations that can hold, consider the following assertions that can be made about the natural numbers:

(a) every even number is the sum of two odd numbers,

(b) every odd number is the sum of two even numbers,

(c) every even number greater than 2 is the sum of two primes.

Of these assertions, (a) is true, (b) is false, while for (c) it is not known whether this is true or false ((c) was conjectured by Goldbach[2] in 1742 and has so far resisted all attempts to prove or disprove it).

If our mathematical system is to serve as a model of reality we must know how to recognize true assertions, at least in principle (even though in practice some may be hard to prove). When the object of discussion is intuitively familiar to us—as in the case of the natural numbers—we take certain assertions recognized to be true as our axioms and try to derive all other assertions from them. Once that is done, we can forget the intuitive interpretation and regard our objects as abstract entities subject to the given axioms. When we come to apply our system to a concrete case, we need to find an interpretation for each notion introduced and verify that each axiom holds in the interpretation; we are then able to conclude that all the assertions derived from the axioms also hold. This underlines the need to keep the axiom system as small as

① 本节课文摘自:P. M. Cohn. Algebra. New York: John Wiley & Sons, 1982.

possible.

The advantage of this axiomatic method of study is that we can examine the effect on our system of varying the axioms and that the proofs become more transparent the more abstract the system. On the other hand, it takes a little time to familiarize oneself with the abstract notions; here the (more or less concrete) model on which it was based will help, although it is not strictly necessary and certainly no part of the theory.

Studying these abstract notions is rather like learning a new language; as in that case we shall find that as our knowledge widens we recognize more landmarks; this makes learning very much easier. But whereas we use a foreign language to talk about the same concepts as in our native tongue, the purpose of the mathematical language is to talk about new ideas which can be expressed only with difficulty (or not at all) in a natural language like English.

There is another respect in which the process differs from learning a language: we shall need to reason about the new concepts and this will require careful attention to the logical interrelation of statements. Of course it is true that even in everyday affairs we can spurn logic only at our peril, but there the patent absurdity of our conclusion usually forces us to abandon a faulty line of reasoning. By contrast, when we pursue an abstract line of thought, involving unfamiliar concepts, we may reach conclusions by logical reasoning, but we will no longer be able to check these conclusions by commonsense. It is therefore important to be fully aware of the rules of logic we need and to realize that these rules can be applied without regard for the actual meaning of the statements on which they are used. For this reason we begin by describing very briefly some concepts and notations from logic.

Propositional logic describes ways in which true statements (also called assertions or propositions) can be combined to produce other true statements. E. g., if it is asserted that 'Jack was running' and 'Jill was singing', then we may conclude that

$$\text{'Jack was running and Jill was singing'}. \qquad (1)$$

On the other hand, if Jack was not running then statement (1) is false irrespective of what Jill was doing. By enumerating further possibilities we can thus give a precise description of the way the word 'and' is used to link assertions. In order to do this concisely, let '$A$' stand for an assertion, such as 'Jack was running', and '$B$' for a second assertion, not necessarily different from $A$. Then we can form the expression '$A$ and $B$', also written '$A \wedge B$' and called the *conjunction* of $A$ and $B$, and make a table which indicates in which cases $A \wedge B$ is true, using 'T' for 'true' and 'F' for 'false':

| | | | | |
|---|---|---|---|---|
| $A$ | T | T | F | F |
| $B$ | T | F | T | F |
| $A \wedge B$ | T | F | F | F |

This is called the *truth-table* for conjunction. It shows that $A \wedge B$ is true when $A$ is true and $B$ is true, and false in all other cases. For our purposes we may assume that each statement is either true or false; the relevant value T or F is called the truth-value of the statement. Since there are two possible truth-values for $A$ and two for $B$, we have $2 \times 2 = 4$ possibilities in all, which are listed in the above table.

A second way in which assertions can be combined is by using 'or': 'John went to the cinema last night, or to the theatre'. This is a true statement if in fact John last night went to the cinema, and also true if he went to the theatre; the possibility that he went to both is not really envisaged, but if he did, the statement would still be regarded as true. This causes some ambiguity in everyday life: if $A$ and $B$ are both true, is the statement '$A$ or $B$' to be regarded as true? The situation is usually cleared up by the context (but not always, cf. 'This summer Jane will go to Italy or Austria'). In mathematics the expression '$A$ or $B$' is always taken to mean '$A$ or $B$ or both'; it is written '$A \vee B$' and is called the *disjunction* of $A$ and $B$. Its truth-table is

| | | | | |
|---|---|---|---|---|
| $A$ | T | T | F | F |
| $B$ | T | F | T | F |
| $A \vee B$ | T | T | T | F |

A typical use of disjunction in mathematics is the sentence: If $a$ and $b$ are two real numbers whose product is zero: $ab = 0$, then $a = 0$ or $b = 0$. Clearly we must not exclude the case where $a = 0$ and $b = 0$.

With every statement we can associate its opposite or negation by inserting 'not' in the appropriate place. Thus 'Max is the biggest liar' has the negation 'Max is not the biggest liar'. Generally, if $A$ is any statement, then its negation is 'not $A$', also written '$\neg A$', and it is true precisely when $A$ is false. Its truth-table is

| | | |
|---|---|---|
| $A$ | T | F |
| $\neg A$ | F | T |

Here there are only two possibilities because only one statement is involved.

The notion of implication is particularly important for us and its use in mathematics differs in some ways from everyday usage, though the underlying meaning is of course the same. Thus '$A$ implies $B$' or 'if $A$, then $B$', written '$A \rightarrow B$' means for us: 'either $A$ is false or $B$ is true'. It is expressed in the truth-table.

For example, a mathematical proof might contain the line: 'If $n>5$, then $n>3$'. A parallel use in everyday English would be: 'If this book was influenced by Shakespeare must been written after the Canterbury Tales'.

| | | | | |
|---|---|---|---|---|
| $A$ | T | T | F | F |
| $B$ | T | F | T | F |
| $A \rightarrow B$ | T | F | T | T |

**注释与说明**

1. 本小节介绍数理逻辑的基本概念和四种基本运算的含义。在§2.11已经提到:"and"译成"(逻辑)与","or"译成"(逻辑)或","not"译成"非","imply"译成"蕴涵",它们对应的运算符号分别为"∧","∨","¬"和"→"。"与"的运算又称"合取"(conjuction),"或"的运算又称"析取"(disjuction)。现在,进一步通过真值表(truth-table)来说明四种基本运算的含义。真值表就是文中的四个表格,其中T表示真,即命题成立,F表示假,即命题不成立 。

2. (c) was conjectured by Goldbach。可译成:(c) 是哥德巴赫提出的猜想。

**学习要求**

1. 掌握如下内容:

(1) 全文的主题和中心意思、各段的主要内容。

(2) 15个新旧数学术语。

(3) 理解数理逻辑的基本概念和四种基本运算的含义。

2. 回答如下问题:

(1) 公理方法有什么优点?

(2) 为什么要采用数学语言来代替自然语言?

(3) 真值表在逻辑学中发挥什么作用?

### 3.2.3 Mappings ①

Let $S$ and $T$ be sets; any subset of $S \times T$ is called a *correspondence* from $S$ to $T$.

① 本节课文摘自:P. M. Cohn. Algebra. New York: John Wiley & Sons, 1982.

E. g., let $S$ be the set of all points in the plane and $T$ the set of lines; the relation of *incidence* (the point $P$ is incident with the line $l$ if $P$ lies on $l$) defines a correspondence from points to lines. To each point $P$ correspond all the lines through $P$ and to each line $l$ correspond all the points on $l$. This is an example of a 'many-many correspondence', where to each element of $S$ correspond many elements of $T$ and *vice versa*'. In general, to each element of $S$ there may correspond many, one or no elements of $T$.

An important special case of a correspondence is that of a bijective or one-one correspondence, also called a bijection. Here there corresponds to each $s \in S$ just one $t \in T$ and to each $t \in T$ just one $s \in S$. The following are some examples of bijections: (i) At a gathering of married couples there is a bijection between the set of men and the set of women: to each person there corresponds precisely one spouse of the other sex. (ii) The real numbers may be represented on a line (the $x$-axis, say, in coordinate geometry) so that to each real number corresponds just one point on the line and to each point on the line corresponds one real number, (iii) The correspondence $x \leftrightarrow 2x$ defines a bijection between real numbers; on the other hand (iv) the correspondence $x \mapsto x^2$ does not, because $x$ and $-x$ have the same square for any real $x$.

The last example makes it clear that the notion of bijection is too restrictive to account for such simple functions as $x^2$. But the notion of function, or mapping, in various guises, plays a basic role in mathematics. For this reason the next definition is fundamental in all that follows.

A *mapping* from $S$ to $T$ is a correspondence between $S$ and $T$ such that to each $x \in S$ there corresponds exactly one $y \in T$. If $f$ is the mapping, one writes $f: S \to T$ or $S \xrightarrow{f} T$ and calls $S$ the *domain* and $T$ the *range* of $f$. The unique element $y \in T$ that corresponds to $x \in S$ is called the *image* of $x$ and is written $f(x)$ or $f_x$, or more often $xf$. We also write $x \mapsto y$ to indicate the correspondence between $x$ and its image $y$. Often the set of all images, namely $\{y \in T \mid y = xf \text{ for some } x \in S\}$ is also called the *image* of the mapping $f$ and is written $Sf$ or im$f$; in practice this double use of the term 'image' does not lead to confusion.

*Examples of mappings.* (i) With each newborn baby associate its weight in grams to the nearest gram. This is a mapping from the set of newborn babies to the natural numbers. (ii) Let $S$ be the United Kingdom and $T$ a map of the United Kingdom. There is a 'mapping' which associates with each place in the country a point on the map. (iii) and (iv). The examples (iii) and (iv) of correspondences considered earlier define mappings from **R** to itself, namely $x \mapsto 2x$ and $x \mapsto x^2$ respectively. (v) If $S=(x_\alpha)$ is a family indexed by a set $A$, then we have a mapping $\alpha \mapsto x_\alpha$ from $A$ to $S$. (vi) Given a Cartesian product $P=S\times T$, we can define mappings from $P$ to $S$ and $T$ by the rules $(x,y)\mapsto x$ and $(x,y)\mapsto y$; they are called the *projections* on the factors $S$ and $T$. Similarly, in an $n$-fold product $P=S_1\times\cdots\times S_n$ we have for each $i=1,\cdots,n$ a projection $\varepsilon_i$: $P\to S$, given by $(x_1,\cdots,x_n)\mapsto x_i$.

Clearly a bijection is a particular type of mapping. On closer examination we see that two properties are required for a mapping to be bijective; it is useful to consider them separately. A mapping $f$: $S\to T$ is said to be *injective* or an *injection* or *one-one* if distinct elements of $S$ have distinct images, i. e., $s\neq s'$ implies $sf\neq s'f$. The mapping is called surjective or a surjection or onto $T$ if every element of $T$ is an image, i. e., if $Sf=T$. Thus a mapping is bijective precisely if it is injective and surjective. In the above examples, (i) is neither surjective nor injective (at least if we take enough babies), while (ii) and (iii) are bijective. The mapping $x\mapsto x^2$ of **R** into itself considered in (iv) is neither injective nor surjective, but on the set **C** of complex numbers it defines a mapping from **C** to itself which is surjective, though not injective (the surjectivity is just an expression of the fact that every complex number has a square root).

Let $f$: $S\to T$, $g:T\to U$ be any mappings, then we can compose them to get a mapping $h:S\to U$, given by

$$xh=(xf)g \quad \text{for all } x\in S. \tag{1}$$

A graphic way of expressing this equation is shown in the accompanying diagram. Starting from an element $x \in S$, we reach the same element of $U$ whether we go via $T$, $x\mapsto xf\mapsto(xf)g$ or direct, $x\mapsto xh'$, we express this by saying that the triangle shown *commutes*.

The mapping $h$ defined by (1) is called the composite or product of $f$ and $g$ and is denoted by $fg$; in this notation (1) reads

$$x(fg)=(xf)g. \tag{2}$$

As a rule one omits the parentheses and denotes either side of (2) by $xfg$. We observe that $fg$ is defined only when the range of $f$ is contained in the domain of $g$. Further we note that if we had written mappings on the left, (2) would read: $(fg)x=g(fx)$. It is to avoid this reversal of factors that we put mappings on the right of their arguments.

As an example, let $f,\ g$: $\mathbf{N}\to\mathbf{N}$ be given by $xf=x+1$, $xg=x^2$, then $xfg=(x+1)^2$, $xgf=x^2+1$. We see that $fg\neq gf$, so attention must be paid to the order in which the mappings are composed.

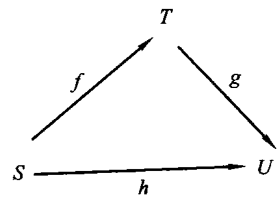


Fig. 3-2-1

When $S$ is a finite set, $f$ and $g$ may be given explicitly. Let us indicate each mapping by writing down the elements of $S$ as a sequence and under each element write its image. Thus if $S=\{1,2,3\}$, and $f$ is given by $\begin{pmatrix}1&2&3\\2&3&1\end{pmatrix}$, while $g$ is $\begin{pmatrix}1&2&3\\1&3&2\end{pmatrix}$, then $fg$ is $\begin{pmatrix}1&2&3\\3&2&1\end{pmatrix}$, and $gf=\begin{pmatrix}1&2&3\\2&1&3\end{pmatrix}$. Again $fg\neq gf$; on the other hand if $h$ is $\begin{pmatrix}1&2&3\\3&1&2\end{pmatrix}$ then $fh=hf$.

An important rule in composing mappings is the associative law: For any mappings $f$: $S\to T$, $g$: $T\to U$, $h$: $U\to V$ we have

$$(fg)h=f(gh). \tag{3}$$

Observe that both sides of (3) are defined, by what was assumed about $f,\ g,\ h$. More generally, the equation (3) holds for any mappings $f,\ g,\ h$ such that both sides of (3) are defined.

To prove (3) we apply each side to an element $x$ of $S$, remembering (2): $x[(fg)h]=[x(fg)]h=((xf)g)h$ and $x[f(gh)]=(xf)(gh)=((xf)g)h$; now a comparison gives (3).

With every set $S$ we can associate the *identity mapping* $1_S$ which maps each element of $S$ to itself: $x\mapsto x$. Clearly this is always a bijection. Further, for any $f$: $S\to T$ and $h$: $U\to S$ we have $1_S f=f$, $h1_S=h$.

Let $S$ be any set and $T$ a subset, then there is a mapping $\iota$ from $T$ to $S$, defined by $x\iota=x$ for all $x\in T$; this is called the *inclusion mapping* of $T$ in $S$. Although $\iota$ and $1_S$ have the same effect wherever they are defined (namely on $T$), they must be carefully distinguished; e. g., whereas $1_S$ is bijective, $\iota$ is injective, but not surjective (except when $T=S$ and so $\iota=1_S$). In fact $\iota$ may be obtained from $1_S$ by restricting the domain to $T$; this is often expressed by writing $\iota=1_S\,|\,T$. Generally, if $f$: $X\to Y$ is any mapping and $X'$ is a subset of $X$, then the *restriction* of $f$ to $X'$, denoted by $f\,|\,X'$, is defined as the mapping $f'$: $X'\to Y$ given by $xf'=xf$ for all $x\in X'$. We observe that this restriction may be written as $f'=\iota f$, where $\iota$ is the inclusion of $X'$ in $X$.

Using the composition of mappings we can describe bijections. In the first place we have

**LEMMA 1** If $f: S \to T$, $g: T \to S$ *are any mappings such that*

$$fg = 1_S, \tag{4}$$

*then $f$ is injective and $g$ is surjective.*

For let $x, y \in S$ and $xf = yf$, then $x = xfg = yfg = y$, hence $f$ is injective. Given any $x \in S$, $x = xfg = (xf)g$ and this shows $g$ to be surjective.

Suppose that $f: S \to T$, $g: T \to S$ satisfy

$$fg = 1_S, \quad gf = 1_T, \tag{5}$$

then $f$ is both injective and surjective, by the lemma, and so is a bijection. Conversely, if $f: S \to T$ is a bijection, then we can always find a unique mapping $g: T \to S$ to satisfy (5). For, given $u \in T$, we know there exists just one $x \in S$ such that $xf = u$. Put $ug = x$, then this defines $g$ on $T$ and $ugf = u$, $xfg = x$, therefore (5) holds. This proves

**THEOREM 2** *A mapping $f: S \to T$ is a bijection if and only if there is a mapping $g: T \to S$ to satisfy* (5).

There can be at most one mapping $g$ to satisfy (5), for any given $f$. For assume that (5) holds and that $g': T \to S$ is another mapping such that $fg' = 1_S$, $g'f = 1_T$, then by the associative law, $g' = g'1_S = g'fg = 1_T g = g$. The unique mapping $g$ satisfying (5) is called the inverse of the bijection $f$ and is written $f^{-1}$. In this notation (5) reads $ff^{-1} = 1_S$, $f^{-1}f = 1_T$.

The distinction between finite and infinite sets is an important one which will be taken up in greater detail in Vol. 2. Here we shall only note one useful property of finite sets (actually it can be used to characterize them):

**LEMMA 3** *An injective mapping from a finite set to itself is also surjective.*

For let $f: S \to S$ be injective and take $a \in S$; we must find $b \in S$ such that

$$a = bf. \tag{6}$$

Consider the effect of performing $f$ repeatedly. Let us write $f^2$ for $ff$ and generally abbreviate $ff\cdots f$ (with $n$ factors) as $f^n$. In the series of elements $a$, $af$, $af^2$, $\cdots$ there must be repetitions, because $S$ is finite, so assume that

$$af^r = af^s, \tag{7}$$

where $r > s$ say. Since $f$ is injective, $xf = yf$ implies $x = y$, so we may cancel $f$ in (7). If we do this $s$ times, we get $af^{r-s} = a$, i. e., (6) holds with $b = af^{r-s-1}$.

Later we shall meet many applications of this lemma. It is not really a surprising result and, to newcomers at least, not as surprising as the fact that it no longer holds for infinite sets. To give an illustration, if in a club each member succeeds in borrowing £1 from another member but no two have borrowed from the same person, then everyone has also had to lend £1 (by the lemma) so no one is any better off. But suppose that we have an infinite club, with members $A_1, A_2, \cdots$ indexed by the positive integers (where it is assumed that $A_m \neq A_n$ for $m \neq n$). If now for each $n$, $A_n$ borrows £1 from $A_{n+1}$, then $A_1$ is £1 better off, while all the others come out even.

The failure of Lemma 3 for infinite sets makes it seem difficult at first sight to extend the notion of counting and cardinality (or 'number of elements') to infinite sets. These difficulties were overcome by Cantor who laid the foundations of set theory in the 1870s. This does not concern us directly as we shall (in this volume) use the notion of cardinality only for finite sets. With every finite set $S$ we associate a natural number $|S|$, the number of its elements (sometimes called the *cardinal* of $S$). In a complete account one would have to show that this is uniquely defined, i. e., that different ways of counting $S$ give the same answer. This will be proved when we come to the axiomatic development of numbers in Vol. 3.

**Exercises**

(1) $S$ is a set of four elements. Find (i) the number of mappings of $S$ into itself; (ii) the number of bijections of $S$ to itself. (Hint. Try sets of two and three elements first.)

(2) If $f$: $A \rightarrow B$ and $g$: $B \rightarrow C$ are both injective (or both surjective), show that $fg$ is so too. If $fg$ is injective (or surjective) what can be said about $f$ and $g$?

(3) If $f$ is any bijection and $f^{-1}$ its inverse, show that the domain and range of $f^{-1}$ are the range and domain respectively of $f$. Show also that $f^{-1}$ is again bijective and $(f^{-1})^{-1} = f$.

(4) If $f$: $A \rightarrow B$, $g$: $B \rightarrow C$, $h$: $C \rightarrow A$ are three mappings such that $fgh = 1_A$, $ghf = 1_B$ and $hfg = 1_C$, show that each of $f$, $g$, $h$ is a bijection and find their inverses.

(5) Let $fg = 1_S$: if $f$ is surjective, or if $g$ is injective, show that both $f$ and $g$ are bijective and inverse to each other.

(6) Let $fg = 1_S$: we say that $g$ is a right inverse of $f$ and that $f$ is a left inverse of $g$. Show that when the domain of $f$ has more than one element, $f$ is bijective iff it has a single right inverse, or also iff it has a single left inverse. (Hint. To get counterexamples, look at mappings from **N** to **N**.)

(7) (i) For any integer $a$ define a mapping $\mu_a$ of **N** into itself by the rule $\mu_a$: $x \mapsto xa$. Show that $\mu_{ab} = \mu_a \mu_b$.[2] (ii) For any integer $a$ define a mapping $\nu_a$ of **N** into itself by $\nu_a$: $x \mapsto x + a$. Show that $\nu_{a+b} = \nu_a \nu_b$.

(8) A mapping $f$: $S \rightarrow T$ is said to be constant if $xf = yf$ for all $x, y \in S$. Show

that for any two distinct constant mappings of $S$ into itself, $fg \neq gf$. What happens when $S$ has only one element?

(9) In an infinite club indexed by the integers, $\{A_1, A_2, \cdots\}$, how much does $A_n$ have to borrow from $A_{n+1}$, in order that each member shall be £1 better off than before?

(10) Let $S$ be the set of finite sequences of 0s and 1s and define a mapping $f$ of $S$ into itself by the rule: If $a = a_1 a_2 \cdots a_n$ ($a_i = 0$ or 1), then $af = a'_1 a'_2 \cdots a'_n$, where $0' = 01$, $1' = 10$. Show that $af$ has no block 000 or 111 and that in $af^2$ ($= aff$) any block of length at least five contains 00 or 11.

(11) Show that $A \times B = B \times A$ only if $A = B$ or one of $A$, $B$ is empty. Is it possible for non-empty sets to satisfy $(A \times B) \times C = A \times (B \times C)$? (Hint. Take $B$ to consist of one element.)

## 注释与说明

1. 本句中 'many-many correspondence' 可译成"多到多的对应","*vice versa*"是拉丁文,表示"反之亦然"。

2. For any integer $a$ define a mapping $\mu_a$ of **N** into itself by the rule $\mu_a: x \mapsto xa$. Show that $\mu_{ab} = \mu_a \mu_b$。阅读数学英语应注意区分英文的文字与数学符号,英文的数学符号通常用斜体表示,如第一句第一个 $a$ 和后面的 $x$。第二句是祈使句,练习题常用这种句型。这两句可译成:对于任意整数 $a$,按照规则 $\mu_a: x \mapsto xa$ 定义了一个 **N** 到自身的映射 $\mu_a$。证明 $\mu_{ab} = \mu_a \mu_b$。注意,符号$\mapsto$和$\rightarrow$的含义不同,前者通过表示元素之间的对应来确定映射,后者通过指明定义域与值域所在的集合来表示映射。

## 学习要求

1. 掌握如下内容:

(1) 全文的主题和中心意思、各段的主要内容。

(2) 20 个新旧数学术语,特别是有关映射的单词和术语。读者学习本小节特别应多记住几个关于映射的数学术语,如:correspondence(对应),mapping(映射),image(像),bijection(双射),injection(内射),surjection(满射),projection(投影),one-one correspondence(一一对应),composing mapping(复合映射),inverse(逆),identity mapping(恒等映射),inclusion mapping(包含映射),projection(投射)等。

(3) 理解各习题的条件、结论及读者的任务。

2. 回答如下问题：

（1）本节是如何定义内射、满射、双射和投射的？

（2）恒等映射在映射的研究中发挥什么作用？

（3）习题 10 按照什么规则来定义映射 $f$？其中 1s 和 0s 分别表示什么？

### 3.2.4 *Equivalence relations* ①

By a relation on a set $S$ we mean a correspondence of $S$ with itself. E. g., 'being related' is a relation on the set of all humans (provided that we are equipped with an exhaustive genealogy)[1]. Let $\omega$ be a relation; we write $x\omega y$ to express the fact that $x$ stands in the relation $\omega$ to $y$, i. e. that the pair $(x, y)$ belongs to $\omega$. Frequently relations are denoted by a symbol such as $\sim$, thus in place of $x\omega y$ we write $x \sim y$.

Many relations have one or more of the following three properties:

E. 1 For every $x \in S$, $x \sim x$ (*reflexive*).

E. 2 For all $x, y \in S$, if $x \sim y$, then $y \sim x$ (*symmetric*).

E. 3 For all $x, y, z \in S$, if $x \sim y$ and $y \sim z$, then $x \sim z$ (*transitive*).

For example, the relation '$x$ is father of $y$' (on the set of all humans) has none of these properties. On the other hand, '$x$ has the same parents as $y$' has all three, '$x$ is ancestor of $y$' is transitive and '$x$ is brother of $y$' is symmetric on the set of all human males, but not on the set of all humans. This last point illustrates that we must always specify the set on which we are operating.

A relation on $S$ which is reflexive, symmetric and transitive is called an *equivalence* on $S$. This is an important notion, which in some ways generalizes the notion of equality, for the relation of equality (on any set) trivially satisfies E. 1–3. An equivalence on $S$ separates the elements of $S$ into classes, grouping together objects which agree in some particular respect.[2] E. g., '$x$ has the same parents as $y$' is an equivalence which groups siblings together. Similarly, the relation '$x$ has the same remainder after division by 2 as $y$' on $\mathbf{N}$ groups all the even numbers together and all the odd numbers.

Let us see how this can be done generally, for any equivalence on a set $S$. For any $x \in S$, we group together all the elements equivalent to $x$ into an *equivalence class* or block $S_x$ i. e. we put

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① 本节课文摘自：P. M. Cohn. Algebra. New York: John Wiley & Sons, 1982。

$$S_x = \{y \in S \mid x \sim y\}.$$

By the reflexivity, $x \in S$, we claim that any two blocks $S_x$ and $S_y$ either are disjoint or coincide. Suppose that $S_x$ and $S_y$ are not disjoint; we must prove that $S_x = S_y$ and we begin by showing that $x \sim y$. Since $S_x \cap S_y \neq \varnothing$, there exists $z \in S_x \cap S_y$, by definition this means that $x \sim z$ and $y \sim z$. By symmetry, $z \sim y$ and hence, by transitivity, $x \sim y$. Now let $u \in S_y$ then $y \sim u$, hence $x \sim u$ (by transitivity) and so $u \in S_x$; this proves that $S_y \subseteq S_x$. A similar argument shows that $S_x \subseteq S_y$ and so $S_x = S_y$, as claimed. Thus the different $S_x$ provide a division of $S$ into non-empty subsets, any two of which are disjoint. This is called a *partition* of $S$.

Given an equivalence '$\sim$' on $S$, we can form a new set $S/\sim$, whose members are the different blocks $S_x$ and we then have a mapping $\lambda: S \to S/\sim$ which assigns to each $x \in S$ the block $S_x$; $\lambda$ is called the natural mapping from $S$ to $S/\sim$. It is surjective, but not injective, unless the equivalence on $S$ was just equality; $S/\sim$ is called a *quotient set*.

We note that conversely, every partition on a set $S$ arises in this way from an equivalence. For suppose that $S$ is partitioned into sets $A$, $B$, $\cdots$. Then each $x \in S$ belongs to just one set of the partition, say $x \in A$. We put $x \sim y$ if $x$ and $y$ lie in the same set. This is an equivalence on $S$ with blocks $A$, $B$, $\cdots$.

The example considered earlier, '$x$ and $y$ leave the same remainder after division by 2' gives a partition of $\mathbf{N}$ into two blocks, the even numbers and the odd numbers. Similarly, in any given year, the relation '$x$ and $y$ fall on the same day of the week' gives a partition of the days of the year into seven blocks, corresponding to the seven days of the week.

Any mapping $f: S \to T$ gives rise to an equivalence on $S$ by the rule: $x \sim y$ if and only if $xf = yf$. The reader should verify that this is indeed an equivalence.

**Exercises**

(1) Which of the following relations between positive integers are reflexive, which are symmetric and which are transitive? (i) $a \neq b$, (ii) $a < b$, (iii) $a$ differs from $b$ by less than 2, (iv) any positive integer dividing $a$ also divides $b$.

(2) Which of the following are equivalence relations? (i) $x$ is within sight of $y$ (where the objects are points on the earth's surface), (ii) $x$ is on the same latitude as $y$, (iii) $x$ has the same number of digits as $y$ (numbers in decimal notation).

(3) Let '$\sim$' be a reflexive relation. Show that '$\sim$' is symmetric and transitive iff $a \sim b$, $a \sim c \Rightarrow b \sim c$.

(4) What is wrong with the following 'proof' that every relation on $S$ that is symmetric and transitive is reflexive? For any $a, b \in S$, $a \sim b$ implies $b \sim a$ (by symmetry) and hence, by transitivity, $a \sim a$. Give a counter-example to the assertion.

**注释与说明**

1. 这句可译成:一个集合 $S$ 上的关系指的是 $S$ 到它自身的一种对应,例如,"亲戚"就是所有的人组成的集合上的一种关系(如果能提供一份完整的家族谱的话)。

2. "An equivalence on $S$ separates the elements of $S$ into classes, grouping together objects which agree in some particular respect."。group 可当动词用,grouping 引起一个现在分词短语,说明如何把 $S$ 中的元素分类。

**学习要求**

1. 掌握如下内容:

(1) 全文的主题和中心意思、各段的主要内容。

(2) 10 个新旧数学术语,包括 reflexive(具有反身性的,反身的), symmetric(对称的),transitive(具有传递性的,传递的),natural mapping(自然映射),quotient set(商集),remainder(余数),division(除法,分法),partition(分划,细分)等。

(3) 理解各习题的含义,特别是要求读者完成的任务。

2. 回答如下问题:

(1) 等价关系是如何定义的?

(2) 什么叫自然映射? 商集是如何定义的? 以整数的除法为例说明之。

(3) 一个映射 $f$: $S \to T$ 能给出 $S$ 上的一个等价关系的充分必要条件是什么?

## 3.2.5 Mathematical methods—teaching and learning ①

### (Ⅰ) How to solve it[1]—Procedure for solving problem

UNDERSTANDING THE PROBLEM

**First. You have to *understand* the problem:**

*What is the unknown? What are the data? What is the condition?* Is it possible to satisfy the condition? Is the condition sufficient to determine the unknown? Or is it

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① 本课三篇短文均选自:G. Polya. How to Solve It: a new aspect of mathematical method. Princeton, N. J.: Princeton Univ. Press, 1985。

insufficient? Or redundant? Or contradictory?

Draw a figure. Introduce suitable notation.

Separate the various parts of the condition. Can you write them down?

## DEVISING A PLAN

**Second. Find the connection between the data and the unknown. You may be obliged to consider auxiliary problems if an immediate connection cannot be found. You should obtain eventually a *plan* of the solution.**

Have you seen it before? Or have you seen the same problem in a slightly different form?

*Do you know a related problem?* Do you know a theorem that could be useful?

*Look at the unknown!* And try to think of a familiar problem having the same or a similar unknown.

*Here is a problem related to yours and solved before.* Could you use it? Could you use its result? Could you use its method? Should you introduce some auxiliary element in order to make its use possible?

Could you restate the problem? Could you restate it still differently?

Go back to definitions.

If you cannot solve the proposed problem try to solve first some related problem. Could you imagine a more accessible related problem? A more general problem? A more special problem? An analogous problem? Could you solve a part of the problem? Keep only a part of the condition, drop the other part; how far is the unknown then determined, how can it vary? Could you derive something useful from the data? Could you think of other data appropriate to determine the unknown? Could you change the unknown or the data, or both if necessary, so that the new unknown and the new data are nearer to each other?

Did you use all the data? Did you use the whole condition? Have you taken into account all essential notions involved in the problem?

## CARRYING OUT THE PLAN

**Third. Carry out your plan.**

Carrying out your plan of the solution, *check each step*. Can you see clearly that the step is correct? Can you prove that it is correct?

LOOKING BACK

**Fourth. Examine the solution obtained.**

Can you *check the result*? Can you check the argument?

Can you derive the result differently? Can you see it at a glance?

Can you use the result, or the method, for some other problem?

( Ⅱ ) **How to solve it — A Dialogue**

Getting Acquainted

Where should I start? Start from the statement of the problem.

What can I do? Visualize the problem as a whole as clearly and as vividly as you can. Do not concern yourself with details for the moment.

What can I gain by doing so? You should understand the problem, familiarize yourself with it, impress its purpose on your mind. The attention bestowed on the problem may also stimulate your memory and prepare for the recollection of relevant points.

Working for Better Understanding

*Where should I start*! Start again from the statement of the problem. Start when this statement is so clear to you and so well impressed on your mind that you may lose sight of it for a while without fear of losing it altogether.

*What can I do*? Isolate the principal parts of your problem. The hypothesis and the conclusion are the principal parts of a "problem to prove"; the unknown, the data, and the conditions are the principal parts of a "problem to find". Go through the principal parts of your problem, consider them one by one, consider them in turn, consider them in various combinations, relating each detail to other details and each to the whole of the problem.

*What can I gain by doing so*? You should prepare and clarify details which are likely to play a role afterwards.

Hunting for the Helpful Idea

*Where should I start*? Start from the consideration of the principal parts of your problem. Start when these principal parts are distinctly arranged and clearly conceived, thanks to your previous work, and when your memory seems responsive.

What can I do? Consider your problem from various sides and seek contacts with your formerly acquired knowledge.

Consider your problem from various sides. Emphasize different parts, examine different details, examine the same details repeatedly but in different ways, combine the details differently, approach them from different sides. Try to see some new meaning in each detail, some new interpretation of the whole.

Seek contacts with your formerly acquired knowledge. Try to think of what helped you in similar situations in the past. Try to recognize something familiar in what you examine, try to perceive something useful in what you recognize.

*What could I perceive*? A helpful idea, perhaps a decisive idea that shows you at a glance the way to the very end.

*How can an idea be helpful*? It shows you the whole of the way or a part of the way; it suggests to you more or less distinctly how you can proceed? Ideas are more or less complete. You are lucky if you have any idea at all.

*What can I do with an incomplete idea*? You should consider it. If it looks advantageous you should consider it longer. If it looks reliable you should ascertain how far it leads you, and reconsider the situation. The situation has changed, thanks to your helpful idea. Consider the new situation from various sides and seek contacts with your formerly acquired knowledge.

*What can I gain by doing so again*? You may be lucky and have another idea. Perhaps your next idea will lead you to the solution right away. Perhaps you need a few more helpful ideas after the next. Perhaps you will he led astray by some of your ideas. Nevertheless you should be grateful for all new ideas, also for the lesser ones, also for the hazy ones, also for the supplementary ideas adding some precision to a hazy one, or attempting the correction of a less fortunate one. Even if you do not have any appreciable new ideas for a while you should be grateful if your conception of the problem becomes more complete or more coherent, more homogeneous or better balanced.

### Carrying Out the Plan

*Where should I start*? Start from the lucky idea that led you to the solution. Start when you feel sure of your grasp of the main connection and you feel confident that you can supply the minor details that may be wanting.

*What can I do*? Make your grasp quite secure. Carry through in detail all the algebraic or geometric operations which you have recognized previously as feasible. Convince yourself of the correctness of each step by formal reasoning, or by intuitive insight, or both ways if you can. If your problem is very complex you may distinguish "great" steps and "small" steps, each great step being composed of several small ones. Check first the great steps, and get down to the smaller ones afterwards.

*What can I gain by doing so?* A presentation of the solution each step of which is correct beyond doubt.

Looking Back

*Where should I start?* From the solution, complete and correct in each detail.

*What can I do?* Consider the solution from various sides and seek contacts with your formerly acquired knowledge.

Consider the details of the solution and try to make them as simple as you can; survey more extensive parts of the solution and try to make them shorter; try to see the whole solution at a glance. Try to modify to their advantage smaller or larger parts of the solution, try to improve the whole solution, to make it intuitive, to fit it into your formerly acquired knowledge as naturally as possible. Scrutinize the method that led you to the solution, try to see its point, and try to make use of it for other problems. Scrutinize the result and try to make use of it for other problems.

*What can I gain by doing so?* You may find a new and better solution, you may discover new and interesting facts. In any case, if you get into the habit of surveying and scrutinizing your solutions in this way, you will acquire some knowledge well ordered and ready to use, and you will develop your ability of solving problems.

**(Ⅲ) Teachers' purpose in the classroom**

1. **Helping the student.** One of the most important tasks of the teacher is to help his students. This task is not quite easy; it demands time, practice, devotion, and sound principles.

The student should acquire as much experience of independent work as possible. But if he is left alone with his problem without any help or with insufficient help, he may make no progress at all. If the teacher helps too much, nothing is left to the student. The teacher should help, but not too much and not too little, so that the student shall have a *reasonable share of the work.*

If the student is not able to do much, the teacher should leave him at least some illusion of independent work. In order to do so, the reader should help the student discreetly, *unobtrusively.*

The best is, however, to help the student naturally. The teacher should put himself in the student's place, he should see the student's case, he should try to understand what is going on in the student's mind, and ask a question or indicate a step that *could have occurred to the student himself.*

2. **Questions, recommendations, mental operations.** Trying to help the student effectively but unobtrusively and naturally, the teacher is led to ask the same questions and to indicate the same steps again and again. Thus, in countless problems, we have to ask the question: *What is the unknown*? We may vary the words, and ask the same thing in many different ways: What is required? What do you want to find? What are you supposed to seek? The aim of these questions is to focus the student's attention upon the unknown. Sometimes, we obtain the same effect more naturally with a suggestion: *Look at the unknown*! Question and suggestion aim at the same effect; they tend to provoke the same mental operation.

It seemed to the author that it might be worth while to collect and to group questions and suggestions which are typically helpful in discussing problems with students. The list we study contains questions and suggestions of this sort, carefully chosen and arranged; they are equally useful to the problem-solver who works by himself. If the reader is sufficiently acquainted with the list and can see, behind the suggestion, the action suggested, he may realize that the list enumerates, indirectly, *mental operations typically useful for the solution of problems.* These operations are listed in the order in which they are most likely to occur.

**注释与说明**

1. 世界著名数学家教育家 G. Polya(波利亚)的名著 How to solve it 有中译本,书名译成《怎样解题》,由科学出版社 1982 年出版。本节第一部分是波利亚精心设计的"解题表",第二部分是对解题表的说明,第三部分叙述教师的教学目的(这里只录其前两小节)。

2. 本节因为是论述数学的教与学的方法论,未涉及具体数学内容,因此它的写作风格与前面各节有明显不同之处,但总体较为通俗易懂。全世界的多数数学家与数学教育家公认波利亚的方法对教学极有帮助。读者不妨认真学习它,理解它,并在实践中加以应用。

**学习要求**

1. 掌握如下内容:

(1) 全文的主题和中心意思、各段的主要内容。

(2) 15 个生词与词组,包括 homogeneous(齐次的,同种的,均衡的),scrutinizing(周密调查的),redundant(多余的),perceive(发觉,理解),auxiliary

problems(辅助问题),recommendations(推荐,褒奖,忠告),mental operations(心算,智力运算)unobtrusively(不显眼的,谨慎的),take into account(对…加以考虑)等。

(3) 理解文中各个问题的含义。

2. 回答如下问题:

(1) 波利亚把解题的过程分成几个步骤,每个步骤的任务各是什么?

(2) 应该如何做好审题? 如何寻求有益的主意(办法)(hunting for the helpful idea)?

(3) 如何制定解题计划,如何执行解题计划? 解完一个题目之后为什么还要反思(looking back)?

(4) 为了提高学生的数学解题能力,教师应该如何帮助学生?

## §3.3 代数、几何与函数论

本节介绍高等数学的一些基础课程的部分内容,分为九小节。前四小节是高等代数的内容,其中第一小节是整数的知识;第二小节是素数的概念;第三小节是多项式与代数函数;第四小节是群的概念。第五小节介绍平行公理,是欧几里得几何的内容。接下来的四小节是解析几何的内容。其中,第六小节介绍 $n$ 元组向量空间;第七小节介绍圆锥曲线;第八小节介绍高维曲线;最后一小节,即第九小节介绍复解析函数零点的性质。

这九篇短文内容丰富,专业性强,但还是较易读懂的。具有大学一年级数学基础和公共英语水平的同学,只要做一些努力,就可以学好。而且,读者若能通过认真地学习本节文章,掌握其中的专业英语知识和阅读理解的方法,今后在阅读有关基础数学的英语文献时就不会有太大的难度了。

本节的"注释与说明"仅在第一小节给出,作为示范或参考。至于其余各小节,请读者自行学会处理阅读中可能遇到的困难和问题。

### 3.3.1 Integers ①

The integers are familiar to us from elementary arithmetic, but here we shall want to express that familiarity in precise terms. We do this by writing down a list of properties which the integers possess and on which we shall base all our deductions. Later, in Vol. 3, we shall see that all the properties listed here can actually be de-

① 本节课文摘自:P. M. Cohn. Algebra. New York: John Wiley & Sons, 1982。

duced from quite a brief list of axioms, but this is immaterial at present.

We denote the set of positive integers (also called natural numbers) 1, 2, 3, ⋯ by **N** and write **Z** for the set of all integers, positive, negative and zero. Here **N** stands for number and **Z** for Zahl, the German for number; both abbreviations are generally used in mathematics.

The set **Z** admits three operations: addition, $x+y$, subtraction, $x-y$, and multiplication, $x \cdot y$ or $xy$. Often it is convenient to express subtraction by adding the negative: $x-y=x+(-y)$. These operations are connected by the following laws:

**Z.**1 *Associative law*: $(x+y)+z=x+(y+z)$, $(xy)z=x(yz)$.

**Z.**2 *Commutative law*: $x+y=y+x$, $xy=yx$.

**Z.**3 *Existence of neutral element*: $x+0=x$, $x1=x$.

**Z.**4 *Existence of (additive) inverse*: $x+(-x)=0$.

The number 0 is said to be *neutral for addition* because adding it to any number $x$ leaves $x$ unchanged; likewise 1 is neutral for multiplication. Every number $x$ has the additive inverse $-x$ (which undoes the effect of adding $x$), but apart from 1 and $-1$, no integer has a multiplicative inverse. However we shall find such inverses once we come to consider rational numbers in 2.4.

In addition to the above laws, there is a further law, relating addition and multiplication:

**Z.**5 *Distributive law*: $x(y+z)=xy+xz$.

A set $R$ with two operations $x+y$, $xy$ and a negative $-x$, satisfying **Z.**1–5 is called a *ring*; more precisely it is a *commutative ring* (because the multiplication is commutative, cf. Ex. (1), 6.1). Thus the set **Z** of all integers is a commutative ring. However, these laws are not yet sufficient to determine **Z**; in Ch. 6 we shall give a general definition of a ring and we shall find that there are many different types.

We now look at some consequences of the above laws. It follows from the distributive law that $x0=0$ for all $x$. By the associative law, the sum of any number of terms is independent of the way in which brackets are placed, and by the commutative law the order of the terms is immaterial. A similar remark applies to multiplication; for the present we shall accept this without proof and return to this point in Ch. 3 to give a general proof.

Thus the sum of numbers $a_1, \cdots, a_n$ may be written $a_1+\cdots+a_n$. Often one abbreviates this expression by writing down the general term $a_\nu$ with a capital sigma, $\sum$, to show that the sum is to be taken, with some indication of the range over which the terms are to be summed (unless this is clear from the context). So instead of $a_1+\cdots+a_n$ we may write

$$\sum_{\nu=1}^{n} a_\nu \quad \text{or} \quad \sum\nolimits_1^n a_\nu \quad \text{or} \quad \sum_\nu a_\nu \quad \text{or} \quad \text{simply} \quad \sum a_\nu,$$

where in each case $\nu$ is a dummy variable (cf. 1.1). When $n=0$, the sum written here is empty and, by convention, this is taken to be 0. This notation is not only briefer; it can also help to make our formulae more perspicuous as well as more accurate. For instance, in the expression

$$1+2+\cdots+n,$$

the reader is expected to guess that he is dealing with an arithmetic progression; the expression $\sum_1^n a_\nu$ removes all doubt. Thus the formula for the sum of the first $n$ natural numbers may be written $\sum_1^n \nu=\frac{1}{2}n(n+1)$. We observe that for $n=0$ the right-hand side reduces to 0, so with our convention about empty sums, this formula still holds for $n=0$.

For another example consider the distributive law. This has a generalized version which reads (cf. Ex. (2))

$$(a_1+\cdots+a_r)(b_1+\cdots+b_s)=a_1b_1+a_2b_2+\cdots+a_rb_s,$$

or in abbreviated form $\sum a_\mu \cdot \sum b_\nu = \sum_{\mu,\nu} a_\mu b_\nu$. Here we have not indicated the precise range of summation, since it is immaterial, but only the indices of summation $\mu$, $\nu$.

A similar abbreviation exists for repeated products, using capital pi, $\Pi$, in place of $\sum$. Thus instead of $a_1a_2\cdots a_n$ we write

$$\prod_{\nu=1}^{n} a_\nu \quad \text{or} \quad \prod_1^n a_\nu \quad \text{or} \quad \prod_\nu a_\nu \quad \text{or} \quad \text{simply} \quad \prod a_\nu.$$

For example, the factorial function may be defined as $n! = \prod n$. An empty product is taken to be 1; thus empty sums and products are neutral for addition and multiplication respectively.

It is an important property of the integers that the product of two non-zero integers is never zero:

**Z.6** *For any integers $a$, $b$, if $a\neq 0$ and $b\neq 0$, then $ab\neq 0$; moreover $1\neq 0$.*
This has the following useful consequence:

Cancellation law: For $a, b, c\in \mathbf{Z}$, if $ca=cb$ and $c\neq 0$, then $a=b$.

This asserts that multiplication by a non-zero integer is an injective mapping of $\mathbf{Z}$ into itself. To prove it, suppose that $a \neq b$, then $a-b \neq 0$ and hence (by **Z**. 6) $c(a-b) \neq 0$, therefore $ca-cb=c(a-b) \neq 0$。

Besides the operations on **Z** we have an order relation, i. e. an ordering on **Z**: $x \leqslant y$ or $y \geqslant x$. If $x \leqslant y$ but $x \neq y$, we write $x<y$ or also $y>x$. This relation satisfies the requirements for a total ordering (see 1.5) and is related to the operations of **Z** by the following rules:

**Z.** 7 *If $x_1 \leqslant x_2$ and $y_1 \leqslant y_2$, then $x_1+y_1 \leqslant x_2+y_2$.*

**Z.** 8 *If $x \leqslant y$ and $z>0$, then $zx \leqslant zy$.*

The presence of these rules means that **Z** is a *totally ordered ring*. Using the ordering we can describe the set **N** of positive integers as

$$\mathbf{N} = \{x \in \mathbf{Z} \mid x>0\}. \quad (1)$$

Later we shall see how to reconstruct **Z** from **N**; for the moment we note that, for every $x \in \mathbf{Z}$, either $x=0$ or $x \in \mathbf{N}$ or $-x \in \mathbf{N}$ and that these three possibilities are mutually exclusive. In fact, this is true in any totally ordered ring, taking **N** to be defined by (1). For we know that just one of the following holds (because we have a total order): $x=0$ or $x>0$ or $x<0$. Now $x+(-x)=0$, hence, if $x<0$, then $0<-x$ by **Z**. 7. Thus either $x=0$ or $x>0$ or $-x>0$, as asserted.

In order to fix **Z** completely, we use the following condition on the set **N** of positive integers:

**I** (*Principle of induction*): Let $S$ be a subset of **N** such that $1 \in S$ and $n+1 \in S$ whenever $n \in S$. Then $S=\mathbf{N}$.

This principle forms the basis of the familiar method of proof by induction. Let $P(n)$ be an assertion about a positive integer $n$, e. g., $P(n)$ might be the sum of the first $n$ positive integers is $n(n+1)/2$. Suppose we wish to prove $P(n)$ for all $n$, i. e. $(\forall n)\ P(n)$. Then by **I** it will be enough to prove (i) $P(1)$ and (ii) $(\forall n)\ (P(n) \Rightarrow P(n+1))$.[2] For this means that the set $S$ of all $n$ for which $P(n)$ holds contains 1 and contains $n+1$ whenever it contains $n$. Hence by **I**, $S=\mathbf{N}$, i. e. $P(n)$ holds for all $n \in \mathbf{N}$.

There are two alternative forms of **I** that are often useful.

**I′**: Let $S$ be a subset of **N** such that $1 \in S$ and $n \in S$ whenever $m \in S$ for all $m<n$; then $S=\mathbf{N}$.

**I″** (Principle of the least element): Every non-empty set of positive integers has a least element.

To prove **I**, **I′** and **I″** equivalent we shall establish the implications **I**⇒**I′**⇒**I″**⇒**I**.

**I⇒I′**. Let $S$ be such that $1 \in S$ and $n \in S$ whenever $m \in S$ for all $m<n$. Define $T$

$= \{x \in \mathbf{N} \mid y \in S$ for all $y \leqslant x\}$, thus $x \in T$ precisely when all the numbers from 1 to $x$ lie in $S$. Clearly $T \subset S$, so it will be enough to show that $T = \mathbf{N}$. Since $1 \in S$, we have $1 \in T$ and, if $n \in T$, then $y \in S$ for all $y \leqslant n$, hence $n+1 \in S$ and so $y \in S$ for all $y \leqslant n+1$; but this means that $n+1 \in T$. Applying **I**, we see that $T = \mathbf{N}$.

**I′⇒I″.** Let $S$ be a set of positive integers without a least element; we shall show that $S$ is empty. Denoting the complement of $S$ by $S'$, we must show that $S' = \mathbf{N}$. Now, since $S$ has no least element, $1 \notin S$, so $1 \in S'$; moreover, if $m \in S'$ for all $m < n$, then $n \in S'$, for otherwise $n$ would be the least element in $S$. Thus by **I′**, $S' =$ $\mathbf{N}$ and $S$ must be empty.

**I″⇒I.** 1 is the least element of **N** (for if the least element $r$ satisfied $r<1$, then $r<r$, a contradiction). Let $S$ be a subset of **N** such that $1 \in S$ and $n+1 \in S$ whenever $n \in S$, then the complement $S'$ of $S$ in **N** has no least element. For $1 \notin S'$ and if $n \in S'$, then $n-1 \in S'$, hence, by **I″**, $S' = \varnothing$ and so $S = \mathbf{N}$ as we wished to show.

We end this section with a practical remark on proofs by induction. Generally a theorem is easier to prove, the stronger the hypothesis and the weaker the conclusion. But in an induction proof the conclusion at the $n$th step becomes the hypothesis at the $(n+1)$th step and the theorem may actually become easier to prove if the conclusion is strengthened; for an instance of this see Lemma 4.3. We also remark that an induction frequently starts at 0 (instead of 1); clearly this does not affect the validity.

**Exercises**

(1) Prove that $(a+b)(c+d) = ac+bc+ad+bd$ from the axioms.

(2) Prove $a\left(\sum b_\nu\right) = \sum ab_\nu$ by induction. Deduce the general distributive law:

$$\left(\sum a_\mu\right)\left(\sum b_\nu\right) = \sum a_\mu b_\nu.$$

(3) Prove that for any integer $a$, $a0 = 0$.

(4) Prove the rule of signs: $(-a)b = -ab$, $(-a)(-b) = ab$.

(5) For any integers $m$, $n$, prove that $m, n \geqslant 1 \Rightarrow mn \geqslant 1$, with '>' unless $m = n = 1$.[3]

**注释与说明**

1. 本小节虽然有些新的数学专业词汇，但多数可从上下文出现的表达式推测其含义。例如：从

**Z.1** *Associative law*: $(x+y)+z = x+(y+z)$, $(xy)z = x(yz)$.

**Z.2** *Commutative law*: $x+y = y+x$, $xy = yx$.

容易想到,*Associative law* 为"结合律";*Commutative law* 为"交换律"。类似地,从上下文推测出:*Distributive law* 为"分配律";*commutative ring* 为"交换环";*neutral element* 为"零元素";(*additive*) *inverse* 为"(加法的)逆元素";*Principle of induction* 为"归纳法原理";Principle of the least element 为"最小数原理";repeated products 为"连乘积";empty product 为"空乘积";indices of summation 为"求和指标"等。

当然如果您对推测无把握,还应查字典加以验证。

2. (i) $P(1)$ and (ii) $(\forall n)(P(n) \Rightarrow P(n+1))$ 是逻辑运算式。$(\forall n)P(n)$ 表示"命题 $P$ 对任意 $n$ 成立",$\Rightarrow$表示"蕴涵",(i) $P(1)$ 和(ii) 一块表示数学归纳法的内容,即该逻辑运算式表示(i) 当 $n=1$ 时命题 $P$ 成立,(ii) 命题 $P$ 对任意 $n$ 成立蕴涵命题 $P$ 对任意 $n+1$ 成立。

3. For any integers $m$, $n$, prove that $m$, $n \geqslant 1 \Rightarrow mn \geqslant 1$, with '$>$' unless $m=n=1$。这里 with 引起的短语做伴随状语。整句可译成:对任意整数,证明 $m,n \geqslant 1$ 蕴涵 $mn \geqslant 1$,而且当不出现 $m=n=1$ 时 $mn>1$ 成立。

**学习要求**

1. 掌握如下内容:

(1) 全文的主题和中心意思、各段的主要内容。

(2) 15 个新旧数学术语,5 ~ 8 个表示数学命题的句型。其中数学术语包括注释与说明之 1 中所列出者。

(3) 理解各习题的含义,特别是要求读者完成的任务。

2. 回答如下问题:

(1) 整数满足哪些运算规律?

(2) 如何用数学归纳法证明命题?

(3) 如何证明数学归纳原理与最小数原理等价?

### 3.3.2 Prime numbers ①

The following classification of the integers according to the integers that they divide, or are divisible by, will greatly facilitate our study. Zero has been defined (Section 1.5) as the identity under addition. The integers $+1$ and $-1$ are called units (Section 2.1). An integer $p$ that is not zero or a unit is said to be *prime* if its only divisors are $\pm p$ and the units. An integer is called *composite* if it has two or more

① 本节课文摘自:B. E. Meserve. Fundamental Concepts of Algebra. New York: Dover Pub. Inc, 1982。

prime divisors (not necessarily distinct). For example, $6=2\cdot 3$ and $121=11^2$ are composite numbers. We shall find that every integer belongs to one of four classes: zero, units, prime numbers, composite numbers. Since zero is neither positive nor negative, this will mean that every positive integer belongs to one of three classes and every positive integer greater than one is either prime or composite. In the following discussion, negative prime numbers are assumed to be expressed in the form $ep$, where $e$ is the unit $-1$ and $p$ is a positive prime. Thus only positive prime numbers need be considered.

We now use these definitions in the proofs of several theorems.

**THEOREM 2.2** *Every integer greater than one has a positive prime divisor.*

Let $m$ be any given integer greater than one. Then $m$ is prime if and only if its only positive divisors are $m$ and 1.

If $m$ is not a prime, it has a positive divisor $m_1$, where $m_1\neq m$ and $m_1\neq 1$. Thus if $m$ is not prime, it may be written as the product of two positive integers, $m=m_1m_2$, where neither $m_1$ nor $m_2$ is a unit. If neither $m_1$ nor $m_2$ is prime, then $m=m_{11}m_{12}m_{21}m_{22}$ where no $m_{ij}$ is a unit. If no $m_{ij}$, is a prime, then $m=m_{111}m_{112}m_{121}m_{122}m_{211}m_{212}m_{221}m_{222}$, where no $m_{ijk}$ is a unit. This process will terminate if and only if at some step at least one of the $m$'s is a prime number. We shall now show that for any given positive integer $m$ the process must terminate, i. e., it cannot continue indefinitely. First, we observe that any positive integer $m_2$ which is not a unit satisfies the order relation $m_2>1$ (Section 1.6). Then we also have $m=m_1m_2>m_1$ and, in general,

$$m>m_1>m_{11}>m_{111}>\cdots$$

for as many steps as the process continues. Thus the process terminates if and only if the set of positive integers $m$, $m_1$, $m_{11}$, $m_{111}$, $\cdots$ is a finite set. However, this set is a subset of the finite set $m$, $m-1$, $m-2$, $\cdots$, 3, 2, 1 and therefore must itself be finite. Thus the above process must terminate after a finite number of steps, and $m$ must have a prime divisor.

We have also proved that any given positive integer $m$ can have only a finite number of positive integral divisors greater than one. Our next theorem indicates which positive integers need to be considered when one seeks the positive divisors of a given integer $m$.

**THEOREM 2.3** *If a positive integer $m$ is composite, it has a positive prime divisor $\leqslant I$, where $I$ is the greatest integer whose square is $\leqslant m$.*

By Theorem 2.2, any positive integer $m$ greater than one has a positive prime divisor $p$, that is, $m=pm_1$. Also, if $m\neq p$, then $m_1$ has a positive prime divisor $\leqslant m_1$. If Theorem 2.3 were false, there would exist a number $m$ that was composite and had no positive prime divisor $\leqslant I$. In this case, we would have $I<p$, $I<m_1$ or $I+1\leqslant p$, $I+1\leqslant m_1$, and $(I+1)^2\leqslant pm_1=m$, contrary to the assumption that $I$ is the greatest integer whose square is $\leqslant m$. Thus Theorem 2.3 must be true (method of indirect proof, Section 1.10).

Before we can use Theorem 2.3 to determine whether or not a given integer $m$, say 359, is prime, we need some method for determining the primes $\leqslant I$ where $I^2\leqslant m<(I+1)^2$. For the case $m=359$ we need to know the prime numbers $\leqslant 18$.

The prime numbers bounded by any finite integer $N$ may be found by a method called the *Sieve of Eratosthenes*: Write down the integers from 1 to $N$, exclude 1 since it is a unit, counting from 2 strike out every second number thereafter, counting from 3 strike out every third number, and, in general, counting from any remaining integer $k$ which is $\leqslant\sqrt{n}$ (Theorem 2.3) strike out every $k$th integer. For example, the prime numbers bounded by $N=18$ are 2, 3, 5, 7, 11, 13, 17, and may be found from the array

$$2, 3, \underline{4}, 5, \underline{6}, 7, \underline{8}, \underline{9}, \underline{10}, 11, \underline{12}, 13, \underline{14}, \underline{15}, \underline{16}, 17, \underline{18}$$

in which it was only necessary to exclude the unit and multiples of 2 and 3 since the next remaining integer, 5, has a square greater than 18.

We now may use Theorem 2.3 and determine whether or not 359 is a prime by testing 359 successively for divisibility by 2, 3, 5, 7, 11, 13, 17. On this basis we may assert that 359 is a prime number.

One reason for considering such mechanical methods as the above for determining primes lies in the fact that no analytical representation or formula for all primes has yet been found. However, we may prove several theorems regarding primes. The following theorem is a modern version of Proposition 20 in Book IX of Euclid's *Elements*.

**THEOREM 2.4** *The set of positive prime numbers is countably infinite.*

Suppose there were a largest prime number, say $P$, then the number $N=P!+1$ must have a prime divisor (Theorem 2.2). But no number $\leqslant P$ divides $P!+1=N$. Thus $N$ has a prime divisor greater than $P$ and there is no greatest prime, i.e., the set of positive prime numbers is countably infinite. For example, if $P=2$, then $N=2!+1=3$, which is prime, if $P=5$, then $N=5!+1=121$, which has $11>5$ as a prime divisor. This process for determining the existence of a prime greater than a given prime $P$ may also be used, together with the existence of a single prime number 2, to prove by mathematical induction (Section 1.4) that there exists a countably infinite subset of the set of positive prime numbers. Then, since the set of all positive prime numbers is a subset of the set of positive integers, which is countably infinite, we have another proof that the set of positive prime numbers is countably infinite.

The best-known properties of primes concern divisibility. Given any integer $m$ and prime $p$, the only positive divisors of $p$, and therefore the olny possible positive common divisors of $p$ and $m$, are $p$ and 1. Thus we have

**THEOREM 2.5** *If $p$ is a prime and $m$ is any integer, then either $p$ divides $m$ or* $(p, m)=1$.

Another common theorem may be proved as follows: Suppose $p$ is a prime number, and $a$ and $b$ are each positive integers less than $p$. We wish to prove that $p$ does not divide the product $ab$, written $p \nmid ab$. We shall use the method of indirect proof and suppose that $p \mid ab$. Furthermore, we shall assume that $b$ is the smallest positive integer such that $p \mid ab$, that is, $ab$ is the least multiple of $a$ such that $p \mid ab$. This last assumption may be made without loss of generality, since if there exists a single integral multiple, there must be a smallest positive integral multiple of $a$ that is divisible by $p$ (Section 2.2). Now by the Division Algorithm, there exists an integer $m$ such that

$$mb \leqslant p<(m+1)b, 0 \leqslant p-mb<b.$$

Actually $mb \neq p$ since $1<b<p$ and $p$ is prime. By assumption, $p \mid ab$ and thus $p \mid mab$. Then from $p \mid ap$ we have $p \mid (ap-mab)$ *and* $p \mid a(p-mb)$, whence $a(p-mb)$ is a multiple of $a$ which is divisible by $p$. But also $a(p-mb)<ab$, contrary to the assumption that $ab$ is the least multiple of $a$ that is divisible by $p$ Thus $p$ does not divide $ab$, and we have given an indirect proof of the following theorem.

**THEOREM 2.6** *If $p$ is a prime, and $a$ and $b$ are two positive integers each less than $p$, then $p \nmid ab$.*

This theorem may be extended to include any two positive integers $a$ and $b$ such that $p \nmid a$ and $p \nmid b$. Let $a=mp+r$, $b=np+s$, $0<r<p$, $0<s<p$. Now if $p \mid ab$, we also have $p \mid rs$, contrary to Theorem 2.6. Thus if $p \nmid a$ and $p \nmid b$, then $p \nmid ab$. In other words, if $p \mid ab$, then $p \mid a$ or $p \mid b$. Since the product of two integers is an integer, we may also take $a_1 \cdot a_2=a$, $a_3=b$ and prove that if $p \mid a_1a_2a_3$, then $p$ divides at least one of the numbers $a_1$, $a_2$, $a_3$. By repeated application of this process, we have

**THEOREM 2.7** *If $p$ is a prime and $p \mid a_1 a_2 \cdots a_n$, then $p$ divides at least one of the integers $a_1, a_2, \cdots, a_n$, where $n$ is any positive integer.*

A very important application of this property of prime numbers is found in the factorization of all positive integers as products of powers of prime numbers (Section 2.4). Throughout the remainder of this text we shall make extensive use of the properties of prime numbers and the analogous properties of irreducible polynomials (Section 3.6).

**Exercises**

1. Find the prime numbers less than 200, using the Sieve of Eratosthenes.

2. Determine which of the following are prime numbers:

(a) 85, 103, 179, 539,

(b) 267, 781, 859, 937,

(c) 1245, 2287.

3. Write out a formal proof of Theorem 2.7, using mathematical induction.

4. Is $n^2-n+41$ a prime number for all positive integral values of $n$? Explain.

5. Give four numerical examples illustrating Theorem 2.5.

6. Repeat Exercise 5 for Theorems 2.6 and 2.7.

7. Given any integer $N$, how could you find all its positive prime divisors?

8. Prove that $n^3+1$ is a composite number if $n$ is greater than one.

9. Prove that $3^n-1$ and, in general, $m^n-1$ is composite if $n$ is greater than one and $m$ is greater than 2 (see Exercise 7, Section 1.4).

10. A number of the form $2^p-1$ that is prime is called a *Mersenne prime*. Find five such numbers.

11. Prove that $2^n-1$ is composite if $n$ is composite (see Exercise 9, Section 1.4). Give an example of a composite number of the form $2^p-1$ where $p$ is a prime.

**学习要求**

1. 掌握如下内容：

(1) 全文的主题和中心意思、各段的主要内容。

(2) 15 个新旧数学术语,5 ~ 8 个表示数学命题的句型。其中数学术语包括 prime(素数/素数的), composite(合数/合数的),prime divisor(素因子),divisibility(可除性),countably infinite(可数无穷),sieve of Eratosthenes(埃拉托色尼筛法),strike out(排除)等。

(3) 理解各习题的含义,特别是要求读者完成的任务。

2. 回答如下问题：

(1) 本文提出哪些概念,得出哪些主要结论?

(2) 证明定理2-2的主要方法和步骤是什么?

(3) 如何求出不大于某个正整数 $N$ 的所有素数? 请描述埃拉托色尼筛法的内容。

## 3.3.3 Polynomials and Algebraic Functions ①

### 1. Polymomials

The positive integers have been used in Chapter 1 to define rational, algebraic, transcendental, and real numbers. Properties of the ring of integers have been discussed in Chapter 2. In this chapter we shall use a ring of polynomials in one variable to define rational, algebraic, transcendental, and analytic functions. Divisibility, the Division Algorithm, the Euclidean Algorithm and properties in the ring of polynomials corresponding to prime numbers, bases, and congruence's in the ring of numbers will be discussed. Our purpose is threefold: to understand the basic properties of polynomials, to see the relationships between polynomials and other common functions, and to introduce a few concepts that will be needed in our discussion of the theory of equations in Chapter 4.

In the first two chapters, we have been primarily concerned with numbers: integers, rational numbers, real numbers, and complex numbers. We now introduce a new set of symbols $x$, $y$, $t$, and consider equality, addition, subtraction, multiplication, and division in the total set composed of the new symbols and the complex numbers. The new symbols may be considered simply as symbols without any sets of values or assumed relations. In this case they are called indeterminates. The new symbols may also be considered as variables that take on values from a subset of the set of complex numbers. We shall usually call the symbols variables, although we shall at times mention corresponding properties of indeterminates. A great deal of the theory considered in this chapter will apply to both variables and indeterminates.

Given any indeterminate $x$, we define the symbol $x^n$ for any positive integer $n$ to represent the product of $n$ factors $x$, $x^0 = 1$, $x^{-n}x^n = 1$, and $(x^{1/n})^n = x$. Similar definitions hold for any variable $x$, with the exception that $x^n$ and $x^{-n}$ are undefined when $x = 0$. Addition and multiplication of the new symbols and complex numbers may be defined such that they are unique, commutative, associative, and satisfy the distribu-

① 本节课文摘自:B. E. Meserve. Fundamental Concepts of Algebra. New York: Dover Pub. Inc, 1982.

tive laws. Thus $ax+bx=(a+b)x$ and $(ax)(bx)=abx^2$ for any complex numbers $a$, $b$.

The product of any set of complex numbers and the new symbols is called a monomial. For example, 15, $x$, $2x$, $5x^2y^3t$, and $3\sqrt{2}\,xy$ are monomials. The sum of two monomials is called a binomial. A sum of three monomials is called a trinomial and, in general, a sum of one or more monomials is called a polynomial.

A monomial of the form $bx^m$, where $m$ is a nonnegative integer and $b$ is a complex number, is called a monomial in $x$ with coefficient $b$ and, when $b \neq 0$, of *degree m*. Any complex number $b$ is itself a monomial. The monomial 0 has no degree. When $b \neq 0$, the monomial $b=bx^0$ has degree zero.

A polynomial of the form

$$a_0x^m + a_1x^{m-1} + \cdots + a_{m-1}x + a_m, \tag{3.1}$$

where the $a_j$ are complex numbers and $a_0 \neq 0$, is called a polynomial of degree $m$ in $x$. The $a_j$ are called the coefficients of the polynomial. The nonzero leading coefficient $a_0$ is called the initial of the polynomial. Since indeterminates do *not* take on numbers as values, two polynomials in an indeterminate $x$ are equal if and only if the coefficients of corresponding powers of $x$ are equal. Thus, for an indeterminate $x$, the equation

$$ax^2+bx+c=x+2$$

implies that $a=0$, $b=1$, and $c=2$. Two polynomials in a variable $x$ may be equal for any nonnegative integral number of values of the variable $x$. Thus for a variable $x$, the equation $x^2-2x-3=0$ implies $x=3$ or $x=-1$.

The degree of a polynomial depends upon the variable under consideration. For example, $3x^2y^5$ is of degree two in $x$ with coefficient $3y^5$ and of degree five in $y$ with coefficient $3x^2$. The polynomial $10x^6$ may be considered as a polynomial of degree six in $x$ with coefficient 10, a polynomial of degree two in $2x^3$ with coefficient $\frac{5}{2}$, a polynomial of degree twelve in $\sqrt{x}$ with coefficient 10, and in many other ways. We shall use the notation $p(x)$ to indicate a polynomial in $x$, $p(\sqrt{2x})$ to indicate a polynomial in $\sqrt{2x}$.

**2. Algebraic functions**

A polynomial equation $f(x,y)=0$, where $f(x,y)$ is considered as a polynomial in $y$ with coefficients from the ring of polynomials in $x$ with complex coefficients, defines $y$ as an algebraic function (Section 3.16) and has as its real graph an *algebraic* *plane curve*. The concept of an algebraic function is an extension of the concept of a polynomial in that every polynomial $p(x)$ satisfies $f(x,y)=y-p(x)$, that is, a polynomial is a special case of an algebraic function that arises when $f(x,y)$ has the form $y-p(x)$. Similarly, a rational function (Section 7.6) is a special case of an algebraic function that arises when $f(x,y)$ has the form $p(x)y-q(x)$.

We have seen (Section 3.10) that any polynomial $p(x)$ is a single valued function of $x$. Also, a rational function of $x$ is single-valued whenever it is defined (Section 3.3). However, an algebraic function defined by a polynomial equation $f(x,y)=0$ of degree $n$ in $y$ may have $n$ values of $y$ corresponding to a given value of $x$. For example, $y^2-x=0$ associates two real values of $y$ with each positive value of $x$. Our discussion in this section consists of a very brief introduction of a graphical representation (the Riemann surface) of the $n$ values of the algebraic function $y$( not necessarily real or distinct) corresponding to each value of $x$( Theorem 4.2).

**学习要求**

1. 掌握如下内容:

(1) 全文的主题和中心意思、各段的主要内容。

(2) 10 个新出现的数学单词与术语,3～6 个表示数学命题的句型。其中数学单词与术语包括:congruence(同余),indeterminate(未定元),symbols variables (符号变量),polynomial(多项式),a polynomial in $x$(关于 $x$ 的多项式),algebraic plane curve(代数平面曲线),single valued function(单值函数),Riemann surface (黎曼曲面),algebraic function(代数函数)等。

2. 回答如下问题:

(1) 什么叫做单项式、二项式、三项式、多项式?

(2) 怎样理解未定元与变量的异同点?

(3) 如何判断两个多项式相等?

(4) 代数函数是如何定义的? 为什么说它是多项式的推广?

## 3.3.4 Groups ①

### 1. Monoid

The number systems described in Ch. 2 had two basic operations: addition and multiplication, and sometimes their inverses: subtraction and division. We now want

① 本节课文摘自:P. M. Cohn. Algebra. New York: John Wiley & Sons, 1982。

to study the general properties of an associative but not necessarily commutative operation. The method of doing this is not to look at a particular system such as the integers or the rational numbers, but to assume that we have an arbitrary set with an operation satisfying the associative law and see what consequences can be derived in this way. It will be useful to begin with a formal definition.

By a *monoid* we understand a set $S$ with an element $e$ and a mapping $\mu$: $S^2 \to S$ such that if $\mu(x,y)$ is the result of applying $\mu$ to the pair $x,y \in S$, then

**M.1** $\mu(x, \mu(y,z)) = \mu(\mu(x,y), z)$ for all $x,y,z \in S$.

**M.2** $\mu(e,x) = \mu(x,e) = x$ for all $x \in S$.

Note that by definition, a monoid is never empty. A mapping such as $\mu$ which acts on pairs of elements of $S$ is called a binary operation, with values in $S$, and an element $e$ satisfying **M.** 2 is said to be a neutral element for $\mu$. There cannot be more than one neutral, for if we also have $\mu(e', x) = \mu(x, e') = x$, then $e = \mu(e, e') = e'$.

Clearly both addition and multiplication are instances of such a binary operation, and in fact **Z** is a monoid under addition (with 0 as neutral) as well as multiplication (with 1 as neutral). This example illustrates that in discussing particular instances of monoids one must name not only the set but also the operation unless this is clear from the context.

## 2. Groups; the axioms

By a *group* one understands a monoid in which every element is invertible.

The commutative law is not assumed; when it holds, i.e. if $xy = yx$ for all $x,y$. the group is said to be *commutative* or *abelian* (after N. H. Abel). Thus a group $G$ is defined by the following four conditions:

**G.1** *$G$ has a binary operation $xy$ defined on it.*

**G.2** *The operation is associative*: $(xy)z = x(yz)$ for all $x,y,z, \in G$.

**G.3** *$G$ has a neutral element* 1: $1x = x1 = x$ *for all* $x \in G$.

**G.4** *Every element $x \in G$ has an inverse* $x^{-1}$: $xx^{-1} = x^{-1}x = 1$.

It is a remarkable fact that the whole of group theory, which includes many deep results and is far from being fully explored yet, rests ultimately on this simple set of axioms. Of course we shall only be able to touch on the most basic properties in this chapter and in Ch. 9, but it is hoped that even this brief account will convey something of the inherent simplicity and beauty of the theory.

We begin by observing that the axioms **G.** 1–4 are to some extent redundant: less is needed to verify that we have a group. This remark is of practical use as it may help to shorten our verifications. Before proving it we note another useful consequence of the group axioms:

**G.5** *Given $a$, $b \in G$, the equations $bx = a$, $yb = a$ each have a unique solution, namely $x = b^{-1}a$, $y = ab^{-1}$.*

For if $bx = a$, then $b'a = b'bx = 1x = x$, so there is at most one solution and, in fact, $b(b^{-1}a) = bb^{-1}a = 1a = a$. Thus $bx = a$ has exactly one solution; by symmetry the same holds for $yb = a$.

**THEOREM 1.** *Let $G$ be a set with a binary operation which is associative. Then the following conditions on $G$ are equivalent:*

(a) *$G$ is a group,*

(b) *$G$ is not empty and for all $a$, $b \in G$, the equations $bx = a$, $yb = a$ each have a solution,*

(c) *there exists $e \in G$ such that $xe = x$ for all $x \in G$ and if we fix such $e$, then for each $x \in G$ there exists $x' \in G$ such that $xx' = e$.*

Note that we do not assume that the solutions in (b) are unique.

We shall prove the theorem according to the scheme (a)$\Rightarrow$(b)$\Rightarrow$(c)$\Rightarrow$(a). The remarks preceding the theorem show that (a)$\Rightarrow$(b). Now assume (b), take $a \in G$ and find $e \in G$ to satisfy $ae = a$. For any $b \in G$ there exists $x \in G$ such that $xa = b$ (by (b)) and hence $b = xa = xae = be$; moreover, $aa' = e$ always has a solution $a'$, therefore (c) follows.

Next assume (c); we must show that $G$ is a group. Given $x \in G$, we can find $x' \in G$ with $xx' = e$ and then $x'' \in G$ with $x'x'' = e$. Hence $x'x = x'xe = x'xx'x'' = x'ex'' = x'x'' = e$; thus $xx' = e = x'x$. Moreover, $x = xe = xx'x = ex$, hence $e$ is neutral and $x'$ is an inverse of $x$, i.e. $G$ is indeed a group.

As a first application we have

**PROPOSITION 2** A finite monoid $S$ with either of the properties

$$xz = yz \Rightarrow x = y \text{ for all } x, y, z \in S \text{ (right cancellation)}, \tag{1}$$

$$zx = zy \Rightarrow x = y \text{ for all } x, y, z \in S \text{ (left cancellation)} \tag{2}$$

is a group.

*Proof.* Assume that one of (1), (2) holds, say (2); by (2) the mapping $\lambda_z$: $x \to zx$ of $S$ into itself is injective, hence bijective (by Lemma 3, 1.3, because $S$ is finite). Thus $bx = 1$ has a solution for any $b \in S$; hence $S$ satisfies (c) of Th. 1 and is therefore a group.

The notions of homomorphism, isomorphism, endomorphism and automorphism defined for monoids carry over to groups as a special case. We note: to verify that a mapping $f$: $G \to H$ between groups is a homomorphism we need only check that

$(xy)f=xf\cdot yf$, for if this holds then $(1_G f)^2=1_G f$, where $1_G$ is the neutral of $G$; by division it follows that $1_G f$ is the neutral of $H$. Since homomorphisms preserve neutrals it will cause no confusion to denote the neutral in any group by the same symbol 1. We also note that for any group homomophism $f$, we have $(xf)(x^{-1}f)=(xx^{-1})f=1f=1$, and similarly $(x^{-1}f)(xf)=1$, hence

$$(xf)^{-1}=x^{-1}f. \qquad (3)$$

Let $G$ be a group, then by a subgroup of $G$ one understands a subset $H$ of $G$ which is a group relative to the operations in $G$. Thus $H$ is a subgroup of $G$ iff (i) $1\in H$, (ii) $x,y\in H\Rightarrow xy\in H$, (iii) $x\in H\Rightarrow x^{-1}\in H$. Alternatively, $H$ is a subgroup of $G$ iff (iv) $H\neq\varnothing$ and $x,y\in H\Rightarrow xy^{-1}\in H$. Clearly (iv) holds for any subgroup of $G$; conversely, when it holds, take $a\in H$, then $1=aa^{-1}\in H$, hence for any $x\in H$, $x^{-1}=1x^{-1}\in H$, and whenever $x,y\in H$ then $y^{-1}\in H$ by what has just been proved, and so by (iv), $xy=x(y^{-1})^{-1}\in H$, therefore $H$ is indeed a subgroup of $G$.

In discussing groups the following notation is often useful. Let $A$, $B$ be any subsets of a group $G$ and write

$$AB=\{xy\mid x\in A,y\in B\},\qquad A^{-1}=\{x^{-1}\mid x\in A\}.$$

If $A$ reduces to a single element: $A=\{a\}$, we write $aB$ instead of $\{a\}B$, and similarly for $B$. To give an example, the condition for a non-empty subset $H$ of $G$ to be a subgroup may be restated as $HH\subseteq H$ and $H^{-1}\subseteq H$, or equivalently, $HH^{-1}\subseteq H$. Some care must be taken to write $HH$ and not $H^2$ (which is sometimes taken to mean $\{x^2\mid x\in H\}$).

**PROPOSITION 3** Let $G$ be a group. If $H$ and $K$ are any subgroups of $G$, then so is their intersection $H\cap K$.

## 学习要求

1. 掌握如下内容:

(1) 全文的主题和中心意思、各段的主要内容。

(2) 10 个新出现的或常用的数学术语,3~6 个表示数学命题的句型。其中数学术语包括:monoid(幺半群),abelian group(阿贝尔群),subgroup(半群),homomorphism(同态),isomorphism(同构),endomorphism(自同态),automorphism(自同构),equivalent(等价的),intersection(交集),binary operation(二元运算)等。

2. 回答如下问题:

(1) 本文提出哪些概念,得出哪些主要结论?

(2) 群与幺半群分别是如何定义的,二者之间有什么关系?

(3) 阿贝尔群必需满足哪些条件?

## 3.3.5 The parallel postulate ①

Euclid's first four postulates have always been readily accepted by mathematicians. The fifth (parallel) postulate, however, was highly controversial. In fact, as we shall see later, consideration of alternatives to Euclid's parallel postulate resulted in the development of non-Euclidean geometries.

At this point we are not going to state the fifth postulate in its original form, as it appeared in the *Elements*. Instead, we will present a simpler postulate (which we will later show is logically equivalent to Euclid's original). This version is sometimes called *Playfair's postulate*.

Because it appeared in John Playfair's presentation of Euclidean geometry, published in 1795, although it was referred to much earlier by Proclus (A. D. 410–485). We will call it *the Euclidean parallel postulate* because it distinguishes Euclidean geometry from other geometries based on parallel postulates. The most important definition in this book is the following:

**DEFINITION.** Two lines $L$ and $m$ are *parallel* if they do not intersect, i. e., if no point lies on both of them. We denote this by $L // m$.

Notice first that we assume the lines lie in the same plane (because of our convention that all points and lines lie in one plane, unless stated otherwise; in space there are noncoplanar lines which fail to intersect and they are called *skew lines*, not "parallel"). Notice secondly what the definition does *not* say: it does not say that the lines are equidistant, i. e., it does not say that the distance between the two lines is everywhere the same. Don't be misled by drawings of parallel lines in which the lines appear to be equidistant. We want to be rigorous here and so should not introduce into our proofs assumptions that have not been stated explicitly. At the same time, don't jump to the conclusion that parallel lines are *not* equidistant. We are not committing ourselves either way and shall reserve judgment until we study the matter further. At this point, the only thing we know for sure about parallel lines is that they do not meet.

**THE EUCLIDEAN PARALLEL POSTULATE.** *For every line L and for ev-*

① 本节课文摘自:D. V. Singer. Geometry. Berlin: Springer, 1998。

*ery point P that does not lie on L there exists a unique line m through P that is parallel to L.* (See Figure 3-3-1).

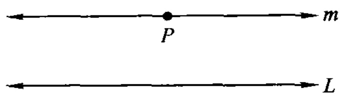


Fig. 3-3-1 Lines $L$ and $m$ are parallel.

Why should this postulate be so controversial? It may seem "obvious" to you, perhaps because you have been conditioned to think in Euclidean terms. However, if we consider the axioms of geometry as abstractions from experience, we can see a difference between this postulate and the other four. The first two postulates are abstractions from our experiences drawing with a straightedge; the third postulate derives from our experiences drawing with a compass. The fourth postulate is perhaps less obvious as an abstraction; nevertheless it derives from our experiences measuring angles with a protractor (where the sum of supplementary angles is 180°, so that if supplementary angles are congruent to each other, they must each measure 90°).

The fifth postulate is different in that we cannot verify empirically whether two lines meet, since we can draw only segments, not lines. We can extend the segments further and further to see if they meet, but we cannot go on extending them forever. Our only recourse is to verify parallelism indirectly, by using criteria other than the definition.

What is another criterion for $L$ to be parallel to $m$? Euclid suggested drawing a transversal (i.e., a line $t$ that intersects both $L$ and $m$ in distinct points), and measuring the number of degrees in the interior angles $\alpha$ and $\beta$) on one side of $t$. Euclid predicted that if the sum of angles $\alpha$ and $\beta$ turns out to be less than 180°, the lines (if produced sufficiently far) would meet on the same side of $t$ as angles $\alpha$ and $\beta$ (see Figure 3-3-2). This, *in fact*, is the content of Euclid's fifth postulate.

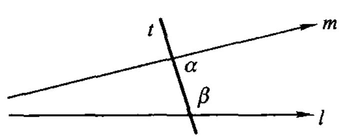


Fig. 3-3-2

The trouble with this criterion for parallelism is that it turns out to be logically equivalent to the Euclidean Parallel postulate that was just stated (see the section Equivalence of Parallel Postulates in Chapter 2). So we cannot use this criterion to convince ourselves of the correctness of the parallel postulate—— that would be circular reasoning. Euclid himself recognized the questionable nature of the parallel postulate, for he postponed using it for as long as he could (until the proof of his 29th proposition).

**学习要求**

1. 掌握如下内容:

(1) 全文的主题和中心意思、各段的主要内容。

(2) 10 个新出现的或常用的数学术语,3 ~6 个表示数学命题的句型。其中数学术语包括:postulate(公设), parallel postulate(平行公设), supplementary angles(补角), congruent to each other(互相全等), transversal(横截的,截线), criterion(准则,判别法), non-Euclidean geometries(非欧几何)等。

2. 回答如下问题

(1) 本文提出哪些概念,得出哪些主要结论?

(2) 欧几里得平行公设为什么总是引起争议(controversial)?

(3) 在给平行下定义时,为什么不把平行说成两条直线的距离处处相等?

## 3.3.6 Vector space of n-tuples ①

The collection of all $n$-dimensional vectors is called the *vector space of n-tuples*, or simply *n-space*. We denote this space by $V_n$.

We introduce now a new kind of multiplication called the dot product or scalar product of two vectors in $V_n$.

**DEFINITION** *If $A = (a_1, \cdots, a_n)$ and $B = (b_1, \cdots, b_n)$ are two vectors in $V_n$, their dot product is denoted by $A \cdot B$ and is defined by the equation*

$$A \cdot B = \sum_{k=1}^{n} a_k b_k.$$

Thus, to compute $A \cdot B$ we multiply corresponding components of $A$ and $B$ and then add all the products. This multiplication has the following algebraic properties.

**THEOREM 1** *For all vectors $A$, $B$, $C$ in $V_n$ and all scalars $c$, we have the following properties:*

(a) $A \cdot B = B \cdot A$ (*commutative law*),

(b) $A \cdot (B+C) = A \cdot B + A \cdot C$ (*distributive law*),

(c) $c(A \cdot B) = = (cA) \cdot B = A \cdot (cB)$ (*homogeneity*),

(d) $A \cdot A > 0$ if $A \neq O$ (*positivity*),

(e) $A \cdot A = 0$ if $A = O$.

① 本节课文摘自:T. M. Apostol. Calculus, Vol. 1. New York: John Wiley & Sons Inc. 1987.

*Proof.* The first three properties are easy consequences of the definition and are left as exercises. To prove the last two, we use the relation $A \cdot A = \sum a_k^2$. Since each term is nonnegative, the sum is nonnegative. Moreover, the sum is zero if and only if each term in the sum is zero and this can happen only if $A = O$.

The dot product has an interesting geometric interpretation which will be described in Section 5. Before we discuss this, however, we mention an important inequality concerning dot products that is fundamental in vector algebra.

**THEOREM 2** THE CAUCHY-SCHWARZ INEQUALITY. *If A and B are vectors in $V_n$, we have*

$$(A \cdot B)^2 \leqslant (A \cdot A)(B \cdot B). \tag{6.1}$$

*Moreover, the equality sign holds if and only if one of the vectors is a scalar multiple of the other.*

*Proof.* Expressing each member of (6.1) in terms of components, we obtain

$$\left(\sum_{k=1}^{n} a_k b_k\right)^2 \leqslant \left(\sum_{k=1}^{n} a_k^2\right)\left(\sum_{k=1}^{n} b_k^2\right),$$

which is the inequality proved earlier in Theorem 1.4.

We shall present another proof of (6.1) that makes no use of components. Such a proof is of interest because it shows that the Cauchy-Schwarz inequality is a consequence of the five properties of the dot product listed in Theorem 1 and does not depend on the particular definition that was used to deduce these properties.

To carry out this proof, we notice first that (6.1) holds trivially if either $A$ or $B$ is the zero vector. Therefore, we may assume that both $A$ and $B$ are nonzero. Let $C$ be the vector

$$C = xA - yB, \qquad \text{where } x = B \cdot B \qquad \text{and } y = A \cdot B.$$

Properties (d) and (e) imply that $C \cdot C \geqslant 0$. When we translate this in terms of $x$ and $y$, it will yield (6.1). To express $C \cdot C$ in terms of $x$ and $y$, we use properties (a), (b) and (c) to obtain

$$C \cdot C = (xA - yB) \cdot (xA - yB) = x^2(A \cdot A) - 2xy(A \cdot B) + y^2(B \cdot B).$$

Using the definitions of $x$ and $y$ and the inequality $C \cdot C \geqslant 0$, we get

$$(B \cdot B)^2(A \cdot A) - 2(A \cdot B)^2(B \cdot B) + (A \cdot B)^2(B \cdot B) \geqslant 0.$$

Property (d) implies $B \cdot B > 0$ since $B \neq O$, so we may divide by $(B \cdot B)$ to obtain

$$(B \cdot B)(A \cdot A) - (A \cdot B)^2 \geqslant 0.$$

which is (6.1). This proof also shows that the equality sign holds in (6.1) if and only if $C = O$. But $C = O$ if and only if $xA = yB$. This equation holds, in turn, if and only if one of the vectors is a scalar multiple of the other.

The Cauchy-Schwarz inequality has important applications to the properties of the length or norm of a vector, a concept which we discuss next.

Figure 3-3-3 shows the geometric vector from the origin to a point $A = (a_1, a_2)$ in the plane. From the theorem of Pythagoras, we find that the length of $A$ is given by the formula

$$\text{length of } A = \sqrt{a_1^2+a_2^2}.$$

A corresponding picture in 3-space is shown in Figure 3-3-4. Applying the theorem of Pythagoras twice, we find that the length of a geometric vector $A$ in 3-space is given by

$$\text{length of } A = \sqrt{a_1^2+a_2^2+a_3^2}.$$

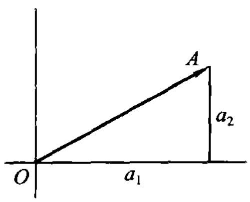

Fig. 3-3-3

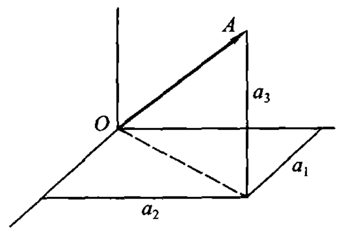

Fig. 3-3-4

Note that in either case the length of $A$ is given by $(A \cdot A)^{1/2}$, the square root of the dot product of $A$ with itself. This formula suggests a way to introduce the concept of length in $n$-space.

**DEFINITION** *If $A$ is a vector in $V_n$, its length or norm is denoted by $\|A\|$ and is defined by the equation*

$$\|A\| = (A \cdot A)^{1/2}.$$

The fundamental properties of the dot product lead to corresponding properties of norms.

**THEOREM 3** *If $A$ is a vector in $V_n$ and if $c$ is a scalar, we have the following properties:*

(a) $\|A\| > 0$ if $A \neq O$ (*posilivity*),

(b) $\|A\| = 0$ if $A = O$,

(c) $\|cA\| = |c| \cdot \|A\|$ (*homogeneity*).

*Proof.* Properties (a) and (b) follow at once from properties (d) and (e) of Theorem 1. To prove (c), we use the homogeneity property of dot products to obtain

$$\|cA\| = (cA \cdot cA)^{1/2} = (c^2 A \cdot A)^{1/2} = (c^2)^{1/2}(A \cdot A)^{1/2} = |c| \cdot \|A\|.$$

The Cauchy-Schwarz inequality can also be expressed in terms of norms. It states that

(6.2) $$(A\cdot B)^2 \leqslant \|A\|^2\|B\|^2.$$

Taking the positive square root of each member, we can also write the Cauchy-Schwarz inequality in the equivalent form

(6.3) $$|A\cdot B| \leqslant \|A\|\cdot\|B\|.$$

Now we shall use the Cauchy-Schwarz inequality to deduce the triangle inequality.

**THEOREM 4** TRIANGLE INEQUALITY. *If A and B are vectors in* $V_n$, *we have*

$$\|A+B\| \leqslant \|A\| + \|B\|.$$

*Moreover, the equality sign holds if and only if* $A=O$, *or* $B=O$, *or* $B=cA$ *for some* $c>0$.

*Proof.* To avoid square roots, we write the triangle inequality in the equivalent form

(6.4) $$\|A+B\|^2 \leqslant (\|A\|+\|B\|)^2.$$

The left member of (6.4) is

$$\|A+B\|^2=(A+B)\cdot(A+B)=A\cdot A+2A\cdot B+B\cdot B=\|A\|^2+2A\cdot B+\|B\|^2,$$

whereas the right member is

$$(\|A\|+\|B\|)^2=\|A\|^2+2\|A\|\cdot\|B\|+\|B\|^2.$$

Comparing these two formulas, we see that (6.4) holds if and only if we have

(6.5) $$A\cdot B\leqslant \|A\|\cdot\|B\|.$$

But $A\cdot B\leqslant |A\cdot B|$ so (6.5) follows from the Cauchy-Schwarz inequality, as expressed in (6.3). This proves that the triangle inequality is a consequence of the Cauchy-Schwarz inequality.

The converse statement is also true. That is, if the triangle inequality holds then (6.5) also holds for $A$ and $-A$, from which we obtain (6.2). If equality holds in (6.4), then $A\cdot B=\|A\|\cdot\|B\|$, so $B=cA$ for some scalar $c$. Hence $A\cdot B=c\|A\|^2$ and $\|A\|\cdot\|B\|=c\|A\|^2$. If $A\neq O$ this implies $c=|c|\geqslant 0$. If $B\neq O$ then $B=cA$ with $c>0$.

## 学习要求

1. 掌握如下内容：

（1）全文的主题和中心意思、各段的主要内容。

（2）15 个新出现的或常用的数学术语，5～8 个表示数学命题的句型。其中数学术语包括：$n$-dimensional vector（$n$ 维向量），vector space（向量空间），$n$-tuple（$n$ 元组），multiplication（乘法），dot product（点乘），scalar product（标量积），homogeneity（齐性），positivity（正性），scalar multiple（纯量倍数），the Cauchy-Schwarz inequality（柯西-施瓦兹不等式），norm（范数），square root（平方根），the triangle inequality（三角不等式）等。

2. 回答如下问题：

（1）本文提出哪些概念，得出哪些主要结论？

（2）两个向量的点乘是如何定义的？

（3）柯西-施瓦兹不等式是如何证明的？（谈谈证明思路。）

（4）三角不等式是指什么？如何证明它？

### 3.3.7 Conic sections ①

The general real quadratic equation in two variables has the form

(7.1) $$Ax^2+Bxy+Cy^2+Dx+Ey+F=0,$$

where the coefficients are assumed to be real and $A^2+B^2+C^2\neq 0$. The graphs of equations of the form (7.1) are called *conic sections*, since for every set of real coefficients for which (7.1) has a nonvacuous graph, the graph may be obtained as the intersection of a plane and a right circular cone (possibly degenerate). In this section we shall discuss briefly the development of the hyperbola, parabola, ellipse, and circle as the intersections of planes with a right circular cone, i. e., as plane sections of a right circular cone. We shall also mention several of the properties of these conic sections as a review of analytic geometry.

A circle may be defined as the locus of points in a plane equidistant from a given fixed point $Q$ of the plane. When there is a coordinate system $(x,y)$ and a distance relation in the plane, the circle is the graph of a polynomial $(x-h)^2+(y-k)^2-r^2$, where its center $Q$ has coordinates $(h, k)$ and the distance is $r\geqslant 0$. A circle with $r=0$ is called a *point circle* and is classified as a degenerate form of the circle.

Consider a nondegenerate real circle with center $Q$ and let $P\neq Q$ be an arbitrary real fixed point on the line that passes through $Q$ and is perpendicular to the plane of the circle. The locus of points on the set of lines joining $P$ to points of the circle is called a *right circular cone* (Figure 3-3-5). The cone has two *nappes* meeting at $P$. The locus of points on the set of lines perpendicular to the plane of the circle and through points of the circle is called a *right circular cylinder* (Figure 3-3-6)

① 本节课文摘自：B. E. Meserve. Fundamental Concepts of Algebra. New York: Dover Pub. Inc, 1982.

and may be considered as a limiting case of the cone as the distance $PQ$ increases without bound.

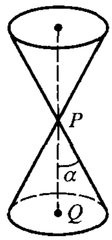


Fig. 3-3-5

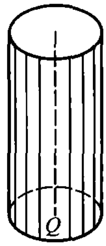


Fig. 3-3-6

Let $\Pi$ be an arbitrary real plane and consider the right circular cone generated by lines through the fixed point $P$ and a given nondegenerate circle with center $Q$ as above. Let the constant angle between the generating lines and $PQ$ be $\alpha$. A graph of the quadratic equation (7.1) is said to be degenerate if and only if it is obtained by passing the plane $\Pi$ through $P$. This condition may also be expressed algebraically as follows: The graph is degenerate if and only if $\Delta = 0$, where

$$\Delta = \begin{vmatrix} 2A & B & D \\ B & 2C & E \\ D & E & 2F \end{vmatrix}.$$

A nondegenerate graph of (7.1) is a hyperbola, parabola, or ellipse according as $B^2-4AC>$, $=$, or $< 0$. Geometrically, these three cases, respectively, arise according as the smallest angle $\theta$ between the plane $\Pi$ and the line $PQ$ is $<$, $=$, $> \alpha$ (Figure 3-3-7). (The normal to the plane makes an angle with $PQ$ equal to the complement of $\theta$.) For example, consider the cone $3x^2+3y^2-z^2=0$, with $\alpha=30°$. For any real number $k$ the plane $z=k$, with $\theta$ equal to a right angle, intersects the cone in a circle $3x^2+3y^2-k^2=0$; the plane $z=x+k$ with $\theta=45°>\alpha$ intersects the cone in an ellipse $2x^2+3y^2-2xk-k^2=0$; the plane $z=x\sqrt{3}+k$ with $\theta=30°=\alpha$ intersects the conic in a parabola $3y^2-2xk\sqrt{3}-k^2=0$; and the plane $z=2x+k$ with $\theta<\alpha$ intersects the cone in a hyperbola $3y^2-x^2-4xk-k^2=0$.

The degenerate conics may be identified algebraically or geometrically. From a geometric point of view (Exercise 1), a hyperbola may degenerate into two intersecting lines, a parabola into two coincident lines or two parallel lines (using a right circular cylinder), and an ellipse into a point. The ellipse becomes a circle when it is a right angle.

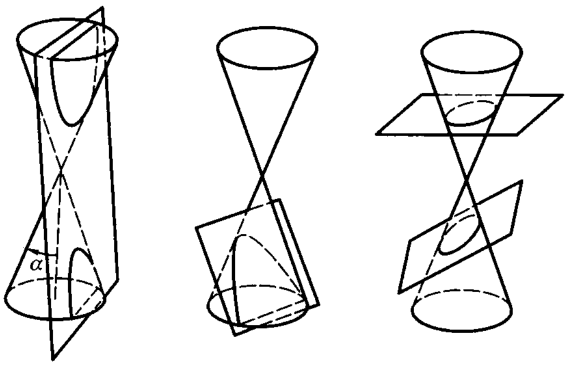


Fig. 3-3-7

If the coordinate axes are rotated (Section 5.15) through an angle $\psi$ where $\tan 2\psi = B/(A-C)$ if $A \neq C$ and $\psi = 45°$ if $A = C$, it is shown (Exercise 6) in most analytic geometry texts that the equation (7.1) takes on the form

(7.2) $$A'x^2+C'y^2+D'x+E'y+F'=0.$$

It can also be shown (Exercise 7) that the numbers $A+C$, $B^2-4AC$, and $A$ are unchanged by a rotation or translation of the coordinate axes. Thus $B^2-4AC=-4A'C'$ and the graph, possibly degenerate, of (7.2) is a hyperbola, parabola, or ellipse according as $A'C'$ <, =, or > 0. Since $A'$ and $C'$ cannot both be zero in the quadratic equation (7.2), the general equation of a nondegenerate parabola may be written in one of the forms

(7.3) $$(y-k)^2=2p(x-h) \quad \text{or} \quad (x-h)^2=2p(y-k).$$

Similarly, if $A'C'\neq 0$, let $h=-D'/2A'$ and $k=-E'/2C'$. Then a nondegenerate ellipse has an equation of the form

(7.4) $$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1, \qquad 0<a, 0<b,$$

and a nondegenerate hyperbola has an equation of the form

(7.5) $$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1 \quad \text{or} \quad \frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1.$$

The first parabola in (7.3) has axis $y=k$, vertex $(h, k)$, and passes through the points $(h+p/2, k\pm p)$. It may be defined in the plane as the locus of points equidistant from the line $x=h-p/2$ (called the directrix) and the point $(h+p/2, k)$ (called the focus).

The ellipse (7.4) has center $(h, k)$ and, assuming $a^2>b^2$, the ends of its major axis are at $(h\pm a, k)$, the ends of its minor axis are at $(h, k\pm b)$, and its foci are at $(h\pm\sqrt{a^2-b^2}, k)$. If $a^2=b^2$, it is a circle. The ellipse may be defined in the plane as the locus of points $P$ such that $PF_1+PF_2=2a$, where $F_1$ and $F_2$ are the foci.

The first hyperbola in (7.5) has center $(h, k)$, ends of its major axis at $(h \pm a, k)$, foci at $(h\pm\sqrt{a^2+b^2}, k)$, and asymptotes $b(x-h)=\pm a(y-k)$. It may be defined in the plane as the locus of points $P$ such that $PF_1-PF_2=\pm 2a$.

Thus the real graph (when such exists) of the general quadratic equation (7.1) may be obtained as the section of a right circular cone (possibly degenerate) by a plane and is called a conic section. The form of the graph may be specified in terms of the quantity $B^2-4AC$ and the rank (Section 5.10) of the determinant $\Delta$. The definitions of hyperbola, parabola, and ellipse as loci on a plane can be proved to be equivalent to the definitions as sections of a right circular cone. A very readable treatment of conic sections may be found in [38; 102–138], a more complete treatment in [18; 171–236], a history of conic sections and quadric surfaces in [11].

**Exercises**

1. Draw figures illustrating how each of the following nonvacuous degenerate conics may be obtained as a plane section of a right circular cone or a right circular cylinder: (a) two intersecting lines, (b) two coincident lines, (c) two distinct parallel lines, (d) a point.

2. Graph the following conic sections:

(a) $x^2+y^2=25$, (b) $9x^2+4y^2=36$,

(c) $9x^2-4y^2=36$, (d) $x^2=y+2$,

(e) $r^2-2x=y$, (f) $x=y^2-2y+5$,

(g) $y=x^2-6x+7$.

## 学习要求

1. 掌握如下内容：

（1）全文的主题和中心意思、各段的主要内容。

（2）15 个新出现的或常用的数学术语，5～8 个表示数学命题的句型。其中数学术语包括：conic sections（圆锥曲线），nonvacuous（非空的），nondegenerate（非退化的），arbitrary（任意的），parabola（抛物线），right circular cone（正圆锥），

right circular cylinder(正圆柱), the generating line (母线), ellipse(椭圆), hyperbola(双曲线), directrix(准线), focus(焦点), asymptone(渐近线)等。

(3) 各道练习题的含义。

2. 回答如下问题:

(1) 本文提出哪些概念,得出哪些主要结论?

(2) 一个平面与一个正圆锥相交,何时产生非退化的椭圆、双曲线或抛物线?

(3) 什么叫做退化圆锥曲线?

(4) 如何求双曲线的准线和焦点?

### 3.3.8 Higher plane curves ①

The graphs in the $xy$-plane of polynomials $f(x,y)$ of degree greater than two are called *higher plane curves*. These graphs have been completely classified when $f(x, y)$ has degree three or four, i. e., for cubic and quartic curves. Many other curves have been extensively studied. In this section we shall define a singular point and, in particular, a double point. Then we shall classify double points and finally classify cubic plane curves in terms of their double points. A few general properties of higher plane curves will be mentioned.

A point $P$ of a curve such that every line through $P$ intersects the curve with a multiplicity (Section 8.2) at least two at $P$ is called a singular point of the curve. If some line through a singular point $P$ intersects the curve with multiplicity two at $P$, then $P$ is a double point. Any line is either entirely on (i. e., a component of) a curve of degree $n$ or intersects the curve in at most $n$ points. This can be proved using the fact that if an equation $f(x,y)=0$ of degree $n$ and the equation of the line are solved simultaneously, then the resulting equation in one variable is either identically zero or of degree at most $n$.

Similarly, two curves of degree $m$ and $n$ either have a component in common or intersect in at most $mn$ points. Using such arguments, it can be shown that a curve of degree $n$ can have at most $\frac{1}{2}(n-1)(n-2)$ double points [23; 41-42]. In particular, a cubic curve has at most one double point (Exercise 1).

At an ordinary (nonsingular) point, a curve of degree $n$ has a unique tangent;

① 本节课文摘自:B. E. Meserve. Fundamental Concepts of Algebra. New York: Dover Pub. Inc, 1982。

at a double point it has two tangents; and, in general, at a singular point $P$ it has $k$ tangents when every line through $P$ intersects the curve with multiplicity at least $k$ at $P$ and some line intersects the curve with multiplicity exactly $k$ at $P$.

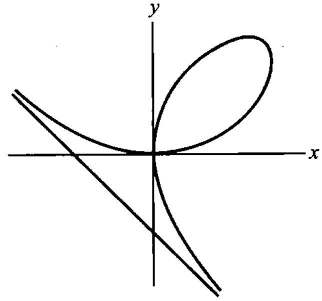


FIG. 3-3-8

Double points are classified as nodes when the tangents are distinct, cusps when they coincide. When the tangents are conjugate imaginary lines, the double point is called a node or isolated point. The Folium of Descartes, $x^3+y^3=3axy$ (FIG. 3-3-8), has a node at the origin and the line $x+y+a=0$ as an asymptote (Section 6), the semicubical parabola $y^2=x^3$ (FIG. 3-3-9) has a cusp at the origin, and the curve $y^2=x^2(x-1)$ (FIG. 3-3-10) has an isolated point at the origin.

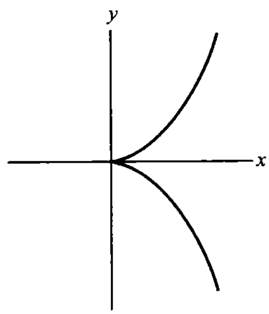


FIG. 3-3-9

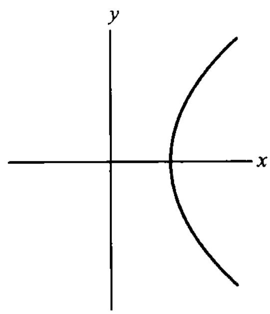


FIG. 3-3-10

Cubic curves are generally classified as follows, in terms of their double points:

(i) *Cubic curves without double points*: *elliptic cubics*,

(ii) *Cubic curves with a node*: *nodal cubics*,

(iii) *Cubic curves with a cusp*: *cuspidal cubics.*

Many of the properties of cubic curves may be found in [23; 139–243] and [26;

201–263].

Quartic curves may also be classified in terms of their singular points. Such a classification and many of the properties of quartic curves may be found in [23; 244–328] and [26; 264–349].

Plane curves of any degree $n$ may be considered in terms of their double points and other singular points. In particular, any irreducible curve (Section 7–9) having its maximum number of singular points, i. e., *zero deficiency*, is called a *unicursal curve*. A unicursal curve is characterized by the fact that the coordinates of every point on the curve may be expressed rationally in terms of a single parameter. Unicursal curves are important in several mathematical theories.

The next two sections contain further details on the graphing of higher plane curves.

**学习要求**

1. 掌握如下内容：

（1）全文的主题和中心意思、各段的主要内容。

（2）10个新出现的或常用的数学术语，5～8个表示数学命题的句型。其中数学术语包括：higher plane curves（高阶平面曲线），cubic curves（三次曲线），quartic curves（四次曲线），double points（二重点），elliptic cubic（椭圆三次曲线），node（结点），nodal cubic（结点三次曲线），cusp（尖点），cuspidal cubic（尖点三次曲线），unicursal curve（单行曲线，有理曲线），singular point（奇异点）等。

2. 回答如下问题：

（1）本文提出哪些概念，得出哪些主要结论？

（2）三次曲线含有哪些类型，如何判别不同类型？

（3）如何区分正常点与奇异点、二重点、尖点？

## 3.3.9 Zeros of an analytic function ①

If $p(z)$ and $q(z)$ are two polynomials then it is well known that $p(z)=s(z)q(z)+r(z)$ where $s(z)$ and $r(z)$ are also polynomial and the degree of $r(z)$ is less than the degree of $q(z)$. In particular, if $a$ is such that $p(a)=0$ then choose $(z-a)$ for $q(z)$. Hence, $p(z)=(z-a)s(z)+r(z)$ and $r(z)$ must be a constant polynomial. But letting $z=a$ gives $0=p(a)=r(a)$. Thus, $p(z)=(z-a)s(z)$. If we also

① 本节课文摘自：J. B. Conway, Functions of one complex variable (2th. Edt), Berlin: Springer, 2002.

have that $s(a) = 0$ we can factor $(z-a)$ from $s(z)$. Continuing we get $p(z) = (z-a)^m t(z)$ where $1 \leqslant m \leqslant$ degree of $p(z)$, and $t(z)$ is a polynomial such that $t(a) \neq 0$. Also, degree $t(z)$ = degree $p(z) - m$.

**Definition 1**. If $f: G \to \mathbf{C}$ is analytic and $a$ in $G$ satisfies $f(a) = 0$ then $a$ is a zero of $f$ of multiplicity $m \geqslant 1$ if there is an analytic function $g: G \to \mathbf{C}$ such that $f(z) = (z-a)^m g(z)$ where $g(a) \neq 0$.

Returning to the discussion of polynomials, we have that the multiplicity of a zero of a polynomial must be less than the degree of the polynomial. If $n$ = the degree of the polynomial $p(z)$ and $a_1, \cdots, a_k$ are all the distinct zeros of $p(z)$ then $p(z) = (z-a_1)^{m_1} \cdots (z-a_k)^{m_k} s(z)$ where $s(z)$ is a polynomial with no zeros. Now the Fundamental Theorem of Algebra says that a polynomial with no zeros is constant. Hence, if we can prove this result we will succeeded in completely factoring $p(z)$ into the product of first degree polynomials. The reader might be pleasantly surprised to know that after many years of studying mathematics he is right now on the threshold of proving the Fundamental Theorem of Algebra. But first it is necessary to prove a famous result about analytic functions. It is also convenient to introduce some new terminology.

**Definition 2**. An entire function is a function which is defined and analytic in the whole complex plane $\mathbf{C}$ (The term "integral function" is also used.)

The following result follows from Theorem 2. 8 and the fact that $\mathbf{C}$ contains $B(0;R)$ for arbitrary large $R$.

**Proposition.** If $f$ is an entire function then $f$ has a power expansion

$$f(z) = \sum_{n=0}^{\infty} a_n z^n$$

with infinite radius of convergence.

In light of the proceeding proposition, entire functions can be considered as polynomials of "infinite degree". So the question arises: can the theory of polynomials be generalized to entire functions? For example, can an entire function be factored? The answer to this difficult and is postponed to Section VII. 5. Another property of polynomials is that no non constant polynomial is bounded. Indeed, if $p(z) = z^n + a_{n-1}z^{n-1} + \cdots + a_0$ then $\lim\limits_{z \to \infty} p(z) = \lim\limits_{z \to \infty} z^n [1 + a_{n-1}z^{-1} + \cdots + a_0 z^{-n}] = \infty$. The fact that also holds for entire functions is an extremely useful result.

**Liouville's Theorem**. If $f$ is a bounded entire function the $f$ is constant.

***Proof***. Suppose $|f(z)| \leqslant M$ for all $z$ in $\mathbf{C}$. We will show that $f'(z) = 0$ for all

$z$ in **C**. To do this use Cauchy's estimate (corollary 2.14). Since $f$ is analytic in any disk $B(z;R)$ we have that $|f(z)| \leqslant M/R$. Since $R$ was arbitrary, it follows that $f'(z) = 0$ for each $z$ in **C**.

The reader should not be deceived into thinking that this theorem is insignificant because it has such short proof. We have expended a great deal of effort building up machinery and increasing our knowledge of analytic functions. We have plowed, planted, and fertilized; we shouldn't be surprised if, occasionally, something is available for easy picking.

Liouville's Theorem will be better appreciated in the following application.

**Fundamental Theorem of Algebra**. If $p(z)$ is a non constant polynomial then there is a complex number $a$ with $p(a) = 0$.

***Proof.*** Suppose $p(z) \neq 0$ for all $z$ and let $f(z) = [p(z)]^{-1}$; then $f$ is an entire function. If $p$ is non constant then, as was shown above, $\lim_{z\to\infty} p(z) = \infty$; so $\lim_{z\to\infty} f(z) = 0$. In particular, there is a number $R > 0$ such that $|f(z)| < 1$ if $|z| > R$. But $f$ is continuous on $B(0;R)$ so there is a constant $M$ such that $|f(z)| \leqslant M$ for $|z| \leqslant R$. Hence $f$ is bounded and, must be constant by Liouville's Theorem. It follows that $p$ must be constant, contradicting our assumption.

**Corollary**. If $p(z)$ is a polynomial and $a_1, \cdots, a_m$ are its zeros with $a_j$ having multiplicity $k_j$ then $p(z) = c(z-a_1)^{k_1}\cdots(z-a_m)^{k_m}$ for some constant $c$ and $k_1+\cdots+k_m$ is the degree of $p$.

### 学习要求

1. 掌握如下内容：

(1) 全文的主题和中心意思、各段的主要内容。

(2) 10 个新出现的或常用的数学术语,5 ~ 8 个表示数学命题的句型。其中数学术语包括:zero(零,零点),entire function(整函数,有时也用 integral function 表示),factor(n. 因素、因子,v. 因式分解),non constant = non-constant(非常数的),the multiplicity of a zero(零点的重数),Liouville's Theorem(刘维尔定理),Fundamental Theorem of Algebra(代数基本定理),Cauchy's estimate(柯西估计),analytic function(解析函数)。

2. 回答如下问题：

(1) 解析函数零点的重数如何定义？什么叫整函数？

(2) 本文如何把多项式的性质推广到整函数？

(3) 为什么说 Liouville 定理的结论很重要？

(4) 代数学基本定理是什么？它是如何获得证明的？

## §3.4 数学的应用与应用数学

本节介绍数学的一些基本的应用和应用数学的若干基础知识，分为五小节。其中第一小节介绍牛顿的力学定律；第二小节介绍非线性常微分方程的应用；第三小节介绍图论的用途；第四小节介绍线性规划的非标准问题；第五小节介绍数学模型的构建。

一般说来，介绍数学的应用和应用数学要牵涉别的学科，因此生词多了，句型也复杂了。不过，作为基础训练，这几篇短文都选得相对简单些，涉及的数学内容的难度不大，相关的学科也是大家较为熟悉的，因此仍是较易读懂的。

随着数学的应用越来越广泛和深入，我们将面临掌握更多应用数学的知识和信息的客观需要，有更多的英语文献将等待着我们去查阅和学习。

### 3.4.1 Newton's law ①

To begin our investigations of mathematical models, a problem with which most of you are somewhat familiar will be considered. We will discuss the motion of a mass attached to a spring as shown in Fig. 3-4-1:

Observations of this kind of apparatus show that the mass, *once set in motion*, moves back and forth (oscillates). Although few people today have any intrinsic interest in such a spring-mass system, historically this problem played an important part in the development of physics. Furthermore, this simple spring-mass system exhibits behavior of more complex systems. For example, the oscillations of a spring-mass system resemble the motions of clock-like mechanisms and, in a sense, also aid in the understanding of the up-and-down motion of the ocean surface.

*Physical problems cannot be analyzed by mathematics alone.* This should be the first fundamental principle of an applied mathematician (although apparently some mathematicians would frequently wish it were not so). A spring-mass system cannot be solved without formulating an equation which describes its motion. Fortunately many experimental observations culminated in **Newton's second law of motion** describing how a particle reacts to a force. Newton discovered that the motion of a point mass is well described by the now famous formula

① 本节课文摘自：R. Haberman. Mathematical Modeles. New York: Prentice-Hall. Inc, 1977。

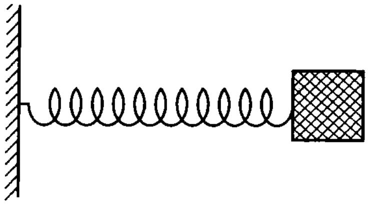

Fig. 3-4-1 Spring-mass system

$$\vec{F}=\frac{\mathrm{d}}{\mathrm{d}t}(m\vec{v}) \tag{1.1}$$

where $\vec{F}$ is the vector sum of all forces applied to a point mass $m$. The forces $\vec{F}$ equal the rate of change of the **momentum** $m\vec{v}$, where $\vec{v}$ is the velocity of the mass and $\vec{x}$ its position:

$$\vec{v}=\frac{\mathrm{d}\vec{x}}{\mathrm{d}t}. \tag{1.2}$$

If the mass is constant (which we assume throughout this text), then

$$\vec{F}=m\frac{\mathrm{d}\vec{v}}{\mathrm{d}t}=m\vec{a},$$

where $\vec{a}$ is the vector acceleration of the mass

$$\vec{a}=\frac{\mathrm{d}\vec{v}}{\mathrm{d}t}=\frac{\mathrm{d}^2\vec{x}}{\mathrm{d}t^2}. \tag{1.3}$$

Newton's second law of motion (often referred to as just **Newton's law**), equation 1.3, states that the force on a particle equals its mass times its acceleration, easily remembered as "*F* equals *ma*." The resulting acceleration of a point mass is proportional to the total force acting on the mass.

At least two assumptions are necessary for the validity of Newton's law. There are no point masses in nature. Thus, this formula is valid only to the extent in which the finite size of a mass can be ignored. For our purposes, we will be satisfied with discussing only point masses. A second approximation has its origins in work by twentieth century physicists in which Newton's law is shown to be invalid as the velocities involved approach the speed of light. However, as long as the velocity of a mass is significantly less than the speed of light, Newton's law remains a good *approximation.* We emphasize the word approximation, for although mathematics is frequently treated as a science of exactness, mathematics is applied to models which only approximate the real world.

## Exercises

**1.1** Consider Fig. 3-4-2, which shows two masses ($m_1$ and $m_2$) attached to the opposite ends of a rigid (and massless) bar:

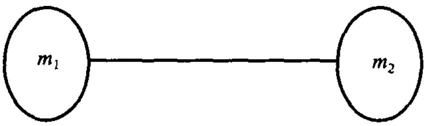


Fig. 3-4-2


Fig. 3-4-3

$m_1$ is located at $\vec{x}_1$, and $m_2$ is located at $\vec{x}_2$, The bar is free to move and rotate due to imposed forces. The bar applies a force $\vec{F}_1$ to mass $m_1$ and also a force $\vec{F}_2$ to $m_2$ as seen in Fig. 3-4-3.

Newton's third law of motion, stating that the forces of action and reaction are equal and opposite, implies that $\vec{F}_2 = -\vec{F}_1$.

(a) Suppose that an external force $\vec{G}_1$ is applied to $m_1$, and $\vec{G}_2$ to $m_2$. By applying Newton's second law to each mass, show the law can be applied to the rigid body consisting of both masses, if $\vec{x}$ is replaced by the center of mass $\vec{x}_{cm}$ [i. e., show $m(\mathrm{d}^2\vec{x}_{cm}/\mathrm{d}t^2) = \vec{F}$, where $m$ is the total mass, $m = m_1 + m_2$, $\vec{x}_{cm}$ is the center of mass, $\vec{x}_{cm} = (m_1\vec{x}_1 + m_2\vec{x}_2)/(m_1 + m_2)$, and $\vec{F}$ is the sum of forces applied, $\vec{F} = \vec{G}_1 + \vec{G}_2$]. The motion of the center of mass of the rigid body is thus determined. However, its rotation remains unknown.

(b) Show that $\vec{x}_{cm}$ lies at a point on the rigid bar connecting $m_1$ to $m_2$.

**1.2** Generalize the result of exercise 1.1 to a rigid body consisting of $N$ masses.

**1.3** Fig. 3-4-4 shows a rigid bar of length $L$:

(a) If the mass density $p(x)$ (mass per unit length) depends on the position along the bar, then what is the total mass $m$?

(b) Using the result of exercise 1.2, where is the center of mass $\vec{x}_{cm}$? [Hint: Divide the bar up into $N$ equal pieces and take the limit as $N \to \infty$.]

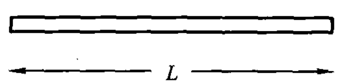


Fig. 3-4-4

(c) If the total force on the mass is $\vec{F}$, show that $m(d^2\vec{x}_{cm}/dt^2)=\vec{F}$.

**学习要求**

1. 掌握如下内容：

(1) 全文的主题和中心意思、各段的主要内容。

(2) 10 个新出现的或常用的数学与物理术语,5~6 个表示数学与物理命题的句型。其中术语包括：mathematical model (数学模型), spring-mass system (弹簧质量系统), oscillates (振动), clock-like mechanisms (钟状机械), Newton's second law of motion (牛顿第二运动定律), momentum (动量), approximation (近似), the finite size of a mass (质量(点)的有限体积), density (密度), rigid body (刚体)等。

2. 回答如下问题：

(1) 本文提出哪些概念,得出哪些主要结论?

(2) 牛顿第二运动定律和牛顿第三运动定律的内容各是什么? 为了使牛顿第二运动定律成立,必须做什么样的假定?

(3) 如何理解"*Physical problems cannot be analyzed by mathematics alone*"?

(4) 习题 1.1 和 1.3 分别研究什么问题?

## 3.4.2 Applications in the Biological Sciences ①

Nonlinear differential equations occur widely and are very interesting. It is sometimes quite a challenge to explain why some behavior that the solutions of a problem exhibit is not behavior that would be expected of the physical system that the equation purports to model[1]. Often the defect can be remedied at the expense of clarity and ease of understanding. This can be an expensive trade-off: simplicity and clarity with possible fallacious behavior, versus obscurity and accuracy of behavior[2]. Here are several applications of nonlinear differential equations to the biological, the physical, and the social sciences. Most of these equations have variables separable, but others are examples of special classes of differential equations that can be studied productively.

At first we consider the applications in the Biological Sciences. One of the typical examples is the following Logistic Equation.

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① 本节课文摘自：C. C. Ross. Differential Equations — An Introduction with Mathmatica. Berlin: Springer. 1995。

The nonlinear differential equation $dy/dt=y(b-ay)$ is called the logistic equation. This name is from the Greek word λογιστικοζ (logistikos), which means "skilled in calculating." Presumably this is because this equation works so well in a wide range of applications. Here the topic is populations where the growth is restricted, rather than unrestricted as it was in Section 1.2. But differential equations similar to the logistic equation, such as the equation

$$\frac{dy}{dt}=(d-cy)(b-ay), \tag{2.1}$$

which is clearly a generalization of the logistic equation, govern such other processes as bimolecular chemical reactions and the spread of flu epidemics. Both of these equations are a special type of separable differential equation where $dy/dt=F(y)$. Notice that the independent variable $t$ is not present on the right hand side. Such equations are called autonomous, and their solutions behave in special ways. For instance, if $y(t)$ is a solution, then so is $y(t+k)$ for any $k$. (Any horizontal translation of a solution is also a solution. This is left as an exercise.)

Autonomous differential equations are solved in the standard way for separable equations to obtain

$$G(y)=\int\frac{1}{F(y)}dy=t+c,$$

which we want to solve for $y$. Sometimes $G$ has an identifiable inverse so that we can explicitly find $y(t)=G^{-1}(t+c)$.

In the last section we saw that the differential equation for unrestricted growth was $dy/dt=ry$. However, it is not reasonable to think that any support system can sustain unrestricted growth for any extended period of time. Early in a process, the growth may appear to be unrestricted, but since the earth and every habitat in it is finite in extent, restrictions have to appear. A closer look at the equation for unrestricted growth indicates that we have successfully accounted for deaths in our population in this way: Suppose that the death rate is $d$ per unit population and the birth rate is $b$ per unit population. Then the rate of population change is $r=b-d$. This says that the effective growth rate is the difference between the birth rate and the death rate. If the birth rate and death rate are equal, $dp/dt=0$, and the population is stable. If the birth rate is numerically greater than the death rate, the population is increasing. If the death rate exceeds the birth rate, the population is in decline. But all of this assumes that any possible growth is unrestricted if $b>d$. How does one account for the fact that natural resources, be they an agar medium in a Petri dish, or a tropical rain forest, or the Pacific Ocean, are finite?

The standard technique for accounting for restricted growth is to assume that the death rate is not a constant, but depends on the size of the population. That is we take as our equation of growth, $dy/dt = by - (ay)y = by - ay^2 = y(b - ay)$. Here $b$ is the birthrate, but the death rate is $(ay)$, which depends on $y$. Strange as it may seem at first glance, this equation has served very well at predicting the size of populations living in restricted environments. Note that if $y$ is small, the equation is essentially $dy/dt = by$, which is the equation of unrestricted growth. But as $y$ increases, no matter how small $a$ is, so long as it is nonzero, the term $(ay)$ begins to have an effect, and the rate of population growth begins to slow. We assume an initial population of $y_0$.

The two constant solutions, $y=0$ and $y=b/a$ have significance. If $y=0$ the population has become **extinct**. No further growth of this population is possible. The other constant value $y = b/a$ is called the **carrying capacity** of the environment. The carrying capacity of the environment is the maximum population that the environment can sustain. If the population reaches this level then it cannot increase further because the death rate becomes larger than the birth rate. We will solve the logistic equation and observe how that equation models the phenomenon of restricted growth. It is of critical importance for a population (such as humanity) to know the value of $a$. Part of the problem of estimating $a$ is that there are competitors for all natural resources, and the equation as formulated does not take this explicitly into account. All of the effects of competition are incorporated into the one coefficient $a$, so $a$ is more than merely an observable "death rate."

Since the portion of the curve in which we have special interest (because it describes most populations living in restricted environments) lies between $y=0$ and $y=b/a$, this interval will be implicitly used in our solution. For $0<y<b/a$, separate variables to get

$$\int \frac{dy}{y(b-ay)} = \int dt + c_0.$$

Then integrate to get

$$\frac{1}{b}\ln y - \frac{1}{b}\ln(b-ay) = t + c_0.$$

When $t=0$, $y(0)=y_0$, so

$$c_0 = \frac{1}{b}\ln\left(\frac{y_0}{b-ay_0}\right).$$

Multiply through by $b$, combine logarithms, and exponentiate to get

$$\frac{y}{b-ay}=\frac{y_0}{b-ay_0}e^{bt}. \tag{2.2}$$

Solving for $y$ gives

$$y(t)=\frac{by_0}{ay_0+(b-ay_0)e^{-bt}}.$$

Notice that $y(t) \to b/a$ as $t \to +\infty$, and that $y(t) \to 0$ as $t \to -\infty$. See Figure 3-4-5.

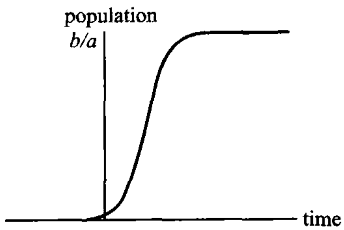


Fig. 3-4-5 A typical logistic curve.

A quick calculation reveals that $y''=(b-2ay)(y)(b-ay)$, so that $y$ is concave up between 0 and $b/(2a)$, and $y$ is concave down between $b/(2a)$ and $b/a$. When $y=b/(2a)$, which is half-way between extinction and the carrying capacity, $y''=0$, so the population curve $y$ has its maximum slope there. If earth's population is following a logistic curve, then we are not yet at one-half of the carrying capacity of the earth, because the rate of increase of population is still increasing. If we were past half-way, then the rate of increase would be decreasing. Of course the flaw in this reasoning is that we are probably not following a logistic curve, and we may be in greater trouble than this simple analysis would dictate. There certainly are regions of the earth that have essentially reached the carrying capacity of those regions to support their own populations.

Observe the disturbing fact that for most of its early history a population is small. It then experiences a short period of very rapid growth that increases the size of the population to near its possible maximum, and then the size of the population becomes nearly constant at this very large size (the carrying capacity). The population of the earth has dramatically increased during this century, so it would appear that mankind is in the short period of rapid growth.

**注释与说明**

1. It is sometimes quite a challenge to explain why some behavior that the solutions of a problem exhibit is not behavior that would be expected of the physical system that the equation purports to model. 这句可译成：有时会遇到这样一个很有挑战性的问题，即如何解释一个问题（方程）的解的性质不是人们所期望刻画的物理系统的性质，尽管该方程被称为那个物理系统的模型。本句 purport 是动词，意思为“宣称”，“据称是”。

2. This can be an expensive trade-off: simplicity and clarity with possible fallacious behavior, versus obscurity and accuracy of behavior. 这句可译成：这可能是一个昂贵的交易：其一方是简单易懂而描述的行为可能不可靠，另一方是复杂难理解但描述的行为准确。

**学习要求**

1. 掌握如下内容：

(1) 全文的主题和中心意思、各段的主要内容。

(2) 12 个新出现的或常用的数学术语，5～6 个表示数学命题的句型。其中数学术语包括：nonlinear differential equation（非线性方程），horizontal translation（平移），separable equation（变量分离方程），identifiable（可识别的，本文指“可求出的”），inverse（逆，反函数），maximun slope（最大斜率），concave up（凹向上），cancave down（凹向下），autonomous differential equation（自控微分方程）等。

2. 回答如下问题：

(1) 本文提出哪些概念，得出哪些主要结论？

(2) 自控微分方程（autonomous differential equation）是如何定义的？

(3) (2.2)式之前的命令式短语“Multiply through by $b$, combine logarithms, and exponentiate”的含义是什么？

(4) 对用数学推导方法求得的解(2.2)，如何给出进一步的分析？其实际意义是什么？

### 3.4.3 The utility of graphs ①

This book is about a branch of discrete mathematics called *graph theory*. Discrete mathematics — the study of discrete structures (usually finite collections) and

① 本节课文摘自：G. Agnarsson, R. Greenlaw, Graph Theory: Modeling, Applications, and Algorithms. London: Pearson Education, 2007.

their properties — includes combinatorics (the study of combination and enumeration of objects), algorithms for computing properties of collections of objects, and graph theory (the study of objects and their relations). Many problems in discrete mathematics can be stated and solved using graph theory. Therefore graph theory is considered by many to be one of the most important and vibrant fields within discrete mathematics.

The best way to illustrate the utility of graph is via a "*Cook's tour*" of several simple problems that can be stated and solved via graph theory. We do this in an intuitive manner prior to presenting formal definitions in § 1.4. Graph theory has many practical applications in various disciplines, including, to name a few, biology, computer science, economics, engineering, informatics, linguistics, mathematics, medicine, and social science. As will become evident after reading this chapter, graphs are excellent modeling tools. We now look at several classic problems.

We begin with the Bridges of Königsberg. This problem has a historical significance, as it was the first problem to be stated and then solved using what is now known as graph theory.

**Problem 1. The Bridges of Königsberg**

Figure 3-4-6 shows the layout of the Königsberg Bridges. The Pregel river in the Konigsberg has two banks (labeled $B_1$ and $B_2$), and its splitting forms two islands (labeled $I_1$ and $I_2$). These islands were connected to each other and to the banks with seven bridges as shown in Fig. 3-4-6. The problem was to make a round trip through downtown Königsberg, traversing each bridge exactly once.

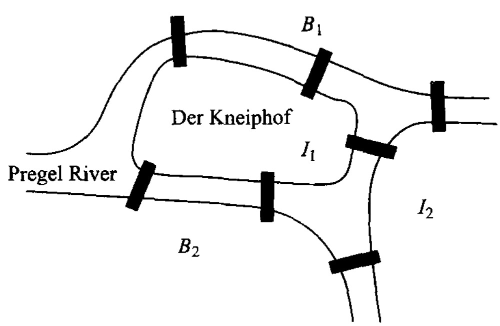


Fig. 3-4-6 The bridges of Königsberg.

Euler distilled the problem into its simplest form using the diagram (that is, *graph*) shown in Fig. 3-4-7. The black dots (that is, *vertices*) represent the areas of land, and the line (that is, *edge*) represent the bridge.

Looking at Figure 3-4-7, we see that the problem now is to start at some black dot, go along each line exact once, and end up at the starting dot. As we shall see in Chapter 5, Euler gave an exact characterization of when such problems have solutions, in term of each dot having an even number of line-ends going into it, thereby showing in particular that the problem of the bridges of Königsberg has no solution.

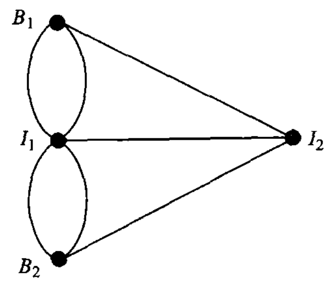


Fig. 3-4-7 The graph of the bridges of Königsberg problem.

**Problem 2. World Wide Web Communities**

The World Wide Web Communities can be modeled as a graph, where the Web pages are represented by dots or vertices and the hyperlinks between them are represented by lines or edges in the graph. One can discover interesting information by examining this "Web graph." As an example, the graph shown in Fig. 3-4-8 is termed a *Web community*. This name is bestowed because the vertices represent two different classes of objects, and each vertex representing one type of object is connected to every vertex representing the other kind of objects. In graph theory such a graph is called a *complete bipartite graph*.

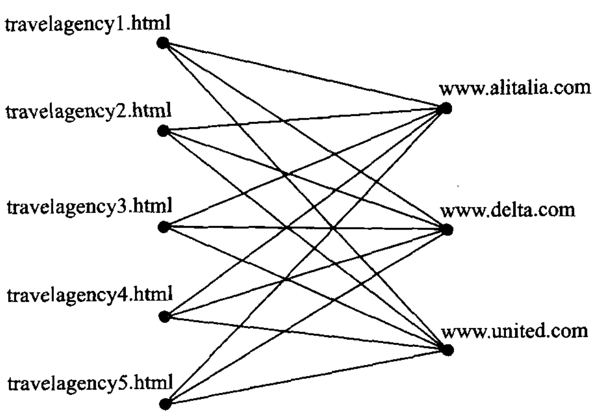


Fig. 3-4-8 World Wide Web community.

On the left-hand side of Figure 3-4-8, the five vertices are displayed with labels travelagency$X$. html, where $1 \leqslant X \leqslant 5$. Each such label travelagency$X$. html represents an HTML file for a given travel agency. On the right-hand side of the figure, the three vertices have labels www. alitalia. com, www. delta. com, and www. united. com. Each of these labels is a Web site of a well-known airline. Note that there is an edge between each HTMH file and each airline, but there are no edges between two HTML file, nor are there any edges between airlines. Such a graph is common on the Web between competitors in the same industry. Rival companies do not have hyperlinks on their Web pages to their competitors. For example, Delta has no hyperlink to United Airlines. To be able to offer the best deals for their customers, travel agents often have hyperlinks from their Web pages to all of the airlines, as shown in Figure 3-4-8. The travel agencies are also competing among themselves, so they too have no hyperlinks from one agency to any other.

Such Web communities can be discovered by finding *complete bipartite subgraghs* in the Web graph. Web community information can be used for marketing purposes or for examining the relationships among companies in a given industry.

**学习要求**

1. 掌握如下内容：

（1）全文的主题和中心意思、各段的主要内容。

（2）15 个新出现的或常用的数学术语，8～10 个表示数学命题的句型。其中数学术语包括：combinatorics（组合学），graph theory（图论），Discrete mathematics（离散数学），Bridges of Königsberg（哥尼斯堡桥[问题]），vertex（顶点，复数为 vertces），distil（抽取），hyperlink（超链接），line-end（线端），complete bipartite graph（完全二部图），prior to（在……之前）。

2. 回答如下问题：

（1）离散数学的研究对象是什么，它包括哪些数学分支？这些分支分别研究什么？作者如何评价图论在离散数学中的地位？

（2）作者认为介绍图论的用途的最好办法是什么？

（3）哥尼斯堡桥[问题]是怎样提出的？大数学家欧拉是如何解决这个问题的？

（4）如何用图论的方法来描述万维网？类似的图还可用于何处？

## 3.4.4 Nonstandard Problems in Linear Programming ①

Thus far, we have seen how to solve a problem of standard maximum form by the simplex algorithm, and we have seen how to solve a standard minimum problem by solving the dual maximum problem, and applying the Strong Duality Theorem. In this section, we discuss a technique for nonstandard LP problems[1]. The approach requires two phases of computation. The first phase involves reformulating the original objective function, and performing the simplex algorithm on the new problem. Its goal is to produce a basic feasible solution. The second phase uses the final system of the first phase, together with the original objective function, to produce an optimal solution by the ordinary simplex method.

The following proposition, whose proof is requested in Exercise 5, allows us to treat maximum problems only.

**4.1 PROPOSITION** Suppose that $g(y) = \boldsymbol{b}' \cdot \boldsymbol{y}$ is objective function for a minimum problem with some nonempty feasible region $F$. Define $f(\boldsymbol{x}) = -g(\boldsymbol{x}) = -(\boldsymbol{b}' \cdot \boldsymbol{x})$. The problem of maximizing $f$ over $F$ has an optimal solution $\boldsymbol{x}^*$ if and only if $\boldsymbol{x}^*$ is also optimal for the problem of minimizing $g$, and in addition,

$$\min g = -(\max f).$$

Therefore, given a minimum problem, we may solve instead the problem of maximizing the negative of the objective over the same feasible region. It should be pointed out that there are other ways of solving minimum problems directly, and it may not be computationally most efficient to translate all minimum problems into maximum problems in this way. But this proposition does allow us to focus our attention only on solving (nonstandard) maximum problems, and therefore it has the advantage of simplifying the exposition.

**4.2 EXAMPLE** Consider the two-variable problem:

(4.3)
$$\begin{aligned} \text{maximize:} \quad & f = 4x_1 + x_2 \\ \text{subject to:} \quad & -x_1 + x_2 \geq -1 \\ & x_1 + x_2 \geq 2 \\ & 2x_1 + x_2 \leq 8 \end{aligned}$$

(4.4)
$$x_1, x_2 \geq 0$$

This is not of standard form, because of the presence of a negative number on the right

① 本节课文摘自 Kevin J Hastings, Introduction to the mathematics of operations research, New York: Marcel Dekker INC, 1989.

side of the first constraint, and inequalities in the "$\geqslant$" direction in the first two constraints. The feasible region is depicted in Figure 3-4-9. Notice that the origin is not feasible. This is in contrast to the situation for standard maximum problems, in which the origin is always feasible. Recall also that in the simplex algorithm, the initial system represented the origin, which was feasible, and at each succeeding step another basic feasible solution was produced. The main difficulty with nonstandard linear programs is the need to produce an initial simplex system that represents a basic feasible solution. Once that is done, the simplex algorithm can finish the problem.

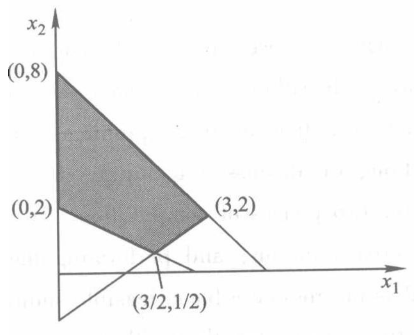


Figure 3-4-9

The first step of our Phase algorithm is to remove negative signs on the constant constraint bounds by multiplying through by $-1$, if necessary. Referring to the problem of Example 1.2, the first constraint in (4.4) becomes

$$x_1 - x_2 \leqslant 1$$

Now introduce slack variables into all "$\leqslant$" onstraints. At the same time, introduce *surplus variables* into all "$\geqslant$" constraints—e. g., in the second constraint of (4.4) here exists a nonnegative surplus variable $x_4$ such that $x_1 + x_2 - x_4 = 2$. We should note there that if a constraint is already written in "$=$" form, then no slack or surplus variables are necessary, and the constraint may simply be left untouched. Thus, the first step of Phase 1 produces the following system of equality constraints for Example 4.2:

(4.5)
$$x_1 - x_2 + x_3 = 1$$
$$x_1 + x_2 - x_4 = 2$$
$$2x_1 + x_2 + x_5 = 8$$
$$x_i \geqslant 0 \text{ for all } i$$

**4.6 REMARK** We have generalized the standard maximum problem by allowing negative entries in the vector $\boldsymbol{b}$, allowing "$\geqslant$" constraints, and allowing "$=$" constraints. Theres is one other direction that we could take, namely, to permit negative values of the variables $x_i$. We show briefly how to convert such a problem into the current form. If a certain variable $x_i$ is bounded form below by some number $L < 0$, then

$$x_i \geqslant L$$
$$\Rightarrow x_i - L \geqslant 0$$

We see that if we change variables in the constraints and the objective function by $x_i' = x_i - L$, then the new variable $x_i'$ is constrained to be nonnegative. If no such lower bound L is present, then $x_i$ may be split into two nonnegative variables $x_i^+$ and $x_i^-$ by

$$x_i = x_i^+ - x_i^- \qquad x_i^+, x_i^- \geqslant 0$$

where

$$x_i^+ = \begin{cases} x_i & if\ x_i \geqslant 0 \\ 0 & \text{otherwise} \end{cases}$$

$$x_i^- = \begin{cases} 0 & if\ x_i \geqslant 0 \\ -x_i & \text{otherwise} \end{cases}$$

This introduces one extra variable into the system for every such $x_i$ that is unbounded below, and increases the complexity of the computation. We will not discuss this issue further here; for more information, the reader may see Hillier and Lieberman [31] or Gribik and Kortanek [26][2].

The second step in Phase 1 is to solve a different maximization problem. To be specific, insert a new *artificial variable* $a_i$ into each equality constraint of the current system, and define a new objective function to be the sum of the negatives of the $a_i$ terms. In the problem of Example 4.2, we produce:

(4.7) maximize: $g = -a_1 - a_2 - a_3$

(4.8) subject to: $x_1 - x_2 + x_3 + a_1 = 1$

$x_1 + x_2 - x_4 + a_2 = 2$

$2x_1 + x_2 + x_5 + a_3 = 8$

$x_i, a_i \geqslant 0$ for all $i$

The reason for doing this is given in the following theorem.

**4.9 THEOREM** Let $\boldsymbol{A}$ be an $m \times n_0$ matrix with $n_0 \geqslant m$, let $\boldsymbol{c}$ be a vector of $n_0$ entries, let $\boldsymbol{b}$ be a vector of $m$ nonnegative components, let $\boldsymbol{I}$ be the $m \times m$ identity matrix, and denote by $\mathbf{1}$ the vector all of whose $m$ entries are equal to 1. Consider the following two LP problems:

(LP1) $\underset{x}{\text{maximize}}$: $\boldsymbol{c} \cdot \boldsymbol{x}$

subject to: $\boldsymbol{Ax} = \boldsymbol{b}, \boldsymbol{x} \geqslant \mathbf{0}$

(LP2) $\underset{x,a}{\text{maximize}}$: $\mathbf{0} \cdot \boldsymbol{x} + (-\mathbf{1}) \cdot \boldsymbol{a}$

subject to: $\boldsymbol{Ax} + \boldsymbol{Ia} = \boldsymbol{b}, \quad \boldsymbol{x}, \boldsymbol{a} \geqslant \mathbf{0}$

We have the following.

(a) Problem LP2 is both feasible and bounded hence the simplex algorithm produces an optimal solucion $\boldsymbol{x}^*$, $\boldsymbol{a}^*$.

(b) The optimum value of LP2 is strictly less than zero if and only if LP1 is infeasible.

(c) If the optimum value of LP2 is zero, taken on at $\boldsymbol{a}^* = \boldsymbol{0}$, then $\boldsymbol{x}^*$ is a basic feasible solution of LP1. Also, the final simplex system of constraints for LP2, with $\boldsymbol{a} = \boldsymbol{0}$, is equivalent to the initial constraints $\boldsymbol{Ax} = \boldsymbol{b}$ for LP1.

**注释与说明**

1. LP 是 Linear Programming 的缩写;LP problem 就是"线性规划问题"。

2. We will not discuss this issue further here; for more information, the reader may see Hillier and Lieberman[31] or Gribik and Kortanek [26]. 这里[31]和[26]分别代表列在书末的序号为 31 和 26 的参考文献. 全句可译成:对这个问题,我们这里不做进一步的讨论;想获得更多信息的读者可参阅 Hillier 和 Lieberman 的著作[31]或 Gribik 和 Kortanek 的著作[26]。

**学习要求**

1. 掌握如下内容:

(1) 全文的主题和中心意思、各段的主要内容。

(2) 15 个新出现的或常用的数学术语,8~10 个表示数学命题的句型。其中数学术语包括:linear programming(线性规划),optimal(最优的),dual(对偶的),Strong Duality Theorem(强对偶定理),simplex(单[纯]形),simplex method(单纯形法),nonempty(非空的),feasible region(可行域),minimum problem(最小值问题),maximum problem(最大值问题),objective function(目标函数),subject to(服从于),surplus variable(剩余变量),artificial variable(人工变量)。

2. 问答如下问题:

(1) 线性规划的非标准问题与标准问题有什么不同?

(2) 解非标准问题的主要困难是什么? 如何克服这个困难?

(3) LP2 与 LP1 有什么不同,它们有什么联系?

(4) 附注(REMARK)部分主要用来说明什么问题?

## 3.4.5 Construction of Mathematical Models ①

In the preceding discussion we viewed modeling as a process and considered

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① 本节课文摘自:F. R. Giordano, M. D. Weir, W. P. Fox. A First Course in Mathematical Modeling. 3rd edt. Beijing: China Machine Press, 2003.

briefly the form of the model. Now let's focus attention on the construction of mathematical models. We begin by presenting an outline of a procedure that is helpful in constructing models. In the next section, we illustrate the various steps in the procedure by discussing several real-world examples.

***STEP* 1 Identify the problem.** What is the problem you would like to explore? Typically this is a difficult step because in real-life situations no one simply hands you a mathematical problem to solve. Usually you have to sort through large amounts of data and identify some particular aspect of the situation to study. Moreover, it is imperative to be sufficiently precise (ultimately) in the formulation of the problem to allow for translation of the verbal statements describing the problem into mathematical symbology. This translation is accomplished through the next steps. It is important to realize that the answer to the question posed might not lead directly to a usable problem identification.

***STEP* 2 Make assumptions.** Generally, we cannot hope to capture in a usable mathematical model all the factors influencing the identified problem. The task is simplified by reducing the number of factors under consideration. Then, relationships among the remaining variables must be determined. Again, by assuming relatively simple relationships, we can reduce the complexity of the problem. Thus the assumptions fall into two main activities:

a. *Classify the variables*:

What things influence the behavior of the problem identified in Step 1? List these things as variables. The variables the model seeks to explain are the dependent variables and there may be several of these. The remaining variables are the independent variables. Each variable is classified as dependent, independent, or neither.

You may choose to neglect some of the independent variables for either of two reasons. First, the effect of the variable may be relatively small compared to other factors involved in the behavior. Second, a factor that affects the various alternatives in about the same way may be neglected, even though it may have a very important influence on the behavior under investigation. For example, consider the problem of determining the optimal shape for a lecture hall, where readability of a chalkboard or overhead projection is a dominant criterion. Lighting is certainly a crucial factor, but it would affect all possible shapes in about the same way. By neglecting such a variable, possibly incorporating it later in a separate, more refined model, the analysis can be simplified considerably.

b. *Determine interrelationships among the variables selected for study*:

Before we can hypothesize a relationship between the variables, we generally must make some additional simplifications. The problem may be sufficiently complex so that we cannot see a relationship among all the variables initially. In such cases it may be possible to study submodels. That is, we study one or more of the independent variables separately. Eventually we will connect the submodels together. Studying various techniques, such as proportionality, will aid in hypothesizing relationships among the variables.

***STEP* 3 Solve or interpret the model.** Now put together all the submodels to see what the model is telling us. In some cases the model may consist of mathematical equations or inequalities that must be solved to find the information we are seeking. Often, a problem statement requires a best or *optimal solution* to the model. Models of this type are discussed later.

Often, we will find that we are not quite ready to complete this step, or we may end up with a model so unwieldy we cannot solve or interpret it. In such situations we might return to Step 2 and make additional simplifying assumptions. Sometimes we will even want to return to Step 1 to redefine the problem. This point will be amplified in the following discussion.

***STEP* 4 Verify the model.** Before we can use the model, we must test it out. There are several questions to ask before designing these tests and collecting data — a process that can be expensive and time-consuming. First, does the model answer the problem identified in Step 1, or did it stray from the key issue as we constructed the model? Second, is the model usable in a practical sense: that is, can we really gather the data necessary to operate the model? Third, does the model make common sense?

Once the commonsense tests are passed, we will want to test many models using actual data obtained from empirical observations. We need to be careful to design the test in such a way as to include observations over the same range of values of the various independent variables we expect to encounter when actually using the model. The assumptions made in Step 2 may be reasonable over a restricted range of the independent variables but very poor outside of those values. For instance, a frequently used interpretation of Newton's second law states that the net force acting on a body is equal to the mass of the body times its acceleration. This law is a reasonable model until the speed of the object approaches the speed of light.

Be careful about the conclusions you draw from any tests. Just as we cannot prove a theorem simply by demonstrating many cases that support the theorem, likewise, we cannot extrapolate broad generalizations from the particular evidence we gather about our model. A model does not become a law just because it is verified repeatedly in some specific instances. Rather, we corroborate the reasonableness of our model through the data we collect.

***STEP* 5 Implement the model.** Of course, our model is of no use just sitting in a filing cabinet. We will want to explain our model in terms that the decision makers and users can understand if it is ever to be of use to anyone. Furthermore, unless the model is placed in a user-friendly mode, it will quickly fall into disuse. Expensive computer programs sometimes suffer such a demise. Often the inclusion of an additional step to facilitate the collection and input of the data necessary to operate the model determines its success or failure.

***STEP* 6 Maintain the model.** Remember that the model is derived from a specific problem identified in Step 1 and from the assumptions made in Step 2. Has the original problem changed in any way, or have some previously neglected factors become important? Does one of the submodels need to be adjusted?

We summarize the steps for constructing mathematical models in Figure 3-4-10. We should not be too enamored with our work. Like any model, our procedure is an approximation process and therefore has its limitations. For example, the procedure seems to consist of discrete steps leading to a usable result, but that is rarely the case in practice. Before offering an alternative procedure that emphasizes the iterative nature of the modeling process, let' s discuss the advantages of the methodology depicted in Figure model.

The process shown in Figure 3-4-10 provides a methodology for progressively focusing on those aspects of the problem we wish to study. Furthermore, it demonstrates a curious blend of creativity with the scientific method used in the modeling process. The first two steps are more artistic or original in nature. They involve abstracting the essential features of the problem under study, neglecting any factors judged to be unimportant, and postulating relationships precise enough to help answer the questions posed by the problem. However, these relationships must be simple enough to permit the completion of the remaining steps. Although these steps admittedly involve a degree of craftsmanship, we will learn some scientific techniques we can apply to appraise the importance of a particular variable and the preciseness of an assumed relationship. Nevertheless, when generating numbers in Steps 3 and 4, remember that the process has been largely inexact and intuitive.

| Construction of a mathematical model |
| --- |
| Step 1. Identify the problem.<br>Step 2. Make assumptions.<br>a. Identify and classify the variables.<br>b. Determine interrelationships between the variables and submodels.<br>Step 3. Solve the model.<br>Step 4. Verify the model.<br>a. Does it address the problem?<br>b. Does it make common sense?<br>c. Test it with real-world data.<br>Step 5. Implement the model.<br>Step 6. Maintain the model. |

Fig. 3-4-10

**学习要求**

1. 掌握如下内容：

(1) 全文的主题和中心意思、各段的主要内容。

(2) 15 个生词与数学术语，8 ~ 10 个表示数学命题的句型。其中生词与数学术语包括：model（模型），modeling process（建模过程），submodel（子模型），optimal solution（最优解），key issue（主要题目），empirical（经验主义的），interrelationship（相互关系），dependent variables（因变量），independent variables（自变量）等，net force（净力，纯力，此时 net 作形容词解），approximation（近似，逼近），postulate（假设，假定），amplify（详细陈述），stray（偏离（话题）），be enamored with（沉迷于），extrapolate（推广，外推）。

2. 回答如下问题：

(1) 数学模型的构建分成几个主要步骤，每个步骤的主要任务是什么？

(2) 建模过程为什么要先确认问题(identify the problem)，而后又要做许多假设？

(3) 在执行第三个步骤时，在什么情况下要重新回到第二个步骤？

(4) 如何检验一个模型？为什么不能随意推广一个不成熟的模型？

(5) 如何理解"一个束之高阁的模型是无用的(Our model is of no use just sitting in a filing cabinet)"？如何使之变得能用？

(6) 如何维护(完善)模型？如何评估模型？

# §3.5 计算数学与计算机科学

本节介绍计算数学与计算机科学，后者涉及该学科的一个重要组成部分——计算理论(theory of computation 或 theory of computing)。大体上说，计算数学研究如何尽可能准确而高效(包括省时间和省空间)地求出数学问题的解；计算机科学不仅研究如何让计算机算得快，而且研究哪些问题可用计算机计算，哪些问题不可。本节含有五个小节，第一、二小节分别介绍数值方法和微分方程数值解，属于计算数学的内容；此后几个小节都是计算机科学的基本知识，包括介绍计算理论的背景、基本概念、P 与 NP 问题及人工智能等。

这些基本知识不仅信息与计算数学专业和计算机专业的同学要懂，其他数学专业的同学也应有所了解。不过，从第二小节起的内容对于未学过有关专业知识的读者来说可能较难理解。同学们不应过于焦急，可先粗略读一下，然后再逐步推进。要知道，困难是对您的勇气和能力的一种挑战。相信有志气的读者是乐于迎难而上的，他们将在奋斗中不断提高自己的能力与水平。经过几次攻克难关之后，必将有“会当凌绝顶，一览众山小”的体会。

## 3.5.1 Introduction to numerical methods ①

The subject of numerical analysis is concerned with the *derivation*, *analysis* and *implementation* of methods for obtaining *reliable* numerical answers to mathematical problems. We use the adjective ‘reliable’ to indicate that it is essential to have confidence in any answers produced and an assessment of reliability can form part of the analysis of a method. In subsequent chapters we shall apply numerical methods to a variety of problems including finding some or all of the roots of an algebraic equation, solving a set of linear simultaneous equations and calculating the value of a definite integral. We can at this stage make a number of preliminary observations on the words ‘derivation’, ‘analysis’ and ‘implementation’ which appear at the beginning of this paragraph.

The ‘derivation’ stage is concerned with deriving and describing the sequence of numerical steps which is expected will eventually lead to the required numerical answers. The complete description of these steps, perhaps written in some pseudo-programming language, is called an *algorithm*. This stage may or may not be easy

① 本节课文摘自：C. Phillips & B. Cornlius. Computational Numerical Methods. New York: John Wiley and Sons, 1986.

and often intuition and experience will play an important role. As a simple example of a derivation, we may cite the so-called trapezium rule in which we try estimate the value of the definite integral

$$I = \int_a^b f(x)\,dx, b>a. \qquad (1.1)$$

In Fig. 3-5-1 (a) we suppose the curve $y=f(x)$ is plotted. Then, remembering the interpretation of a definite integral, if $A$ is the point $(a, f(a))$ and $B$ is the point $(b, f(b))$, we can say that an approximation to $I$ is given by the shaded area. This corresponds to the area of the trapezium with base $b-a$ and vertical sides $f(a)$ and $f(b)$; that is,

$$I \approx \frac{b-a}{2}[f(a)+f(b)]. \qquad (1.2)$$

We ought now to ask immediately about the accuracy of the approximation (1.2). Trying to find an answer to this question forms a part of the 'analysis' stage to which we have referred. For the present we observe that the error is represented by the unshaded part of the area under the curve. It may be 'small' in the sense that the trapezium rule approximation is sufficiently accurate for the purpose in hand, or it may be unacceptably large as illustrated in Fig. 3-5-1(b). It is important to realize that, in general, we shall not be able to find this error. What we can do, however, is to try to find a bound for it; that is, if $E$ denotes the error, we try to find a positive number bound $M$ for which we can assert that $|E| \leqslant M$.

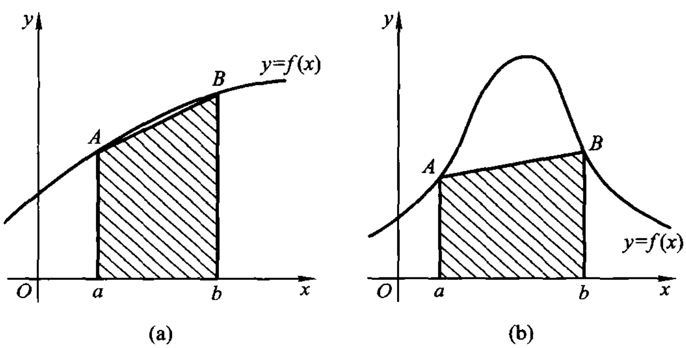


Fig. 3-5-1

Intuitively, we would expect that an improvement in accuracy can be obtained by dividing the interval of integration into a number of subintervals and then applying the trapezium rule to each sub-interval in turn. In section 6.8 we describe a method which employs interval subdivision to estimate a definite integral correct to some specified tolerance.

*In producing a computer program which implements a numerical method*, due attention must be paid to efficiency; by this we mean that time and storage requirements must not be excessive. Suppose that we wish to estimate (1.1) using the trapezium rule with $f(x)=2x^3-3x^2+4x+1$. Clearly, we need to evaluate $2a^3-3a^2+4a+1$ (and, of course, $2b^3-3b^2+4b+1$) and this would appear to involve six multiplications, one subtraction and two additions. However, by expressing $f(a)$ as $((2a-3)a+4)a+1$ we can reduce the number of multiplications to just three. Although this may *seem a fairly small reduction in the number of operations (and hence the time taken* to evaluate the expression) for high degree polynomials the savings achievable are very large indeed.

**学习要求**

1. 掌握如下内容：

（1）全文的主题和中心意思、各段的主要内容。

（2）15个新出现的或常用的计算数学术语，5～8个表示命题的句型。其中术语包括：numerical analysis（数值分析），derivation, analysis and implementation（推导、分析与实现），assessment（估计），algorithm（算法），trapezium rule（梯形法则），vertical side（竖直边），accuracy（精确度），efficiency（有效性），storage（存储器，存储量），savings achievable（可达节省），high degree polynomials（高阶多项式），reduction（简化），pseudo-programming language（伪程序语言）等。

2. 回答如下问题：

（1）本文提出哪些概念，得出哪些主要结论？

（2）本文对“reliable（可靠的）”这一概念是如何解释的？

（3）本文如何解释 derivation, analysis and implementation（推导、分析与实现）？

（4）在编制实现数值方法的计算机程序时特别要注意什么事情？

## 3.5.2 Prelude to Numerical partial differential equations ①

Numerical partial differential equations is a large area of study. The subject includes components in the areas of applications, mathematics and computers. These

① 本节课文摘自：J. W. Thomas. Numerical Partial Differential Equations. Berlin: Springer, 1995.

three aspects of a problem are so strongly tied together that it is virtually impossible to consider one in isolation from the other two. It is impossible (or at least, fool-hardy) to consider the applied aspect of a problem without considering at least some of the mathematical and computing aspects of that problem. Often, the mathematical aspects of numerical partial differential equations can be developed without considering applications or computing, but experience shows that this approach does not generally yield useful results.

Of the many different approaches to solving partial differential equations numerically (finite differences, finite elements, spectral methods, collocation methods, etc.), we shall study **difference methods.** We will provide a review of a large range of methods. A certain amount of theory and rigor will be included, but at all times implementation of the methods will be stressed. The goal is to come out of this course with a large number of methods about which you have **theoretical knowledge** and with which you have **numerical experience.**

Often, when numerical techniques are going to be used to solve a physical problem, it is not possible to thoroughly analyze the methods that are used. Any time we use methods that have not been thoroughly analyzed, we must resort to methods that become a part of **numerical experimentation.** As we shall see, often such experimentation will also become necessary for linear problems. In fact, we often do not even know what to try to prove analytically until we have run a well-designed series of experiments.

As in other areas of experimental work, the experiments must be care-fully designed. The total problem may involve the physics that describes the original problem; the mathematics involved in the partial differential equation, the difference equation and the solution algorithm for solving the difference equation; and the computer on which the numerical work will be done. Ideally, before experiments are designed, one should know as much as possible about each of the above aspects of the problem.

Several times throughout the text we will assign small experiments that will hopefully clarify some aspects of the schemes we will be studying. It is hoped that these experiments will begin to teach some useful experimental techniques that can be used in the area of numerical solution of partial differential equations. In other parts of the text, it will be necessary or best for the reader to decide that a small experiment is needed, and then design and run that experiment. It is hoped that one thing the reader will learn from this text is that *numerical experimentation is a part of the subject* *of numerical solution of partial differential equations.*

Numerical methods for solving partial differential equations, like the analytic methods, often depend on the type of equation, *i. e.* elliptic, parabolic or hyperbolic. A slight background in the methods and theory of partial differential equations will be assumed. An elementary course in partial differential equations is generally sufficient. In addition, since part of the emphasis of the text is on the implementation of the difference schemes, it will be assumed that the reader has sufficient programming skills to implement the schemes and appreciate the implementations that are discussed.

The methods that we develop, analyze and implement will usually be illustrated using common model equations. Most often these will be the heat equation, the one way wave equation or the Poisson equation. We will generally separate the methods to correspond to the different equation types by first doing methods for parabolic equations, then methods for hyperbolic equations and finally methods for elliptic equations. However, we reserve the right to introduce an equation of a different type and a method for equations of different types at any time.

Much of the useful work in numerical partial differential equations that is being done today involves nonlinear equations. The one class of nonlinear equations that we will study is in Chapter 9, Volume 2, ref. [13], where we consider the numerical solution of conservation laws. Often an analysis of the method developed to solve a nonlinear partial differential equation is impossible. In addition, the discrete version of the nonlinear problem may not itself be solvable. (Of course, methods like Newton's method can be used to solve the nonlinear problems. But Newton's method is really an iteration of a series of linear problems.) A common technique is to develop and test the schemes based on model linear problems that have similarities to the desired nonlinear problem. We will try to illustrate some methods that can be used to solve nonlinear problems. Below we include four model nonlinear problems. The reader should become familiar with these problems. When we develop linear methods that might be applicable to these nonlinear problems, a numerical experiment should be conducted using the linearized method to try to solve the nonlinear problem. Hints and discussions of methods will be given when we think we should be trying to solve problems HW0.0.1–0.0.4.

**HW 0.0.1 (Viscous Burgers' Equation)**

$$v_t+vv_x=\nu v_{xx}, \quad x\in(0,1),\ t>0, \tag{0.0.1}$$

$$v(0,t)=v(1,t)=0,\ t>0, \tag{0.0.2}$$

$$v(x,0)=\sin 2\pi x,\ x\in[0,1]. \tag{0.0.3}$$

We wish to find solutions for $\nu=1.0$, 0.1, 0.01, 0.001, 0.0001 and 0.00001.

The reader should be aware that there has been a large amount of work done on the above problem. There is an exact solution to the problem which is not especially useful to us. It is probably better to try to work this problem without knowing the exact solution since that is the situation in which a working numerical analyst must function.

**HW 0.0.2 (Inviscid Burgers' Equation)**

$$v_t+vv_x=0,\quad x\in(0,1),\ t>0, \tag{0.0.4}$$

$$v(0,t)=v(1,t)=0,\ t>0, \tag{0.0.5}$$

$$v(x,0)=\sin 2\pi x,\ x\in[0,1]. \tag{0.0.6}$$

We note that the only difference between this problem and HW0.0.1 is that the $v_{xx}$ term is not included. As we shall see later, this difference theoretically changes the whole character of the problem. We also note that this problem is well-posed with a boundary condition at both $x=0$ and $x=1$.

**HW 0.0.3 (Shock Tube Problem)**

Consider a tube filled with gas where a membrane separates the tube into two sections. For the moment, suppose that the tube is infinitely long, the membrane is situated at $x=0$, for $x<0$ the density and pressure are equal to 2, for $x>0$ the density and pressure are equal to 1 and the velocity is zero everywhere. At time $t=0$, the membrane is removed and the problem is to determine the resulting flow of gas in the tube. We assume that the gas is inviscid, the flow is one dimensional and write the conservation laws for the flow (mass, momentum and energy) as

$$\rho_t+(\rho v)_x=0, \tag{0.0.7}$$

$$(\rho v)_t+(\rho v^2+p)_x=0, \tag{0.0.8}$$

$$E_t+[v(E+p)]_x=0 \tag{0.0.9}$$

where $\rho$, $v$, $p$ and $E$ are the density, velocity, pressure and total energy, respectively. In addition, we assume that the gas is polytropic and write the equation of state as

$$p=(\gamma-1)(E-\rho v^2/2), \tag{0.0.10}$$

where $\gamma$ is the ratio of specific heats which we take to be equal to 1.4. Clearly, both as a physical problem and as a practical numerical problem, a finite domain and boundary conditions are necessary. These conditions will be discussed in Chapter 6 when we prod you into beginning to solve this problem.

**HW 0.0.4 (Thin Disturbance Transonic Flow Equation)**

Consider potential flow over a symmetric, thin, circular arc airfoil. Expansions based on the thickness of the airfoil reduce the problem to one involving the thin disturbance transonic flow equation. For example we consider the following problem describing the transonic flow past a 5% half circular arc airfoil.

$$[1-M_\infty^2-(\gamma+1)M_\infty^2\phi_x]\phi_{xx}+\phi_{yy}=0,$$

$$(x,y)\in R=(-1,2)\times(0,1), \tag{0.0.11}$$

$$\phi_x=0 \text{ on } \partial R-\{(x,0):0\leqslant x\leqslant 1\}, \tag{0.0.12}$$

$$\phi_y=\frac{-(x-1/2)}{\sqrt{6.375\,625-(x-1/2)^2}} \quad \text{on} \quad \{(x,0):0\leqslant x\leqslant 1\}, \tag{0.0.13}$$

(i) Solve this problem for $\gamma=1.4$ and $M_\infty=0.7$ and

(ii) $\gamma=1.4$ and $M_\infty=0.78$.

Though nonlinear, the partial differential equation above will look like either an elliptic equation or a hyperbolic equation, depending on the sign of the term $[1-M_\infty^2-(\gamma+1)M_\infty^2\phi_x]$.

As you will see as you attempt to solve problems HW 0.0.1-0.0.4, when we apply schemes developed for model linear problems to nonlinear problems, the results are often not what we anticipated. For this reason, much care must be taken when we work the above problems.

**学习要求**

1. 掌握如下内容:

(1) 全文的主题和中心意思、各段的主要内容。

(2) 20 个新出现的或常用的数学与物理学的术语,8~10 个专业常用的句型。其中术语包括: finite differences (有限差分), finite elements (有限元), spectral methods (谱方法), collocation methods (配置法), partial differential equation(偏微分方程), numerical experimentation (数值实验), programming skill (编程技巧), wave equation (波动方程), Poisson equation (泊松方程), polytropic (多变的), viscous (黏滞性的), inviscid (非黏滞性的), iteration (迭代), conservation laws (守恒定律), potential flow (位势流), disturbance (扰动), airfoil (翼剖面), transonic flow (超音速流)等。

2. 回答如下问题:

(1) 本文提出哪些概念,得出哪些主要结论?

(2) 数值微分方程问题是由哪几个方面的内容组成的?

（3）为什么数值实验是偏微分方程数值解这个主题的一部分？

（4）你是否理解问题 HW 0.0.1～0.0.4 的含义？在解决这几个问题时应注意什么？

### 3.5.3 Introduction to the theory of Computation ①

Let's begin with an overview of those areas in the theory of computation that we present in this course. Then, you'll have a chance to learn and/or review some mathematical concepts that you will need later.

#### AUTOMATA, COMPUTABILITY, AND COMPLEXITY

This book focuses on three traditionally central areas of the theory of computation: automata, computability, and complexity. They are linked by the question:

*What are the fundamental capabilities and limitations of computers?*

This question goes back to the 1930s when mathematical logicians first began to explore the meaning of computation. Technological advances since that time have greatly increased our ability to compute and have brought this question out of the realm of theory into the world of practical concern.

In each of the three areas — automata, computability, and complexity — this question is interpreted differently, and the answers vary according to the interpretation. Following this introductory chapter, we'll explore each area in a separate part of this book. Here, we introduce these parts in reverse order because starting from the end you can better understand the reason for the beginning.

#### COMPLEXITY THEORY

Computer problems come in different varieties; some are easy and some hard. For example, the sorting problem is an easy one. Say that you need to arrange a list of numbers in ascending order. Even a small computer can sort a million numbers rather quickly. Compare that to a scheduling problem. Say that you must find a schedule of classes for the entire university to satisfy some reasonable constraints, such as that no two classes take place in the same room at the same time. The scheduling problem seems to be much harder than the sorting problem. If you have just a thousand classes, finding the best schedule may require centuries, even with a

① 本节课文摘自：M. Sipser. Introduction to the Theory of Computation. Boston: PES Pub, 1997.

supercomputer.

*What makes some problems computationally hard and others easy?*

This is the central question of complexity theory. Remarkably, we don't know the answer to it, though it has been intensively researched for the past 25 years. Later, we explore this fascinating question and some of its ramifications.

In one of the important achievements of complexity theory thus far, researchers have discovered an elegant scheme for classifying problems according to their computational difficulty. It is analogous to the periodic table for classifying elements according to their chemical properties. Using this scheme, we can demonstrate a method for giving evidence that certain problems are computationally hard, even if we are unable to prove that they are so.

You have several options when you confront a problem that appears to be computationally hard. First, by understanding which aspect of the problem is at the root of the difficulty, you may be able to alter it so that the problem is more easily solvable. Second, you may be able to settle for less than a perfect solution to the problem. In certain cases finding solutions that only approximate the perfect one is relatively easy. Third, some problems are hard only in the worst case situation, but easy most of the time. Depending on the application, you may be satisfied with a procedure that occasionally is slow but usually runs quickly. Finally, you may consider alternative types of computation, such as randomized computation, that can speed up certain tasks.

One applied area that has been affected directly by complexity theory is the ancient field of cryptography. In most fields, an easy computational problem is preferable to a hard one because easy ones are cheaper to solve. Cryptography is unusual because it specifically requires computational problems that are hard, rather than easy, because secret codes should be hard to break without the secret key or password. Complexity theory has pointed cryptographers in the direction of computationally hard problems around which they have designed revolutionary new codes.

## COMPUTABILITY THEORY

During the first half of the twentieth century, mathematicians such as Kurt Gödel, Alan Turing, and Alonzo Church discovered that certain basic problems cannot be solved by computers. One example of this phenomenon is the problem of determining whether a mathematical statement is true or false. This task is the bread and butter of mathematicians. It seems like a natural for solution by computer because it lies strictly within the realm of mathematics. But no computer algorithm can perform this task.

Among the consequences of this profound result was the development of ideas concerning theoretical models of computers that eventually would help lead to the construction of actual computers.

The theories of computability and complexity are closely related. In complexity theory, the objective is to classify problems as easy ones and hard ones, whereas in computability theory the classification of problems is by those that are solvable and those that are not. Computability theory introduces several of the concepts used in complexity theory.

## AUTOMATA THEORY

Automata theory deals with the definitions and properties of mathematical models of computation. These models play a role in several applied areas of computer science. One model, called the *finite automaton*, is used in text processing, compilers, and hardware design. Another model, called the *context-free grammar*, is used in programming languages and artificial intelligence.

Automata theory is an excellent place to begin the study of the theory of computation. The theories of computability and complexity require a precise definition of a *computer*. Automata theory allows practice with formal definitions of computation as it introduces concepts relevant to other nontheoretical areas of computer science.

## 学习要求

1. 掌握如下内容:

(1) 全文的主题和中心意思、各段的主要内容。

(2) 15 个新出现的或常用的数学与计算机科学的术语,5 ~ 8 个专业常用的句型。其中术语包括: automata (自动机), computability (可计算性), complexity (复杂性), ascending order (递增升序), reverse order (逆序), sorting problem (分类问题), reasonable constraints (合理的约束), randomized computation (随机化计算), secret codes (密码), secret key (密钥), cryptography (密码学), compiler (编译器), hardware design (硬件设计), text processing (文本处理), programming language (程序语言), artificial intelligence (人工智能), context-free grammar (上下文无关语法)等。

2. 回答如下问题:

(1) 本文提出哪些概念,得出哪些主要结论?

(2) 传统的计算理论包括哪几个方面的内容?

(3) 计算复杂性理论与可计算理论分别研究什么问题? 二者有何联系?

(4) 自动机理论用来处理什么问题? 它有什么重要作用?

## 3.5.4 P versus NP Question and NP completement ①

### Ⅰ) P class, NP class

For our purposes, polynomial differences in running time are considered to be small, whereas exponential differences are considered to be large. Let's look at why we chose to make this separation between polynomials and exponentials rather than between some other classes of functions.

P is the class of languages that are decidable in polynomial time on a deterministic single-tape Turing machine. In other words,

$$P = \bigcup_k \text{TIME}(n^k).$$

The class P plays a central role in our theory and is important because

1. P is invariant for all models of computation that are polynomially equivalent to the deterministic single-tape Turing machine, and

2. P roughly corresponds to the class of problems that are realistically solvable on computer.

NP is the class of languages that have polynomial time verifiers. (*A verifier* for a language $A$ is an algorithm $V$, where

$$A = \{w \mid V \text{ accepts } \langle w, c\rangle \text{ for some string } c\}.$$

We measure the time of a verifier only in times of length of $w$, so a *polynomial time verifier* runs in polynomial time in the length of $w$. A language $A$ is *polynomially verifiable* if it has a polynomial time verifier.)

The class NP is important because it contains many problems of practical interest. From the preceding discussion, HAMPATH and COMPOSITES are members of NP. The term NP comes from *nondeterministic polynomial time* and derived from an alternative characterization by using nondeterministic polynomial time Turing machine.

### Ⅱ) The P Versus NP Question

As we have been saying, NP is the class of languages that are solvable in polynomial time on a nondeterministic Turing machine, or, equivalently, it is the class of languages whereby membership in the language can be verified in polynomial time. P is the class of languages where membership can be tested in polynomial time. We

① 本节课文摘自:M. Sipser. Introduction to the Theory of Computation. Boston: PES Pub, 1997.

summarize this information as follows, where we loosely refer to polynomial time solvable as solvable "quickly".

P = the class of languages where membership can be decided quickly.

NP = the class of languages where membership can be verified quickly.

We have presented examples of languages, such as HAMPATH and CLIQUE, that are members of NP but that are not known to be in P. The power of polynomial verifiability seems to be much greater than that of polynomial decidability. But, hard as it may be to imagine, P and NP could be equal. We are unable to prove the existence of a single language in NP that is not in P.

The question of whether P = NP is one of the greatest unsolved problems in theoretical computer science and contemporary mathematics. If these classes were equal, any polynomially verifiable problem would be polynomially decidable. Most researchers believe that the two classes are not equal because people have invested enormous effort to find polynomial time algorithms for certain problems in NP, without success. Researchers also have tried proving that the classes are unequal, but that would entail showing that no fast algorithm exists to replace brute-force search. Doing so is presently beyond scientific reach. The following figure shows the two possibilities.

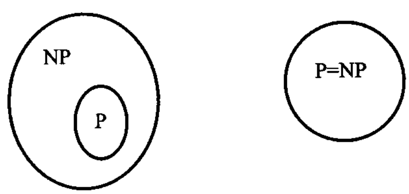


Fig. 3-5-2 One of these two possibilities is correct.

The best method known for solving languages in NP deterministically uses exponential time. In other words, we can prove that

$$NP \subseteq EXPTIME = \bigcup_k TIME(2^{n^k}),$$

but we don't know whether NP is contained in a smaller deterministic time complexity class.

### Ⅲ) NP-Completeness

One important advance on the P versus NP question came in the early 1970s with the work of Stephen Cook and Leonid Levin. They discovered certain problems in NP whose individual complexity is related to that of the entire class. If a polynomial time algorithm exists for any of these problems, all problems in NP would be polynomial time solvable. These problems are called *NP-complete*. The phenomenon of NP-completeness is important for both theoretical and practical reasons.

On the theoretical side, a researcher trying to show that P is unequal to NP may focus on an NP-complete problem. If any problem in NP requires more than polynomial time, an NP-complete one does. Furthermore, a researcher attempting to prove that P equals NP only needs to find a polynomial time algorithm for an NP-complete problem to achieve this goal.

On the practical side, the phenomenon of NP-completeness may prevent wasting time searching for a nonexistent polynomial time algorithm to solve a particular problem. Even though we may not have the necessary mathematics to prove that the problem is unsolvable in polynomial time, we believe that P is unequal to NP, so proving that a problem is NP-complete is strong evidence of its nonpolynomiality.

The first NP-complete problem that we present is called the *satisfiability problem*. Recall that variables that can take on the values TRUE and FALSE are called *Boolean variables* (see Section 0.2). Usually, we represent TRUE by 1 and FALSE by 0. The Boolean operations AND, OR, and NOT, represented by the symbols $\wedge$, $\vee$, and $\neg$, respectively, are described in the following list. We use the overbar as a shorthand for the $\neg$ symbol, so $\bar{x}$ means $\neg x$.

$$
\begin{array}{lll}
0 \wedge 0 = 0, & 0 \vee 0 = 0, & \bar{0} = 1, \\
0 \wedge 1 = 0, & 0 \vee 1 = 1, & \bar{1} = 0, \\
1 \wedge 0 = 0, & 1 \vee 0 = 1, & \\
1 \wedge 1 = 1, & 1 \vee 1 = 1. &
\end{array}
$$

A *Boolean formula* is an expression involving Boolean variables and operations. For example,

$$\phi = (\bar{x} \wedge y) \vee (x \wedge \bar{z})$$

is a Boolean formula. A Boolean formula is *satisfiable* if some assignment of 0*s* and 1*s* to the variables makes the formula evaluate to 1. The preceding formula is satisfiable because the assignment $x = 0$, $y = 1$, and $z = 0$ makes $\phi$ evaluate to 1. We say the assignment satisfies $\phi$. The *satisfiability problem* is to test whether a Boolean formula is satisfiable. Let

$$\text{SAT} = \{ \langle \phi \rangle \mid \phi \text{ is a satisfiable Boolean formula} \}.$$

Now we state the Cook-Levin theorem which links the complexity of the SAT problem to the complexities of all problems in NP.

**Cook-Levin theorem** SAT $\in$ P iff P = NP.

Next, we develop the method that is central to the proof of the Cook-Levin theorem.

**学习要求**

1. 掌握如下内容：

(1) 全文的主题和中心意思、各段的主要内容。

(2) 12 个新出现的或常用的数学与计算科学的术语，5～8 个专业常用的句型。其中术语包括：Turing machine（图灵机）Boolean variable（布尔变量），Boolean formula（布尔公式），evaluate（赋值，求值），nonexistent（不存在的），satisfiability（可满足性），satisfiable（可满足的），polynomial time algorithm（多项式时间算法），nonpolynomiality（非多项式），EXPTIME 即 exponential time（指数时间），deterministically（确定性地），decidability（可判定性），verifier（验证算法，检验器），discourse（谈论，讲述）等。

2. 回答如下问题：

(1) 本文提出哪些概念，得出哪些主要结论？

(2) 什么是 P 类，什么是 NP 类？它们的重要性何在？

(3) 如果把"多项式时间可解"粗略地称作"快速可解"的话，如何描述 P 类和 NP 类？The P Versus NP Question 的含义是什么？

(4) 为什么说"P 是否等于 NP"是最大的未解决问题？

(5) 已经知道的用于解决 NP 语言问题的最好方法是什么？

(6) 什么是 NP 完全性？为什么说它在理论上和实践上都是重要的？

(7) 描述满足性问题的含义及其与 NP 完全性的关系。

(8) 如何用 Boolean formula 描述 Cook-Levin theorem？

## 3.5.5 Artificial intelligence and its tasks? ①

What exactly is artificial intelligence? Although most attempts to define complex and widely used terms precisely are exercises in futility, it is useful to draw at least an approximate boundary around the concept to provide a perspective on the discussion that follows. To do this, we propose the following by no means universally accepted definition. *Artificial intelligence* (AI) is the study of how to make computers do things which, at the moment, people do better. This definition is, of course,

① 本节课文摘自：R. Knight. Artificial Intelligence. New York: McGraw-Hill, Inc. 1991.

somewhat ephemeral because of its reference to the current state of computer science. And it fails to include some areas of potentially very large impact, namely problems that cannot now be solved well by either computers or people. But it provides a good outline of what constitutes artificial intelligence, and it avoids the philosophical issues that dominate attempts to define the meaning of either *artificial* or *intelligence*. Interestingly, though, it suggests a similarity with philosophy at the same time it is avoiding it. Philosophy has always been the study of those branches of knowledge that were so poorly understood that they had not yet become separate disciplines in their own right. As fields such as mathematics or physics became more advanced, they broke off from philosophy. Perhaps if AI succeeds it can reduce itself to the empty set.

What then are some of the problems contained within AI? Much of the early work in the field focused on formal tasks, such as game playing and theorem proving. Samuel wrote a checkers-playing program that not only played games with opponents but also used its experience at those games to improve its later performance. Chess also received a good deal of attention. The Logic Theorist was an early attempt to prove mathematical theorems. It was able to prove several theorems from the first chapter of Whitehead and Russell's *Principia Mathematica*[1]. Gelernter's theorem prover explored another area of mathematics: geometry. Game playing and theorem proving share the property that people who do them well are considered to be displaying intelligence. Despite this, it appeared initially that computers could perform well at those tasks simply by being fast at exploring a large number of solution paths and then selecting the best one. It was thought that this process required very little knowledge and could therefore be programmed easily. As we will see later, this assumption turned out to be false since no computer is fast enough to overcome the combinatorial explosion generated by most problems.

Another early foray into AI focused on the sort of problem solving that we do every day when we decide how to get to work in the morning, often called *commonsense reasoning*. It includes reasoning about physical objects and their relationships to each other (e.g., an object can be in only one place at a time), as well as reasoning about actions and their consequences (e.g., if you let go of something, it will fall to the floor and maybe break). To investigate this sort of reasoning, Newell, Shaw, and Simon built the General Problem Solver (GPS)[2], which they applied to several commonsense tasks as well as to the problem of performing symbolic manipulations of logical expressions. Again, no attempt was made to create a program with a large amount of knowledge about a particular problem domain. Only quite simple tasks were selected.

As AI research progressed and techniques for handling larger amounts of world knowledge were developed, some progress was made on the tasks just described and new tasks could reasonably be attempted. These include perception (vision and speech), natural language understanding, and problem solving in specialized domains such as medical diagnosis and chemical analysis.

Perception of the world around us is crucial to our survival. Animals with much less intelligence than people are capable of more sophisticated visual perception than are current machines. Perceptual tasks are difficult because they involve analog (rather than digital) signals: the signals are typically very noisy and usually a large number of things (some of which may be partially obscuring others) must be perceived at once. The problems of perception are discussed in greater detail in Chapter 2. 1.

The ability to use language to communicate a wide variety of ideas is perhaps the most important thing that separates humans from the other animals. The problem of understanding spoken language is a perceptual problem and is hard to solve for the reasons just discussed. But suppose we simplify the problem by restricting it to written language. This problem, usually referred to as *natural language understanding*, is still extremely difficult. In order to understand sentences about a topic, it is necessary to know not only a lot about the language itself (its vocabulary and grammar) but also a good deal about the topic so that unstated assumptions can be recognized. We discuss this problem again later in this chapter and then in more detail in Chapter 15.

In addition to these mundane tasks, many people can also perform one or maybe more specialized tasks in which carefully acquired expertise is necessary. Examples of such tasks include engineering design, scientific discovery, medical diagnosis, and financial planning. Programs that can solve problems in these domains also fall under the aegis of artificial intelligence. Figure 3-5-3 lists some of the tasks that are the targets of work in AI.

A person who knows how to perform tasks from several of the categories shown in the figure learns the necessary skills in a standard order. First perceptual, linguistic, and commonsense skills are learned. Later (and of course for some people, never) expert skills such as engineering, medicine, or finance are acquired. It might seem to make sense then that the earlier skills are easier and thus more amenable to computerized duplication than are the later, more specialized ones. For this reason, much of the initial AI work was concentrated in those early areas. But it turns out that this naive assumption is not right. Although expert skills require knowledge that many of use do not have, they often require much less knowledge than do the more mundane skills and that knowledge is usually easier to represent and deal with inside programs.

**Mundane Tasks**

- Perception
  - — Vision
  - — Speech
- Natural language
  - — Understanding
  - — Generation
  - — Translation
- Commonsense reasoning
- Robot control

**Formal Tasks**

- Games
  - — Chess
  - — Backgammon
  - — Checkers
  - — Go
- Mathematics
  - — Geometry
  - — Logic
  - — Integral calculus
  - — Proving properties of programs

**Expert Tasks**

- Engineering
  - — Design
  - — Fault finding
  - — Manufacturing planning
- Scientific analysis
- Medical diagnosis
- Financial analysis

Fig. 3-5-3 Some of the Task Domains of Artificial Intelligence

As a result, the problem areas where AI is now flourishing most as a practical discipline (as opposed to a purely research one) are primarily the domains that require only specialized expertise without the assistance of commonsense knowledge. There are now thousands of programs called expert systems in day-to-day operation throughout all areas of industry and government. Each of these systems attempts to solve part, or perhaps all, of a practical, significant problem that previously required scarce human expertise. In Chapter 20 we examine several of these systems and explore techniques for constructing them.

Before embarking on a study of specific AI problems and solution techniques, it is important at least to discuss, if not to answer, the following four questions:

1. What are our underlying assumptions about intelligence?
2. What kinds of techniques will be useful for solving AI problems?
3. At what level of detail, if at all, are we trying to model human intelligence?
4. How will we know when we have succeeded in building an intelligent program?

The next four sections of this chapter address these questions. Following that is a survey of some AI books that may be of interest and a summary of the chapter.

**注释与说明**

1. The Logic Theorist was an early attempt to prove mathematical theorems. It was able to prove several theorems from the first chapter of Whitehead and Russell's *Principia Mathematica*. 这里 The Logic Theorist(理论家)是一种软件系统的名称;*Principia Mathematica* 本是数学软件的名称,这里指介绍该软件的著作。第一句是"主—系—表"结构,表示一种状态或情况。这两句可译成:软件 The Logic Theorist(理论家)是人们早期在数学定理证明方面的一种尝试.它能够证明 Whitehead and Russell 所著《Principia Mathematica》的第一章的几个定理。

2. …Newell, Shaw, and Simon built the General Problem Solver(GPS)… 这里 General Problem Solver 是人工智能软件的名称,由 Newell, Shaw 和 Simon创建。

**学习要求**

1. 掌握如下内容:

(1) 全文的主题和中心意思、各段的主要内容。

(2) 15 个新出现的或常用的数学与计算机科学的术语,8 ~ 10 个表示专业命题的句型。其中数学与计算机科学的术语包括:artificial intelligence(人工智

能),commonsense reasoning(常识推理),first perceptual(第一知觉),expert system(专家系统),entitie(实物),expression(表达式),symbol structure(符号结构),The Physical Symbol System Hypothesis(物理符号系统假定),category(范畴),Robot control(机器人控制),mundane tasks(常规任务),naive assumption(天真的假设),symbolic manipulations(符号操作)等。

2. 回答如下问题:

(1) 作者认为如何给人工智能下定义最合适?为什么?

(2) 早期人工智能主要从事哪些方面的研究?关于人工智能的早期研究,什么样的想法被称为是“天真的设想”并已被证实是错误的?

(3) 人工智能包括哪些任务?在开展人工智能特殊问题研究之前应该先讨论哪些问题?

(4) 作者对知觉的任务(Perceptual tasks)是如何评价的?为什么完成这个任务的难度很大?

## §3.6 新数学分支简介

本节含有三个小节。第一小节介绍小波与傅里叶分析,第二小节介绍分形几何的发现,第三小节介绍模糊集合。模糊数学与分形几何是20世纪60年代新诞生的两个最引人注目的数学分支,而小波分析因其应用之广泛而从80年代起得到迅速发展。希望本节提供的阅读材料有助于读者了解数学最新发展的部分信息。

现代科学技术的进步日新月异,具备了解科学技术的最新发展动态的能力是当代科技人员自身知识持续发展的需要,也是每一个本科生必须具备的基本能力之一,同学们应该加以重视。

由于这三篇短文介绍的是新领域的知识且应用性较强,所以难度较大。特别是第三小节,不仅数学概念不易理解,而且文中有较多晦涩难懂的书面语言。相信此前已做过大量阅读的朋友们,绝不会知难而退的。勇于探索知识奥秘的读者们一定能在阅读中找到许多乐趣。

### 3.6.1 Wavelets and Fourier analysis ①

Fourier series and the Fourier transform have been around since the nineteenth century and many research articles and books (at both the graduate and under-gradu-

① 本节课文摘自:A. Boggess & F. J. Narcowich. A First Course in Wavelets with Fourier Analysis. New York: Printice Hall. 2002.

ate levels) have been written about these topics. By contrast, the development of wavelets has been much more recent. While its origins go back many decades, the subject of wavelets has become a popular tool in signal analysis and other areas of applications only within the last two decades or so partly as a result of Ingrid Daubechies's celebrated work on the construction of compactly supported, orthonormal wavelets.

**Fourier Analysis**

The basic goal of Fourier series is to take a signal, which will be considered as a function of the time variable $t$, and decompose it into its various frequency components. The basic building blocks are the sine and cosine functions:

$$\sin(nt), \qquad \cos(nt),$$

which vibrate at a frequency of $n$ times per $2\pi$ interval. As an example, consider the following function:

$$f(t) = \sin(t) + 2\cos(3t) + 0.3\sin(50t).$$

This function has three components that vibrate at frequency 1 [the $\sin(t)$ part], at frequency 3 [the $2\cos(3t)$ part], and at frequency 50 [the $0.3\sin(50t)$ part].

The graph of $f$ is given in Figure 3-6-1.

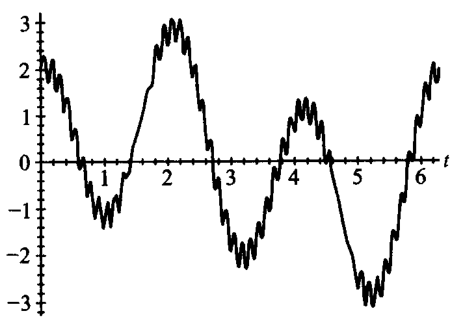


Fig .3-6-1 Plot of $f(t) = \sin(t) + 2\cos(3t) + 0.3\sin(50t)$

A common problem in signal analysis is to filter out unwanted noise. The background hiss on a cassette tape is an example of high-frequency (audio) noise that various devices (Dolby filters) try to filter out. In the preceding example, the component, $0.3\sin(50t)$, contributes the high-frequency wiggles to the graph of $f$ in Figure 3-6-1. By setting the coefficient 0.3 equal to zero, the resulting function is

$$\tilde{f}(t) = \sin(t) + 2\cos(3t)$$

whose graph (given in Figure 3-6-2) is the same as the one for $f$ but without the high-frequency wiggles.

The preceding example shows that one approach to the problem of filtering out unwanted noise is to express a given signal, $f(t)$, in terms of sines and cosines:

$$f(t)=\sum_{n} a_n\cos(nt)+b_n\sin(nt)$$

and then to eliminate (i.e., set equal to zero) the coefficients (the $a_n$ and $b_n$) that correspond to the unwanted frequencies. In the case of the signal $f$ just presented, this process is easy since the signal is already presented as a sum of sines and cosines. Most signals, however, are not presented in this manner. The subject of Fourier series, in part, is the study of how to efficiently decompose a function into a sum of cosine and sine components so that various types of filtering can be accomplished easily.

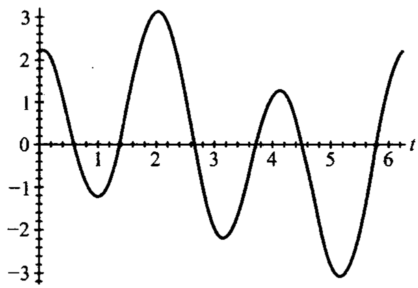


Fig. 3-6-2 Plot of $f(t)=\sin(t)+2\cos(3t)$

Another related problem in signal analysis is that of data compression. Imagine that the graph of the signal $f(t)$ in Figure 3-6-1 represents a telephone conversation. The horizontal axis is time, perhaps measured in milliseconds, and the vertical axis represents the electric voltage of a sound signal generated by someone's voice. Suppose this signal is to be digitized and sent via satellite overseas from America to Europe. One naive approach is to sample the signal every millisecond or so and send these data bits across the Atlantic. However, this would result in thousands of data bits per second for just one phone conversation. Since there will be many such conversations between the two continents, the phone company would like to compress this signal into as few digital bits as possible without significantly distorting the signal. A more efficient approach is to express the signal into its Fourier series: $f(t)=\sum_{n} a_n\cos(nt)+b_n\sin(nt)$ and then discard those coefficients, $a_n$ and $b_n$, that are smaller than some tolerance for error. Only those coefficients that are above this tolerance need to be sent across the Atlantic, where the signal can then be reconstructed. For most signals, the number of significant coefficients in its Fourier series is relatively small.

**Wavelets**

One disadvantage of Fourier series is that its building blocks, sines and cosines, are periodic waves that continue forever. While this approach may be appropriate for filtering or compressing signals that have time-independent wavelike features (as in Figure 3-6-1), other signals may have more localized features for which sines and cosines do not model very well. As an example, consider the graph given in Figure 3-6-3. This may represent a sound signal with two isolated noisy pops that need to be filtered out. Since these pops are isolated, sines and cosines do not model this signal very well. A different set of building blocks, called wavelets, is designed to model these types of signals. In a rough sense, a wavelet looks like a wave that travels for one or more periods and is nonzero only over a finite interval instead of propagating forever the way sines and cosines do [see Figure 3-6-4 for the graph of the Daubechies ($N=2$) wavelet]. A wavelet can be translated forward or backward in time. It also can be stretched or compressed by scaling to obtain low-and high-frequency wavelets (see Figure 3-6-5). Once a wavelet function is constructed, it can be used to filter or compress signals in much the same manner as Fourier series. A given signal is first expressed as a sum of translations and scalings of the wavelet. Then the coefficients corresponding to the unwanted terms are removed or modified.

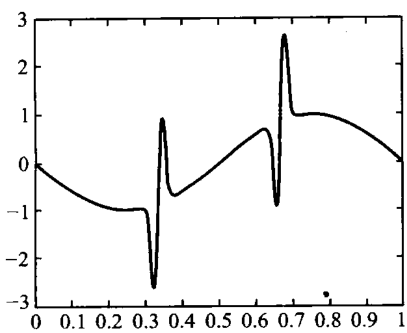


Fig. 3-6-3 Graph of a signal with isolated noise

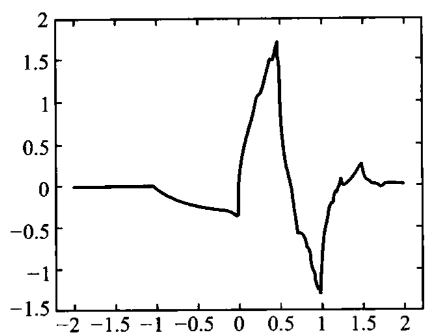


Fig. 3-6-4 Graph of Daubechies wavelet

In order to implement efficient algorithms for decomposing a signal into an expansion (either Fourier or wavelet based), the building blocks (sines, cosines or wavelets) should satisfy various properties. One convenient property is *orthogonality*, which for the sine function states

$$\frac{1}{\pi}\int_0^{2\pi}\sin(nt)\sin(mt)\,\mathrm{d}t=\begin{cases}0 & \text{if } n\neq m,\\ 1 & \text{if } n=m.\end{cases}$$

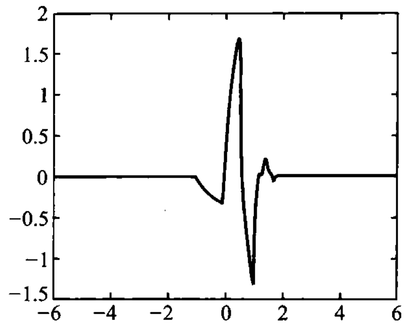

Fig. 3-6-5 High-freequecy Daubechies wavelet

The analogous properties hold for the cosine function as well. In addition, $\int_0^{2\pi}\sin(nt)\cos(mt)\,\mathrm{d}t=0$ for all $n$ and $m$. We shall see that these orthogonality properties result in simple formulas for the Fourier coefficients (the $a_n$ and $b_n$) and efficient algorithms (fast Fourier transform) for their computation.

One of the difficult tasks in the construction of a wavelet is to make sure that its translates and rescalings satisfy analogous orthogonality relationships, so that efficient algorithms for the computation of the wavelet coefficients of a given signal can be found. This is why we cannot construct a wavelet simply by truncating a sine or cosine wave by declaring it to be zero outside of one or more of its periods. Such a function, while satisfying the desired support feature of a wavelet, would not satisfy any reasonable orthogonality relationship with its translates and rescales and thus would not be as useful for signal analysis.

## 学习要求

1. 掌握如下内容：

（1）全文的主题和中心意思、各段的主要内容。

（2）20 个新出现的或常用的数学与应用数学的术语，5 ~ 8 个表示数学命题的句型。其中数学单词与术语包括：wavelet（小波），Fourier analysis（傅里叶分析），vibrate（振动），filter out（过滤掉），high-frequency wiggle（高频抽动），eliminate（排除），data compression（数据压缩），digital bits（数字位），tolerance error（容许误差），time-independent wavelike feature（时间独立的波状特性），algorithm（算法），expansion（展开式），scaling（换算、定标），rescaling（换算），translate（平移），truncate（截断，截尾）等。

2. 回答如下问题：

(1) 傅里叶分析的主要作用是什么，它有什么不足之处？

(2) 与傅里叶级数相比，小波有什么优势？

(3) 您认为在小波分析中通常只是使用小波或同时也考虑正弦波和余弦波？

(4) 构造小波函数的困难是什么？

## 3.6.2 The discovery of fractal geometry ①

The aim of this chapter is to give the reader a short summary of the fundamentals of fractal geometry. There are, of course, excellent works on this, some of which are listed in the bibliography, which will take the reader further with the analysis that is only sketched out here. All we attempt here is an elementary presentation of the relations that have to be known in order to follow, without difficulty and with a critical mind, the scientific literature dealing with *the concept of time in physics in a fractal environment*. We have preferred clarity to mathematical rigour. In this study the reader will find that the key element of fractal geometry, the non-integral dimension, is intimately associated with the way in which we evaluate our space, measure its boundaries and "weigh" its contents, the meaning of the concept of dimension will be seen to be, above all, physical. He will see also that the aim of a measure is to try to free the mind of the paradoxes that characterize "pathological" behavior, whilst using examples of such behavior to enrich our vision of the world. Finally, he will see that this freedom can be exercised at the mathematical level, perhaps more there than elsewhere — something that the engineer too often forgets.

**The surveyor's task: rectifiable curves, measuring by arc lengths**

For most human beings the length of a curve is a primitive notion, acquired in the first years of life. Doubtless Neanderthal man, although never having formalized the knowledge, knew that the "longer" a path the more time it would take him to follow it; and similarly he would know that it was easier to walk on the flat than up a mountain. Even so, he would not know the meaning of rectification, a term invented much later to express our ability to measure the lengths of smooth curves.

Consider this concept for a moment: let us call a certain path an arc $\Gamma$, and suppose that a person walks along this at a "speed" expressed by a certain function

① 本节课文摘自：A. L'mehaate. Fractal Geometries. London: CRC Presss Inc. 1977.

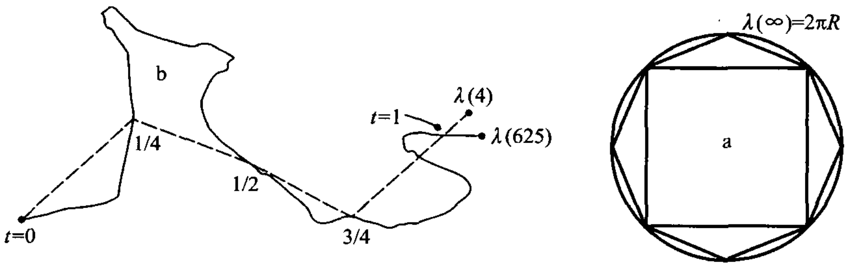


Fig. 3-6-6 Approximation to a curve by steps of length $\eta$, for $p=4$ and $p=625$. We know that the perimeter of the circle is the finite number $\lambda=2\pi R$, where $R$ is its radius; this is the value obtained when $\eta \to 0$.

$v(t)$. Each point on $\Gamma$ will be determined by a function $l(t)$ which gives the distance travelled from the starting point, where "$t$" is the "time" of arrival at that point. We say that the curve is *parametrised* by $l(t)$, which initially we assume to be defined and continuous everywhere. This will be the case if the walker does not jump erratically from place to place, and indeed the principle of the walk requires that he does not. The *step* is the human gauge for measuring lengths along the paths we take. But even without any jumping about the walker will not follow the curve in all its infinitesimal details: implicitly, he will approximate it by a polygonal line whose general form is given by steps of length $\eta$ placed on the ground in succession.

Let $\lambda(\mathscr{P})$ be the length of the plane polygonal line $[f(0), f(t_1), f(t_2), \cdots, f(t_p)]$, where $f(t_i)=[x(t_i), y(t_i)]$ and $\mathscr{P}$ is the finite sequence $[t_1, t_2, \cdots, t_p]$. We say that the arc is rectifiable if the upper bound of the real numbers $\lambda(\mathscr{P})$ when $p \to \infty$ is a finite number $L$, which is then called the length of the curve $\Gamma$.

So far no constraint has been placed on the elementary segments that constitute the polygonal line; suppose now that these are all of the same length $\eta(p)$ and that all the time intervals $t_i - t_{i-1}$ are equal and have the value $\Delta t$. Then the sequence $\mathscr{P}$ is equivalent to giving the number $p$ of measuring steps. If $t$ is the total time needed to make the measure, $p=t/\Delta t$, and if $p=t/\tau_0$ where $\tau_0$ is some constant time then $p$ can be regarded as a generalized frequency and used as a Laplace variable. We have now

$$\lambda(p) = N(p)\cdot\eta(p), \tag{1.1}$$

where $N(p)$ is the number of steps in the polygonal line and $\lambda(p)=\lambda(\mathscr{P})$. Rectifiability can be understood simply from a graph of $\lambda(p)$ as a function of $\eta(p)$: it means that $\lambda(p)$ must tend to a finite limit, $L$, as $\eta(p)$ tends to zero. The necessary analysis can be performed in Laplace or Fourier space.

$\lambda(p) \to L$ when $p \to \infty$ , that is when $\Delta t \to 0$ and $t$ is finite.

Stated otherwise, the space gauge tends to zero when the time gauge tends to zero, or equivalently when the frequency gauge tends to infinity: an infinitely precise measure requires an infinite frequency. Note that if $p$ is the generalised frequency defined above, the number of steps per unit time is proportional to $p$ and also $N(p) \sim p$, where by the symbol " ~ " is meant "behaves like".

The concept of rectifiability is simple, intuitive and almost natural, but does it hold universally? Ten years ago almost everyone except for a few mathematical specialists would have said yes. However, as Mandelbrot recalled in 1975, the true answer is no, a study of Figure 3-6-7 should put us on our guard. Approximating a rectifiable curve assumes some very special properties.

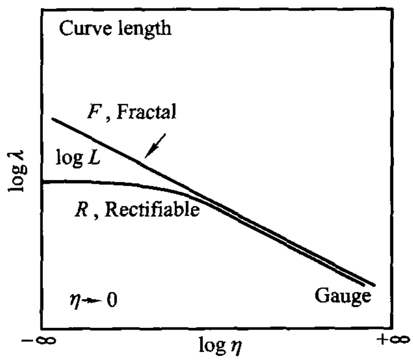


Fig. 3-6-7 Characteristics of rectifiable ($R$) and fractal ($F$) curves. For $R$ the length $\lambda(\eta)$ measured in steps of length $\eta$ approaches a finite limit $L$ as $\eta \to 0$, for $F$ there is no limit. The concept of non-integral dimension is related to the slope of a fractal curve.

There are curves with strange properties, such that the length does not tend to a finite limit as the step length $\eta(p)$ tends to zero. How can we classify these, what properties are hidden in them? The essential aim of this chapter is to show that the concept of "measure", in the physical sense of the term, enables us to bring the properties of these strange objects under our control. But we shall now leave further discussion of the parameter $p$ to the next chapter, and concentrate on the analysis of the space.

During the last century mathematicians established the formal existence of certain pathological curves: for example, Cantor, Peano, Hausdorff, Bouligand among others. But it was the expatriate French geometer Mandelbrot who first understood the depth of this work and made it more widely known, in particular characterizing the strange objects by the term *fractal*. This was not easily achieved, his first book, published in 1975, was effectively ignored and he had to wait until 1983 for recognition of the importance of his contribution. Eight years after the birth of his ideas, and thanks to a generation of young physicists led by Mandelbrot himself, the importance of these pathological objects has at last been acknowledged by the great majority of the scientific community.

**注释与说明**

1. IBM 公司研究员兼哈佛大学数学教授 Mandelbrot, B. B. 于 1975 年创建了分形理论。

2. Cantor, G. (1845—1918)德国数学家; Peano, G. (1858—1932)意大利数学家; Hausdorff, F. (1868—1942)德国数学家; Bouligand, G(1914— )法国数学家。

**学习要求**

1. 掌握如下内容:

(1) 全文的主题和中心意思、各段的主要内容。

(2) 10 个新出现的或常用的数学术语, 5 ~ 8 个表示数学命题的句型。其中数学术语包括: fractal geometry (分形几何), pathological (病态的), rectifiable curves (可求长的曲线), rectifiability (可求长性), approximation (逼近), non-integral dimension (非整数维), parameter (参数), gauge (度规,规范)等。

2. 回答如下问题:

(1) 本文作者的写作目的是什么?

(2) 用步行来测量一条弯曲的路径要遵守什么规则? 什么样曲线叫做可求长曲线?

(3) 如何理解曲线的非整数维的概念? 这个概念在分形几何起什么作用?

(4) 分形几何是如何发现的? 1975 年分形几何刚创立时, 人们对它的态度如何? 后来的情形又如何?

## 3.6.3 *The concept of the fuzzy set* ①

Fuzzy sets and fuzzy logic have become one of the emerging areas in contemporary technologies of information processing. Recent studies spread across various areas, from control, pattern recognition, and knowledge-based systems to computer vision and artificial life. A significant number of direct real-world implementations range from home appliances to industrial installations and involve fuzzy sets, both by them-

① 本节课文摘自: A. Kandel. Fuzzy Mathematical Techniques with Applications. California: Addison-Wesley Pbl. 1986.

selves and hand in hand with other modern approaches, including neural networks[1].

Essentially, fuzziness is a type of imprecision that stems from a grouping of elements into classes that do not have sharply defined boundaries. Such classes — called *fuzzy sets* — arise, for example, whenever we describe ambiguity, vagueness and ambivalence in mathematical models of empirical phenomena. Since certain aspects of reality always escape such models, the strictly binary (and even the ternary) approach to the treatment of physical phenomena is not always adequate to describe systems in the real world; and the attributes of the system variables often emerge from an elusive fuzziness, a readjustment to context, or an effect of human imprecision. In many cases, however, even if the model is precise, fuzziness may be a concomitant of complexity. Systems of high cardinality are rampant in real life and their computer simulations require some kind of mathematical formulation to deal with the imprecise descriptions.

The theory of fuzzy sets has as one of its aims the development of a methodology for the formulation and solution of problems that are too complex or too ill-defined to be susceptible of analysis by conventional techniques.

The theory of fuzzy sets deals with a subset $A$ of the universe of discourse $X$, where the transition between *full membership* and *no membership* is gradual rather than abrupt. The "fuzzy subset" has no well-defined boundaries where the universe of discourse (the universe $X$) covers a definite range of objects. Fuzzy classes of objects are often encountered in the real world. For instance, $A$ may be the set of beautiful women in a town $X$, or $A$ may be the set of long streets in town $X$. Traditionally, the grade of membership 1 is assigned to those objects that fully and completely belong to $A$, while 0 is assigned to objects that do not belong to $A$ at all. The more an object $x$ belongs to $A$, the closer to 1 is grade of membership $\chi_A(x)$.

In abstract (or conventional, or nonfuzzy) set theory, the sets considered are defined as collections of objects having some very general property $P$; nothing special is assumed or considered about the nature of the individual objects. For example, we define a set $X$ as of streets. Symbolically,

$$X = \{x \mid x \text{ is a street}\}.$$

Now what about the "class of long streets"? First all, is it a set in the ordinary sense? Before we answer that, we may first ask: "How long is a long street? Is a one-mile street a long street? If so, then is there any difference between a half-mile street and a one-mile street, etc?" Frankly, we do not know how to answer these questions adequately from the information "long street," because the "class of long streets" does not constitute a set in the usual sense. In fact, most of the classes of objects encountered in the real physical world are of this fuzzy, not sharply defined type. They do not have precisely defined criteria of membership. In such classes, it is not necessary for an object to belong or not belong to a class; there may be intermediate grades of membership. This is thee concept of fuzzy set, which is a "class" with a continuum of grades of membership.

Fuzzy-set theory, introduced by Zadeh[2] in 1965, is a generalization of abstract set theory. In other words, the former always includes the latter as a special case; definitions, theorems, and proofs of fuzzy-set theory always hold for nonfuzzy sets. Because of this generalization, fuzzy-set theory has a wider scope of applicability than abstract set theory in solving problems that involve, to some degree, subjective evaluation.

Intuitively, a fuzzy set is a class that admits the possibility of *partial membership* in it. Let $X = \{x\}$ denote a space of objects. Then a fuzzy set $A$ in $X$ is a set of ordered pairs

$$A=\{x,\ \chi_A(x)\},x\in X,$$

where $\chi_A(x)$ is termed "the grade of membership of $x$ in $A$." We shall assume for simplicity that $\chi_A(x)$ is a number in the interval $[0, 1]$, with the grades 1 and 0 representing, respectively, *full membership* and *nonmembership* in a fuzzy set, as discussed before. We have assumed that an exact comparison is possible for the truths of any two inexact statements "$x\in A$" and "$y\in A$," and that the exact relation so obtained satisfies the minimal consistency requirement of *transitivity* and *reflexivity*; the ordering $x \geqslant y$ means "$x$ is at least as true as $y$" with $x \leqslant y$ denoting "$x$ is not truer than $y$."

The grades of membership reflect an "ordering" of the objects in the universe; it is interesting to note that the grade-of-membership value $\chi_A(x)$ of an object $x$ in $A$ can be interpreted as the degree of compatibility of the predicate associated with $A$ and the object $x$. As will be seen later, it is also possible to interpret $\chi_A(x)$ as the degree of possibility that $x$ is the value of a parameter fuzzy restricted by $A$.

We now give an example of fuzzy sets.

**Example.** Consider the class of all real numbers that are much greater than 1. We can define this set as

$$A=\{x\mid x \text{ is a real number and } x>>1\}.$$

But it is not well-defined set for the reasons mentioned before. This set may be defined subjectively by a membership function such as

$$\chi_A(x)=0 \text{ for } x\leqslant 1;\ \chi_A(x)=\frac{x-1}{x} \text{ for } x>1.$$

The assignment of the membership function of a fuzzy set is subjective in nature and, in general, reflects the context in which the problem is viewed. Although the assignment of the membership function of a fuzzy set $A$ is "subjective," it cannot be assigned arbitrarily. For example, it would be totally wrong to assign the membership function of above example as

$$\chi_A(x)=\begin{cases}\dfrac{x-1}{x}, & x\leqslant 1,\\ 0, & x>1.\end{cases}$$

## 注释与说明

1. A significant number of direct real-world implementations range from home appliances to industrial installations and involve fuzzy sets, both by themselves and hand in hand with other modern approaches, including neural networks. 本句为简单句，range 和 involve 是并列谓语，现在分词短语 including neural networks 是 approaches 的定语（neural network，即神经网络，是计算机人工智能的一个重要组成部分）。全句可意译成：从家用器具到工业设施的各个领域，人们可实际接触到的现实世界中的大量用具，不仅其本身，而且还通过其他许多现代化途径（包括神经网络）与模糊集相关联。

2. 美国加利福尼亚大学 L. A. Zadeh 于 1965 年发表了著名的论文"模糊集"，被公认为模糊数学理论的创始人。

## 学习要求

1. 掌握如下内容：

(1) 全文的主题和中心意思、各段的主要内容。

(2) 20 个新出现的或常用的数学术语，10 ~ 12 个表示数学命题的句型。其中数学术语包括：fuzzy（模糊的，不分明的；模糊度），fuzziness（模糊性），pattern recognition（类型识别），knowledge-based system（基于知识的系统），neural network（神经网络），multiple-valued logic（多值逻辑），artificial intelligence（AI）（人工智能，简称 AI），ambiguity（模棱两可），vagueness（含糊），ambivalence（不明朗），imprecision（不明确性），ill-defined（不确定的），empirical phenomena（以实验为主的现象），binary（二元的），ternary（三元的），readjustment（重新适应，重新整理），simulation（模拟实验），rampant（不能控制的），compatibility（相容性），the *minimal consistency requirement*（最小相容性要求）等。

2. 回答如下问题：

(1) 什么是模糊集？怎样理解隶属度（the grade of memberships）？

（2）模糊集与通常意义下的集有什么不同与联系？为什么说模糊集是通常意义下的集的概念的推广？

（3）在什么情况下会产生模糊集？

（4）模糊集理论和模糊逻辑有什么用途？

（5）最后的例子说明什么问题？

**插页：数学珍言——数学界流传着：**

The mathematical method is the essence of mathematics. He who fully comprehends the method is a mathematician.

Music has much resemblance to algebra.

Pure mathematics is the real magician's wand.

The real mathematician is an enthusiast person; without enthusiasm, no mathematics.

One may be a mathematician of the first rank without being able to compute, it is possible to be a great computer without having the slightest idea of mathematics.

What logarithms are to mathematics, that mathematics are to the other sciences.

**上面引述的六段话，译为汉语是：**

数学方法是数学的本质。数学家是能完全领悟数学方法的人。

音乐与代数很类似。

纯数学是真正的魔术师的魔杖。

真正的数学家，其本质是一个热情洋溢的人，没有热情就没有数学。

一个不擅于计算的人，有可能成为一个第一流的数学家，而一个没有丝毫数学观念的人，充其量只能成为一个大计算家。

对数之于数学，恰如数学之于其他科学。

# 第四章　英语数学论文写作基础

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本章主要介绍用英语写作的数学论文在语言表达与书写格式上的基本要求和应该注意的事项,包括数学论文的组成部分及书写要求、数学论文中的语法与习惯用法问题、数学论文的精练要求、标点与数学符号的正确使用等。应该申明的是,本章所介绍的仅仅是一些基本的要点,给初学者一个大体的导引(有关数学英语的特点可参阅第一章)。至于进一步的提高,还需要读者多学、多读、多练,不断总结和自我完善。

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## §4.1　英语数学论文的组成部分及书写要求

一篇研究性的数学论文主要包含如下六个部分:标题、作者信息、摘要、关键词、正文和参考文献。必要时还可在正文之后加上答谢语句。其中,正文部分可包括引言、论证部分等。关键词之后有时需要添上文章主题分类号。目前国际上一般采用2000年版本的《数学主题分类》(美国数学会主编,见§5.3)。据此,若文章类别为31A05和30D25(这就是分类号),可写成:

"2000 Mathematics Subject Classification 31A05,30D25"(参见附录1),

或写成:　　"MR(2000) MSC: 31A05,30D25"。

现根据各部分内容在论文中出现的先后次序,分别作简单介绍。由于用中文写的论文也经常要求附英文摘要,因此根据毕业生写作的要求,我们特别把摘要的介绍作为一个重点,列举了多个例子,并加上评论和译文供参考。拟在国内发表的论文应该按照我国国家标准局公布的格式和新闻出版署印发的规范来书写,详细规定见书末列出的参考文献[13]和[15]。

不同的数学杂志对格式的要求大同小异,本节介绍的内容是基本的,但实际应用时还得根据准备投稿的杂志的要求做少量适当的修改。

### 4.1.1　标题(title)

1. 一篇论文应有一个既简短明了又能概括全篇中心内容的引人注目的标题。值得注意的是,一个令人满意的命题应该注意做到:(1) 不过于概括以至流

于空泛、一般化;(2) 不过于繁烦,缺乏鲜明的特点或让人难以记忆和引证。在标题写作上,美国数学会曾要求一条标题不要超过 12 个单词,并要求用词质朴、明确、实事求是,避免用广告式的哗众取宠的字眼。这种要求得到多数人的认可,值得参考。

例如,有一篇论文的标题为:

Concerning Some Applications of a Theorem of H. Gauss(关于 H. 高斯定理的一些应用)。

这样的标题就太笼统了,事实上,一个著名的数学家可能创造很多个定理,有很多以他的名字命名的定理。标题写成这样,读者就不知是指哪一个定理,也不知是在什么方面的应用。反之,若改为下面的标题,则比原来的明确多了:

Some Applications of H. Gauss Integration Formula in Fluid Mechanics(高斯积分公式在流体力学中的一些应用).

2. 在英语文字表达上应注意,论文标题一般是一个名词性短语(名词化结构),必要时可在开头加一个介词 On,表示"论……"或"关于……的探讨"。标题通常不写成一个句子或一个不定式短语①,也不出现从句。因此标题常用名词或动名词代替动词。

例如,中文题目为"用积分法证明一个不等式",标题不应写成:

"We prove an inequality by the integration";

或 "Prove an inequality by the integration";

或 "Proving an inequality by the integration";

或 "Use the integration to prove an inequality";

或 "An inequality is proved by the integration".

可写成: "A proof of an inequality by integration".

允许在标题的最前面加上介词 On 的例子如:

On Applications of H. Gauss Integration Formula in Fluid Mechanics

3. 在行文格式上,按惯例,英语标题第一个词和每个实词的第一个字母都要大写,而小词(介词、冠词、连接词)不用大写。不过,有的杂志也允许标题每一个字母全部大写或第一个单词的首字母大写而其余的字母都小写。同时,为了简短,标题开头的冠词可以省略,其他处的冠词也能省则省。

4. 题目的末尾一般不加句点。

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① 偶然可见到以疑问句或疑问代词加不定式的标题,这可能是一篇以提出问题为中心的论文或像 G. Polya 的 How to solve it 之类用以研究数学教育方法的专著。

## 4.1.2 作者姓名及有关信息(about author(s))

题目之后必须列出作者的姓名。文章署名既表示著作权,也表示作者文责自负。作者姓名规定放在标题的下一行中间,末尾一般不加句点;有多个作者时,通常按次序先后排在同一行,有时也分行排列。文章中通常还要附上作者的工作单位、所在城市、邮政编码、国家,必要时还要求写出 Email 地址,以便交流。它们可放在作者姓名的下一行或出版社规定的其他地方。另外,论文投稿时,有的杂志还要求作者提供其他信息,例如作者的职称,是否国家、省、市科研基金资助的项目等,这些通常不与姓名放在一块,而是作为脚注放在文章首页的页脚。

国内杂志要求[15]作者姓名按拼音字母写出,其中表示姓的所有字母都大写,名不止一个字的,要用连字符把字与字隔开;如作者不止一个,则每两个作者的姓名之间要用逗号隔开;工作单位(可缩写),所在城市,邮政编码和国家依次放在另一行的括号中。例如:

ZHANG Shao-ming

(Dept. of Math., Xiamen Univ., Xiamen 361005, China)

国外有的杂志要求依西方国家的习惯,通常把名写在前姓写在后,如 Shao-ming ZHANG;如果姓写在前,则作者的姓之后要加逗号,写成 ZHANG, Shao-Ming 或 ZHANG, Shao Ming 等形式,或将其中 ZHANG 改成 Zhang。

## 4.1.3 摘要(abstract)

论文摘要是关于论文内容的一段简短而精练的表达。

通常数学论文都附有摘要,许多学术刊物要求用中文发表的论文也要附上英文题目、英文摘要和关键词。

论文摘要应简短扼要,中心突出。因为摘要有时要单独使用(如申请参加学术会议或在一些文摘杂志上刊登),因此,要能使那些对所论问题有所了解的读者,读了摘要就能对正文(尽管他们可能未读到正文)的内容或研究进展有个基本的了解和明确的概念。

研究性论文的摘要应概括本文的主要成果和所采用的主要方法,用简短的文字给出作者创新内容的尽可能多的信息;必要时也可加一两句话概述这项研究的背景、重要性及成果的意义。这种摘要称为**报道性摘要**。另一种摘要只点明论文的论题或体现写作目的,称为**提示性摘要**。一些简短摘要可能同时具有上述两类的某些特点。

摘要书写的基本要求:

(1) 文句精练:一篇五页(4000 单词)的论文的摘要通常不超过 120 个单词。摘要中只写论点,不列举例证。

(2) 论点具体鲜明：不笼统地讲“与什么有关”，而直接讲论文“说明了什么”。

(3) 词句表达规范：要使用正规英语和标准术语，避免使用缩写字和非规范用语；通常使用一般现在时，最好使用第三人称来叙述（不过现在也有人使用“we study”等第一人称语句）；提倡用被动语态，也可用主动语态，但在同一小段或仅有两三句的小型摘要中，各个主句的语态一般应尽可能统一（参见下面简短的英文摘要举例）。

(4) 相对完整：摘要本身要相对完整，不需要补充、解释或评论，不在其中引用别的文献或本文中的某一段或某张插图来代替说明。

例文：

STRUCTURAL CHANGE OF SOLUTIONS FOR A SCALAR CURVATURE EQUATION

**Abstract** A semilinear elliptic equation

$$\Delta u+[1+\varepsilon k(|x|)]u^p=0,\ x\in\mathbf{R}^n,$$

is studied, where $n>2$ and $\varepsilon$ is a small parameter. It is known that for $p=(n+2)/(n-2)$ fixed, the structure of radial solutions drastically changes under the perturbation $\varepsilon k(|x|)$. In this paper it is shown that such a change can be understood in a natural way if the exponent $p$ also is taken as a parameter. It is shown that the Pohozaev identity plays an important role in the perturbation analysis.

(**参考译文**：一类纯量曲率方程的解的结构之改变

**摘要** 本文研究半线性椭圆方程

$$\Delta u+[1+\varepsilon k(|x|)]u^p=0,\ x\in\mathbf{R}^n,$$

其中 $n>2$，$\varepsilon$ 是一个小常数。熟知，当 $p=(n+2)/(n-2)$ 固定时，径向解的结构在摄动 $\varepsilon k(|x|)$ 下发生激烈的改变。本文证明，当指数 $p$ 也当成参数时，这种结构的改变可用一种自然的方式来理解。文中表明，Pohozaev 恒等式在摄动分析中起了重要作用。)

**注意** “**Abstract**”之后不加冒号。

**论文标题与简短的英文摘要举例**

论文摘要不宜长，只要把重要的结论或结论的范围和必需交代的主要方法表达清楚即可。下面六个例子短小精悍，无论摘要选用的内容或表达方式都是值得借鉴的。

1. 采用被动语态语句来书写摘要是比较合乎规范的表达方式，也较常见。

**例 1**

On the Uniqueness of Inverse Problems of the Potential Theory

**Abstract** Inverse problems of potential theory are considered for two and three dimensional domains. The uniqueness theorems are proved under specific restrictions on the boundaries of the domains and the densities of potentials.

**短评**：这篇摘要采用被动语态语句。第一句说明在什么区域上研究什么，第二句说明本文在某种特殊条件下得出的结论。

**参考译文**：(题目)关于位势论逆问题的唯一性

**摘要** 本文考虑 2 维和 3 维区域上的位势论逆问题，并在对区域边界和位势密度作特殊限制的条件下证明了唯一性定理。

**例 2**

A Critical Point Theory for Nonsmooth Functionals

**Abstract** *A new generalized notion of $\| df(u) \|$ is introduced, which allows* to prove several results of critical theory for continuous functional. An application to *variational inequalities is shown.*

**短评**：这篇摘要采用被动语态语句。第一句描述方法与结果，第二句指出结论的一个应用。

**参考译文**：(题目)关于非光滑泛函的临界点理论

**摘要** 本文引入关于 $\| df(u) \|$ 的一个新推广的概念，并用以证明关于连续泛函的临界点问题的若干结果，同时还指出了它们在变分不等式中的一个应用。

**例 3**

Lagrange Interpolation on Conics and Cubics

**Abstract** A bivariate polynomial interpolation problem for points lying on an algebraic curves is introduced. The geometric characterization introduced by Chung and Yao, which provides simple Lagrange formulae, is here analyzed for interpolation points lying on a line, a conics, or a cubic.

**短评**：这篇摘要采用被动语态语句。第一句描述论文内容，第二句指出采用的方法。

**参考译文**：(题目)圆锥曲线与三次曲线的拉格朗日插值

**摘要** 本文介绍一种插点位于代数曲线上的二元多项式插值问题。由 Chung 和 Yao 引进的几何特征提供了简单的拉格朗日公式，在这里被用来分析那些位于直线、圆锥曲线与三次曲线上的插点。

**例 4**

Integrable Harmonic Functions on $\mathbf{R}^n$

**Abstract** A class of radial measure $\mu$ on $\mathbf{R}^n$ is defined so that integrable harmonic functions $f$ on $\mathbf{R}^n$ can be characterized as solutions of convolution equations

$f * \mu = f$. In particular, it is proved that $f * e^{-2\pi|x|} = f$, $f \in L^1(e^{-2\pi|x|})$ is harmonic if and only if $n<9$.

**短评**：这篇摘要两句都是被动语态语句，第二句用了形式主语，it is proved that …也是常用句型。

**参考译文**：（题目）$\mathbf{R}^n$上的可积调和函数

**摘要**　本文定义了$\mathbf{R}^n$上的一类径向测度$\mu$，使得$\mathbf{R}^n$上的可积调和函数$f$可特征化为卷积方程$f * \mu = f$的解；特别，证明了满足$f * e^{-2\pi|x|} = f$的函数$f \in L^1(e^{-2\pi|x|})$调和当且仅当$n<9$。

2. 采用第三人称主动语态语句来书写摘要也是合乎要求的。这里第三人称主语常是 this paper 或 the author(s)。

**例 5**

On Multiple Positive Solutions for a Non-Linear Elliptic Problem in $\mathbf{R}^N$

**Abstract**　By making use of variational method, the authors obtain some results about existence of multiple positive solutions and their asymptotic behavior as the parameter $\lambda \to \infty$ for a semilinear elliptic problem in $\mathbf{R}^N$.

**短评**：这篇摘要采用第三人称主动语态语句，说明利用那种研究方法获得什么方面的结果。

**参考译文**：（题目）关于$\mathbf{R}^N$上非线性椭圆问题的多重正解

**摘要**　考虑$\mathbf{R}^N$上一个半线性椭圆问题，本文利用变分方法得到了多重正解的存在性以及解的渐近性质（当参数$\lambda \to \infty$时）的一些结果。

3. 采用第一人称复数（we）的主动语态语句来书写摘要也是允许的。

**例 6**

A Sharp $L^q$-Liouville Theorem for $p$-harmonic Functions

**Abstract**　We study $L^p$-Liouville properties of nonnegative $p$-superharmonic and, respectively, $p$-subharmonic functions on a complete Riemannian manifold $M$. In particular, we prove that every $p$-harmonic function $u \in L^p(M)$ is constant if $q > p-1$。

**短评**：本文采用主动语态语句表述。第一句说明研究的对象，第二句强调其中特别突出的结果。

**参考译文**：（题目）关于$p$-调和函数的一个加强的$L^q$-Liouville 定理

**摘要**　我们在完备的 Riemann 流形$M$上分别研究了非负$p$-上调和函数与$p$-下调和函数的$L^p$-Liouville 性质；特别地，证明了当$q>p-1$时，每一个$p$-调和函

数 $u \in L^p(M)$ 都是常数。

顺便指出，有一部分动词在摘要中出现的频率比较高，例如：discuss，consider，deal with，study，investigate，present，propose，give，develop 等。

### 4.1.4 关键词（key words）

关键词是指从论文的正文、摘要中抽出的，在表达论文的内容、主题等方面具有实际意义并起关键作用的词汇。关键词应尽可能采用规范化的数学词汇，大多是名词性术语，部分是具有检索意义的动词和形容词等。一般每篇论文选用 3～8 个关键词，列于摘要之后。

例如，上面例 6 的关键词可选"Liouville theorem；*p*-harmonic function；Riemann manifold"，并把它们放在摘要之后，另起一行，行首写上"Key words"，即

**Key words** Liouville theorem；*p*-harmonic function；Riemann manifold

**附注** 用中文写作数学论文，在参考文献之后附上英文题目、摘要和关键词时，除参考上述要求外，一般还要求中、英文一致。因此在设计中文的题目、摘要和关键词时还要考虑到翻译上的方便，要注意中、英文表达习惯上的差异和词句表达的准确性。

书写时，"**Key words**"之后不加冒号，每两个关键词之间用分号隔开，最后一个关键词之后不加标点。

考虑到师范类毕业生毕业论文的特点，以下选录了三篇关于数学教育、教学改革的论文的中英文摘要与关键词（原文用中文发表，同时附英文摘要与关键词，这里引用时作了局部修改），供读者参考。

**例 1**

高师数学专业课改革的探索与实践

**摘要** 从分析高师数学专业课程的现状和影响数学专业课程改革的因素入手，对高师数学课程改革进行了探索，提出了一些具体做法。

**关键词** 数学专业课；教育改革；现代技术；创新意识；应用能力

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Exploration and Practice on the Reform in Mathematics Course in Normal Universities

**Abstract** Based on the present state of mathematics course in normal college and university and the factors of the reform of mathematics courses, this paper discusses reformatory plan of mathematics courses in normal college and university and puts forward some reformatory practice.

**Key words** mathematics courses; education reform; modern technique; creating consciousness; practical ability

**例 2**

关于数学教育核心问题的思索

**摘要** 本文分析了当前一些"数学教育核心说"所存在的问题，提出了新的数学教育核心——提高受教育者的数学素质。

**关键词** 数学教育；核心；数学素质

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Thinking on the Kernel Problem of Mathematics Education

**Abstract** Based on the analysis of current kernel problem of mathematics education, this paper puts forward a new kernel of mathematics education: raising students' mathematics diathesis.

**Key words** mathematics education; kernel; mathematics diathesis

**例 3**

基于学生数学能力结构的分析提出的培养建议

**摘要** 公民数学素养的提高无论对于社会发展还是其个人发展都具有重要意义。为了提高中小学数学教育质量，必须明确中小学生数学能力结构。中小学生的数学能力可以分为两个层次，运算能力、空间想象能力、信息处理能力是第一个层次，逻辑思维能力和问题解决能力是第二个层次；模式能力在这两个层次之间起着非常重要的桥梁作用。据此，本文提出了中小学生数学能力培养的几点建议。

**关键词** 中小学生；数学能力结构；模式能力

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Some Suggestions for Fostering of the Mathematical Ability of Students Based on an Analysis of the Ability's Structure

**Abstract** The improvement of citizens' mathematical quality has significant meanings in both social and individual development. To identify the structure of students' mathematical ability is necessary for improving the quality of mathematical education in elementary and secondary schools. Mathematical ability consists of two levels. The first one includes the abilities of operation, spatial visualization and information processing; and the second one includes the abilities of logical thinking and problem solving. The patterning ability plays an important role in bridging the two levels, Based on the analysis, the paper sets forward some suggestions for the fostering of the mathematical ability of these students.

**Key words** students in elementary and secondary schools; structure of mathematical ability; patterning ability

## 4.1.5 引言(introduction)

引言就是论文的开场白,是正文的开头部分。它向读者解释论文的主题、目的和总纲。一篇完整的引言常包括以下几个内容:

1. 研究的理由、目的和背景;
2. 理论依据、实验基础和研究方法;
3. 预期的结果及其作用、意义。

此外,必要时引言还可以介绍本文采用的记号、约定,提示文中各组成部分的内容要点等。

总之,引言向读者交代本项研究的来龙去脉,对论文的总体轮廓作出概述。

引言写作应达到以下要求:

(1) 言简意赅,突出重点;

(2) 开门见山,紧扣主题;

(3) 实事求是,不用客套(在引言中书写过于谦虚被认为是写作上的错误)。

还应注意引言不应与摘要类同,或成为摘要的简单注释。

数学论文的引言部分通常要冠上"引言"(Introduction)小标题;有时可以在这部分兼介绍预备知识,这时可加上小标题"引言与预备知识"(Introduction and preliminaries)。不过,现在也有人提倡,引言部分不用"Introduction"作小标题,即该段无小标题,或采用其他能反映该段中心意思的小标题。

引言在阐述研究的理由、目的和背景时经常要引经据典,正文其他地方在介绍前人提出的概念和结果时也要说明根据。在这些地方要使用"标引"标明出处。标引由方括号和其中的文献序号组成,例如,[6]表示排列在论文末尾 Refferences 处的第六篇文献。标引可出现在被引用文献的作者名字或被引用内容的右上方(以上标的方式出现)或作为句子的某个成分(如…in[6]…,不作上标)。标引方式参见下面例子。

**例** 下面是一篇题为"The boundary Harnack principle for the fractional Laplacian"(分数型拉普拉斯算子的边界哈纳克原理)的引言,作者 K. Bogdan,发表于 Studia Mathematica, **123**(1)(1997)P41 ~ 80.

**The boundary Harnack principle for the fractional Laplacian**

KRZYSZTOF BOGDAN(Wroclaw)

**Abstract** We study nonnegative functions which are harmonic on a Lipschitz domain with respect to symmetric stable processes. We prove that if two such functions vanish continuously outside the domain near a part of its boundary, then their ratio is bounded near this part of the boundary.

**1. Introduction** The boundary Harnack principle (BHP) for nonnegative harmonic functions has inportant applications in probability theory and potential theory. Among these are approximations to excursion laws for the Brownian motion[6], "3G Theorem" and "Conditional Gauge Theorem"[8]. BHP was first proved in [9] for Lipschitz domains by analytic methods (see also [12],[11]). Later, the classical link between harmonic functions and the Brownian motion in $\mathbf{R}^n$ was used to give a probabilistic proof of BHP[2]. The result and generalizations of BHP to elliptic operators and Schrödinger operators have yielded stimulating interplay between probability theory, harmonic analysis and potential theory (see [7],[3],[8],[16],[1]).

The Brownian motion is a particular (and limiting) instance of the standard rotation invariant $\alpha$-stable, process, $\alpha \in (0,2]$. The infinitesimal generator $\Delta^{\alpha/2}$ of the latter and the related class of $\alpha$-harmonic functions have simple homogeneity properties analogous to those of the classical Laplacian and harmonic functions ($\alpha=2$) in $\mathbf{R}^n$. Also, the potential theory of $\Delta^{\alpha/2}(n>\alpha)$ enjoys an explicit formulation in terms of M. Riesz kernels, and is similar to that of the Laplacian in $\mathbf{R}^n$, $n>2$([13]).

The main result of this paper is the following theorem which gives another extension of the classical theory ($\alpha=2$) to the case $\alpha \in (0,2)$.

THEOREM 1. Let $\alpha \in (0,2)$ and $n>2$. Let $D$ be a Lipschitz domain in $\mathbf{R}^n$ and $V$ be an open set. For every compact set $K \subset V$, there exists a positive constant $C = C(\alpha, D, V, K)$ such that for all nonnegative functions $u$ and $v$ in $\mathbf{R}^n$ which are continuous in $V$, $\alpha$-harmonic in $D \cap V$, vanish on $D^c \cap V$, and satisfy $u(x_0)=v(x_0)>0$ for some $x_0 \in D \cap K$, we have

$$C^{-1}u(x) \leqslant v(x) \leqslant Cu(x), \quad x \in D \cap K. \tag{1.1}$$

Moreover, there exists a constant $\eta = \eta(\alpha, D, V, K) > 0$ such that the function $u(x)/v(x)$ is Holder continuous of order $\eta$ in $K \cap D$. In particular, for every $Q \in \partial D \cap V$, $\lim u(x)/v(x)$ exists as $D \ni x \to Q$.

Generally, we follow the approach designed by W. Hansen[7] for elliptic operators (see also [11]). In particular, Lemmas 1,3,4,10,13 and 16 below have their analogues in [12],[11] and [7], with major changes in the proofs. The main obstacle to our development is the non-locality of the integral-differential operator $\Delta^{\alpha/2}$, resulting in non-locality of the definition of $\alpha$-harmonic function and even of the notion of nonnegativity for such functions. This makes many of the arguments essentially different compared with the case of elliptic operators. In reward we are confronted with new concepts shedding new light on the classical theory. To prove results on the class of $\alpha$-harmonic functions, we rely on basic properties of the corresponding $\alpha$-stable process. While a purely analytic approach is possible ([13] provides an analytic introduction to $\alpha$-harmonic functions), the probabilistic methods are very often more natural and convenient.

这是一篇写得相当完整、全面的引言。共分为四段。第一段指出边界哈纳克原理的重要作用，引用了许多文献，从历史的发展和研究的多个侧面来阐述。第二段指出 $\alpha$-稳定过程以及无穷小算子 $\Delta^{\alpha/2}$ 的基本性质类似于经典拉普拉斯算子。第三段给出本文的主要结果，即定理 1，并指出它是经典理论相关结果的推广。第四段，说明获得本成果遇到的主要困难是算子 $\Delta^{\alpha/2}$ 及 $\alpha$-调和函数定义的非局部性，同时指出，之所以能克服这一困难是由于利用了 $\alpha$-稳定过程的基本性质，即指出了本文所采用的基本方法是概率的方法。

**注** 在第三章中，我们提供了多篇来自专著或教材的前言，本章末又附有一篇参考论文。有兴趣的读者可做进一步的阅读和比较。

### *4.1.6* 论证(*proof*)

论证部分是一篇论文的主体，在论文中占主要篇幅。它可以根据需要(比如逻辑顺序)分为若干小部分，每小部分叙述或论证一个或者若干个结论。例如，有的论文的正文可分成以下四个部分：

1. 引言；
2. 预备知识与主要结果；
3. 引理及其证明；
4. 主要结果的证明。

但若在引言中已指出全篇最主要结果，则在第 2 部分就不再罗列主要结果，而是在第 4 部分边罗列主要结果边加以证明。如果主要结果较多且大体可分成两大类或若干类，也可以多分几部分，在每个部分分别论证，每个部分前各加一个小标题。一些应用性论文或关于计算方法研究的论文还加上应用或计算举例、结论等部分。

论证部分写作的总体要求是：

(1) 论点明确，论据充分，论证合理，符合逻辑；

(2) 事实确凿，数据准确，计算无误；

(3) 文字简练，表达确切，条理清楚，层次分明；

(4) 图、表、数量单位、标点字母大小写等使用规范；

(5) 参考文献的引用应是充分且必要的。

### 4.1.7 志谢(acknowledgement)

在论文结束之后,有时可据需要写上一个致谢的语句,感谢对本项研究作出指导或重要帮助的师友,言辞应恳切、实事求是、恰如其分、简明扼要。语句常用第三人称表述。

如:

**Acknowledgement** The author is indebted to Dr. Hansen for suggesting the use of Abel ergodic theorem to simplify the proof.

(本文作者对汉森博士建议使用阿贝尔遍历定理来简化证明一事表示感谢。)

### 4.1.8 参考文献(references)

参考文献是论文的必要组成部分之一。科技论文列举参考文献是传统惯例,它反映作者严肃科学的态度和科研工作的广泛根据,以及对他人工作成果的尊重。正文中引用文献时都应加上标引(见4.1.5),并在全文最后的参考文献(references)中将这些文献依序列出。文献的列出应该符合出版社规定的规格,准确地写出文献的作者、文献名称、文献所载的期刊名(或书名或文集名等)、杂志期卷、年份、页码(或书的出版地点、出版社、年份)。一般的书写形式参见§5.2"英语数学文献的著录(编排)格式",具体书写形式各家杂志都有专门的规定,作者应据杂志"来稿须知"的要求书写。

本章后面的附录2是美国数学会主办的《Journal of mathematical analysis and applications》(数学分析及应用杂志)刊登的来稿须知,它详细描述了该杂志对稿件的要求、投稿的方式、版权的移交等事项,最后还谈到校对和作者预订抽印本的办法等。

## §4.2 英语数学论文中的语法与习惯用法

数学论文属于严肃的书面文体,要求行文简练,语法正确,重点突出。在写作实践中所遇到的语法现象很多,这里主要是针对写作英语数学论文时在语法上应注意的常见问题做一些补充或强调,其中包括常用人称,常用时态和主语与谓语在数上的一致性等问题;另外,也列举少数使用频率较高的单词的习惯用法。

### 4.2.1 常用的人称

传统的主张认为科技论文侧重于叙事和推理,因此观察要准确,叙事要客

观。读者重视的是论文的内容和观点，感兴趣的是作者的发现，不是作者本人，因此应避免第一、第二人称代词，尤其反对使用第一人称单数“I”。由此带来的问题是被动语态用得多，主动语态用得少，有时显得句子不够简练，表达不够生动有力。因此，也有部分人提出另一种主张，强调文章要亲切、自然、直截了当，应多用第一、第二人称。

就数学论文而言，通常采用第一种主张，但在介绍自己的工作时也常用“我们”(we)，而不用第一人称单数“I”。在推理过程中，有时用“we have…”，表示此时得到什么结论或有什么式子成立，未必强调是“谁”得到这一结论或式子。

### 4.2.2 常用的语态与时态

被动语态在科技论文中使用的频率很高。国外有人做了统计，在科技英语中，有三分之一左右的句子是被动句，在英文摘要中用得更多(见上节)。前面(如第一章)已经谈到，这是因为它适用于强调客观事实和行为效果。这里不再多谈。

论文作者要向读者表述研究的各项事实、观点产生的时间以及它们之间的相互的关系，指明哪些是一般真理，哪些只是推断等。因此，时态的正确应用是很重要的。

科技论文中有些时态与内容有密切联系，这也形成一些惯例，最常见的有：

(1) 叙述论文研究之前所进行的工作，用过去完成时：This had been the case before…。

(2) 实验用过去时。

(3) 图表用一般现在时：The second column of Table 2 represents the sum of $A$ and $B$. (表 2 第 2 栏表示 $A$ 与 $B$ 之和)。

(4) 一般真理用一般现在时。一般的数学结论(包括数学概念、已证明成立的引理、定理等)和推导过程都用现在时。因此，一篇研究性的数学论文中引言之外的部分基本上用一般现在时。

(5) 计划要做的工作和预期的结果用将来时。

### 4.2.3 主语和谓语在数上的一致

在简单句中，关于主语与谓语在数上一致的要求通常较易掌握，但遇到复合主语、集合名词和倒装句时，就容易混淆，读者应特别注意防止出错。下面列举若干要点。

(1) 单数主语用单数动词当谓语，复数主语用复数动词当谓语；

**例** $X$ equals to $Y$. ($X$ 等于 $Y$。)

They are equal. (它们相等。)

(2) 复合词用 and 连接时，谓语动词一般用复数；

**例** 3 and 5 are prime numbers.（3 和 5 都是素数。）

(3) 用 or, nor, either … or, neither … nor 连接的两个主语都是单数时，动词也用单数；当两个主语都是复数时，谓语动词也用复数。

**例** Neither 3 nor 5 satiefies the equation.（3 和 5 都不满足这个方程。）

若一个主语为单数，另一个主语为复数，则谓语动词与较接近它的主语的数一致。如：

Neither Einstein nor his students have considered this problem.（爱因斯坦和他的学生们都没有考虑过这个问题。）

(4) 集合名词作主语时，当它代表整体时，谓语动词用单数；当它表示整体内各个组成部分时，谓语动词用复数。

The committee has agreed to the plan.（委员会同意这个计划。）

The committee were at odds over the question.（委员会的委员们对这个问题有争执。）

(5) 用 there 引导一个句子时，谓语动词要与它后面的第一个名词或代词（主语）一致。

There exists a number belonging to $A$ in this interval.（在这区间中存在一个数属于 $A$。）

There exist infinite elements in the neighborhood of $x$.（在 $x$ 的这个邻域中存在着无穷多个元素。）

(6) 先行词、代词、关系代词和谓语动词的数要一致。

This number is the upper bound that has been proved the best.（这个数是已获证明的最佳上界。）

(7) number 作主语且其后接一个 of 引起的定语时，若修饰它的冠词是"the"，则用动词单数，因为它特指某个数字；若修饰它的冠词为"a"，则谓语动词用复数，因为"a number of"是一个短语，意为"有些"。

The number of absentees is small.（缺席人数少。）

A number of students are out today.（有些学生今天出去了。）

代表数或量的复数主语，当把它当作一个单位数值时用单数动词当谓语，如：

Sixty dollars is too much to pay for the unit.（这个单元的费用要 60 美元是太多了。）

### 4.2.4 动词的使用与应注意的若干问题

通常在一篇数学论文中使用的动词词汇量不大。在前言和一般叙述语句中

介绍研究情况和分析、讨论问题时，谓语动词的用法与其他文章用法没有什么根本区别。这里主要是就一些容易产生错误之处给予强调，此外还列举数学表达中的少数习惯用词。

1. 分清及物动词与不及物动词的使用

**注意**(1) 不及物动词不可接宾语，因此一般情况下，不及物动词不用被动语态；

(2) 汉语的表达与英语表达方式有很大不同。

**例 1** 若 $a,b$ 非负，则得到不等式 $\sqrt{ab}\leqslant\frac{a+b}{2}$，而当 $a=b$ 时，得到的是等式。

(If $a,b$ are negative we have the inequality $\sqrt{ab}\leqslant\frac{a+b}{2}$; and the equality occurs when $a=b$. )(注意，occur 是不及物动词。)

**例 2** 有的同学把句子“这个定理容易证明。”译成“The theorem easily prove. ”便错了。应改成“The theorem is proved easily. ”或译成“The theorem is easy to prove. ”

2. 适当采用“动词对应的名词形式+make”之类句型代替由具体行为动词当谓语的句型

上一节在介绍标题时已经指出，数学论文的标题常采用名词性短语(名词化结构)，不用句子(当然也不用动宾结构，也不用命令式)表示。其实，论文的各部分表示动作的谓语动词也常用与它们对应的抽象名词加上 make 或别的可搭配的动词来代替，且有的专家认为这是一种趋势。

**例 1** Irrational numbers are compared with rational numbers here. (这里，我们把无理数与有理数做了比较。)常改为：

A comparison of irrational numbers with rational numbers is made.

**例 2** Fundamentals of a digital computer is briefly introduced in this book. (本书简要介绍数字计算机的基本知识。)常改为：

A briefly introduction is given to fundamentals of a digital computer in this book.

3. 注意分词的正确使用

(1) 过去分词的使用

我们知道，过去分词可用于构造被动句。

**例 1** The function is well defined. (这个函数已经完全确定(定义好了)。)

科技论文中除了常用被动句外，为了使得表达简练，还常用过去分词作定语代替作为定语从句的被动语态语句。

**例 2** 用 required condition 代替 the condition which is required. (所需要的条件。)

用 the normalized polynomial 代替 the polynomial which is normalized.（正规化（了的）多项式。）

过去分词作形容词，有的已成为固定的表达方式：

**例 3** the well-ordered set（良序集）

generalized solutions（广义解）

过去分词作独立结构，表示条件、假定等：

**例 4** Given $\varepsilon>0$, there exists $\delta>0$ such that⋯.（对给出的 $\varepsilon>0$，存在 $\delta>0$ 使得⋯⋯。）

**例 5** The first problem being solved, the second problem will be solved easily.（（若）第一个问题得以解决，（那么）第二个问题就容易处理了。）

**例 6** All things considered, the solution is the best in our problem.（考虑到各个方面，这就是我们这个问题的最佳解答。）

（2）现在分词的使用

在纯数学领域，现在分词构成的独立结构常用以说明经过某种运算（积分、加、减、乘、除和取极限等）后会得到的结果。

**例 7** Setting $n\to\infty$, we obtain the desired result.（令 $n\to0$，就得到所要的结果。）

**例 8** Dividing the same number $a$ in both sides, we deduce the solution of equation 1.（两边同除以数 $a$，就求得方程 1 的解。）

在应用数学领域，使用现在分词来描述研究对象的语句很多。例如，常用现在分词作定语，可以放在所修饰的词之前或之后（构成分词短语）。

**例 9** A moving body always resists being accelerated.（运动着的物体总是与加速相对抗。）

**例 10** A body moving with uniform speed in circle is not in equilibrium.（作匀速圆周运动的物体不处于平衡状态。）

4. 掌握数学文章常用的一些动词

（1）表示假设、设定（包括定义）的动词

对于简单的假设或设定，常用命令格式。如，

Let $x$ be a set.（设 $x$ 是一个集。）

Suppose that $x$ is a non-empty set.（假设 $x$ 是一个非空集合。其中 that 可以省略，Suppose 可换成 Assume。）

Set $\mathbf{N}=\{1,2,3,\cdots\}$. 或 Put $\mathbf{N}=\{1,2,3,\cdots\}$. （令 $\mathbf{N}=\{1,2,3,\cdots\}$。）

Denote by $df$ the differential of the function $f$.（用 $df$ 表示函数 $f$ 的微分。）

Denote 也可以用被动式。例如，上一句也可改为：

The differential of the function $f$ is denoted by $df$.

在第二句及类似的句型中，Suppose 与 Assume 可互相替代；但是，对于附带说明原因的假设，通常采用 assume 为动词的句子。如：

We can assume $A<\infty$, otherwise there is nothing to show.（我们可以假定 $A<\infty$，否则不证自明。）

We can assume $A<\infty$ without losing any generality. 或 Without loss of generality, we may assume $A<\infty$.（不失一般性，我们可以假定 $A<\infty$。）

（2）表示推导出结论或表示证明结束的动词

常用表示“推导”的及物动词有 deduce、imply、obtain、yield、have 等，不及物动词有 follow、occur 等；一些词组，如 arrive at 等也可表示这个意思。如，

We deduce that $x=3$.（我们导出 $x=3$。）

We have the equality $a^2=b^2+c^2$.（我得到等式 $a^2=b^2+c^2$。）

Equalities $A=B$ and $B=C$ imply that $A=C$.（等式 $A=B$ 与 $B=C$ 蕴含 $A=C$。）

From (1) and (2) follows that $x=y+1$.（据(1)和(2)式导出 $x=y+1$。）

The conclusion is obtained.（得出结论。）

It follows that $x^2+y^2\geqslant 2$.（推出 $x^2+y^2\geqslant 2$。）

动词 conclude 和 complete 常用来表示论证的结束。如，

Because $r$ is arbitrary, we conclude the proof of equality（Ⅰ）.（因为 $r$ 是任意的，等式（Ⅰ）证明完毕，或：我们证得了等式（Ⅰ）。）此句中的 conclude 可换成 complete（动词）。不过 complete 出现在表示证明完毕的句子中，更经常作为形容词，即句子改为

Because $r$ is arbitrary, the proof of equality（Ⅰ）is complete.

（3）表示存在的动词与句型

主要是 be 与 exist 构成的句型：there is (are)…和 there exist(s)…。

（4）表示参阅文献的动词

For the detail, we refer the reader to Zhang's paper [2].

（详情请读者参见张的论文[2]（注：其中[2]表示在参考文献中所列的第二个文献。）

（5）表示“把……称为……”，“把……记作……”的动词与词组

例如 call, say, refer to…as…, term as, denote, designate 等，要注意它们的用法。

For simplicity, we will refer to the elements of $L^1(X)$ as function instead of equivalence class.（为了简便起见，我们把 $L^1(X)$ 的元素称为函数，而不称为等价类。）

（6）表示“与……矛盾”的动词

主要是 contradict。

This contradicts the property of $A$. (这与 $A$ 的性质矛盾。)

注意,形容词词组 contradictory to…和名词词组 in contradiction with …也可用于表示同样意思,但是配搭的介词不要选错。

(7) have 句型,用于表示尺寸、大小、比较优缺点,具有的性质等

**例 1** The table has a height of 105 centimeters. (这张桌子高 105 厘米。)

**例 2** This segment has an approximate length of 20 centimeters. (这条线段长约 20 厘米。)

**例 3** This ball has a smaller volume than that cube. (这个球的体积比那个立方体的体积小。)

**例 4** The unit matrix $I$ has the property that $AI=A$ for any matrix $A$ with the same order. (单位矩阵具有这样的性质:对任何同阶矩阵 $A$ 满足 $AI=A$。)

(8) be 句型,用于表示尺寸、大小、比较优缺点,具有某种特点等

上面用 have 句形表示的例 1 ~3 可相应改成:

**例 1** The table is 105 centimeters high.

**例 2** This segment is approximately 20 centimeters long.

**例 3** This ball is smaller in volume than that cube (is).

Be+形容词表语或 Be+of+形容词+名词常用以表示具有某种特点。

**例 4** The set is uncountable. (这个集是不可数的。)

**例 5** Cauchy criterion is of great use. (柯西准则很有用。)

### 4.2.5 副词的位置

使用副词时应该注意它与汉语的异同之处,特别是副词与它所修饰的动词的相对位置,下面是一些例子。

(1) 多数副词可放在被修饰的动词之后。但是,表示频度和不确定时间的副词(如 often, always, never, already, seldom, quite, almost, hardly, generally, usually, frequently 等)通常放在行为动词之前;但若句中有情态动词、助动词或系词 be,则放在这类词(的第一个)之后,其他副词(如 easily, particularly 等)有时也这样放置。

**例 1** They often study this kind of equations. (他们经常研究这类方程。)

**例 2** The conversion is usually effected. (转化通常是有效的。)

**例 3** The conclusion can easily be obtained by using the following methods. (用下面方法容易求得结论。)

(2) 一般地说,不要将副词放在及物动词与它的宾语之间,尤其不要将一个长的状语放在及物动词与它的宾语之间;通常应把它们放在宾语之后,有时也可放在动词之前。

**例 1** They find the solution quickly and accurately.(他们迅速而准确地求出解来。)

**例 2** They correctly interpret the situation that….(他们正确地阐明了……的情形。)

(3) 程度副词和强调副词常放在形容词的前面来修饰形容词,如 infinitely many(无限多的),uniformly bounded(一致有界)。这时,不能随意把副词后缀-ly 去掉改成形容词。此外,应尽量避免把两个带-ly 的副词连接在一起使用,如"正的径向整体解"可以译成 entirely radial positive solution,但不宜译成 entirely radially positive solution。

### 4.2.6 若干常用的连接词、副词和特殊用语

1. "因为……所以……"、"由于……(所以)……"的表示法

在数学结论的推导过程中,数学英语很少用 because,而常用 since(有时也用 as 或 for)引导原因状语从句,作为推理的条件和理由。这类从句可放在主句之前或之后。应注意的是,英语与汉语不同,主句不用表示"所以"或"因此"的连接词来搭配。

**例** Since the hypotheses (2) and (3) are valid, we can construct a convergent subsequence of $\{x_n\}$. (由于条件(2)和(3)成立,我们可以构造$\{x_n\}$的一个收敛子列。)

**注** 如果上述条件(2)和(3)是已知成立的等式,也可以将上句简化为:By the equalities (2) and (3), we...。

2. "所以"、"因此"、"从而"的表示法

Hence, therefore 等有表示"所以","因此","从而"的意思,但它们并不和 since 或 because 搭配,只是用于表示进一步的推导。

**例** From the inequality (3) follows $x^2+y^2=1$, hence the conclusion is valid. (由不等式(3)导出 $x^2+y^2=1$,因此结论成立。)

3. Such that 从句的作用

在数学英语中常用 such that 从句表示对中心词的进一步说明和限制(说明该中心词所表示的数学概念必须满足的条件)。

**例** Suppose $X$ is a topology space such that each close subset of $X$ is a $G_\delta$-set. (假定 $X$ 是这样的拓扑空间:$X$ 的每一个闭子集都是 $G_\delta$-型集。)这一句相当于:

Suppose $X$ is a topology space satisfying that each closed subset of $X$ is a $G_\delta$-set.

4. "若(如果)……则(那么)……"、"如果……就……"的表示法

If 引导的从句在数学英语中广泛地应用,相应的复合句与汉语的句型"若(如果)……则(那么)……"、"如果……就……"类似。但是,英语的主句通常

不出现与汉语“则(那么)”或“就”对应的连接词;在数学英语中,当非常强调结论与条件的关系时,可在主句开头处添上副词 then(很多计算机语言把“if…then…”作为一种语句)。If 从句可以放在主句之前,也可放在主句之后;有时主句的前后各放一个 If 从句,分别表示大小前提。

**例 1** If $\boldsymbol{x} \in V$, we can express $\boldsymbol{x}$ as a linear combination of these basis elements.(若 $\boldsymbol{x} \in V$,我们就可以把 $\boldsymbol{x}$ 表示成这些基元素的线性组合。)

**例 2** If the domain $D$ is regular, then the problem has a unique solution if the given function $f$ on $\partial D$ is continuous.(若区域 $D$ 正则,那么当 $\partial D$ 上给定的函数 $f$ 连续时,该问题就有唯一的解。)

5. “当且仅当”的表示法

When and only when; if and only if 均表示“当且仅当”,有时也省略成 iff。

例:The equality is valid when and only when $p>1$.(这个不等式当且仅当 $p>1$ 时成立。)

6. “总是……除非”的表示法

The term measurable applied to a set in $S$ will always mean $V$-measurable unless stated otherwise.(若无另加说明,可测这术语用于 $S$ 的一个集合时总是指 $V$-可测。)

### 4.2.7 关于正式语体的使用举例

在科技英语中,除了专业技术词汇要规范化外,非技术用词也要求采用正式语体,不使用一般语体。例如,表示问的意思,不使用 ask,而使用 inquire。下面再给出一些正式语体的例子,括号中是相应的一般语体同义词:

开始 commence (begin), 结束 conclude (end),
完成 complete (finish), 最终 eventually (in the end),
使用 employ 或 utilize (use), 足够 sufficient (enough),
许多 many(a lot of), 快 rapid(quick),
当心 caution (care), 试,力求 endeavor (try)等。

不过,在一般的数学文献中,仍允许有少量的一般语体词,如 begin、try 等仍较常用。

### 4.2.8 正确选择专业术语,采用“道地”英语表达

本节最后强调,英语数学论文的写作不是简单地把中文语句直接译成英文。因为道地的英语有许多表述方式不同于中文,所以“直接的”汉译英得到的多半是“中国式的英语”。做好汉译英是一门大学问,对于初学者,这里仅简要强调几个应注意的问题。

1. 谨防逐字直译专业术语引起大错

许多中国人在写作论文时，常写成中文稿，再把它译成英文。这本是允许的。但是常见一些初学者，既未能事先掌握数学专业术语，临场又不仔细查证，而简单地采用直译（其实是逐字翻译）的办法（就像采用一些翻译软件一样），结果出大错，闹了不少笑话。例如，一些毕业论文把“子空间”译成 son space（应为 subspace），把“真子集”译成 true son set 或是 true subspace（应为 proper subset），把“方程组”译成 *equation group*（应为 *system of equations*）等等，屡见不鲜。

因此，读者在专业英语的学习过程中，应努力掌握一些常用的数学术语，并认真区别汉语与英语表达方式的异同。

2. 认真鉴别汉英对译的专业术语在含义上的差别

数学的不同学科分支常采用一些词干类似的术语来表达不同的概念，它们译成汉语时是近义词，甚至可能是同一个词。例如，下面三个词

module, modulo, modulus（复数为 moduli）

译成汉语时都是“模，模数”。但 module 和 modulo 限于代数或有关分支，其中 module 表示“加法半群”之类数学结构；modulo 常出现在 modulo rho-homology group（“模 $\rho$ 同调群”）等专用词组之中；而 modulus 则可能出现在多个数学分支，其含义随分支不同而差别甚大，为示区别，常带定语，如 modulus of continuity（“连续模”，见于逼近论），modulus of convexity（“凸模”，见于凸分析），conformal modulus（“共形模”，见于函数论）。

与此同时，modular（“模的”）是与 module 对应的形容词，用它来修饰的专业术语很多，如 modular ideal（“模理想”），modular ring（“模环”）等。而 modulus 没有对应的形容词，modulus 本身常当名词性定语构成专业术语，如 modulus principle（“模原理”），modulus theory（“模理论”），此外还组成一些逻辑学上的术语。

又，汉语的“模子”（“模具”）对应的英语是 mold 或 mould；“模型”对应的英语是 model；而“模式”对应的英语是 pattern。如此等等，读者不妨仔细揣摩。

3. 重视汉英语言表达方式上的不同

**例 1** “我们的重点放在第二章所介绍的内容。”应表述为：

Our emphasis is put on what is described in chapter 2.（注意“内容”并未直译。）

**例 2** “这个线性空间是 3 维的。”道地英语说成：

The linear space has dimension 3.（这时“是”并不采用“is”来直译。）

**例 3** “变量 $A$ 与变量 $B$ 之间的关系由欧拉公式来表示。”道地英语说成：

Variable $A$ is related to variable $B$ by the Euler formula.（注意“关系”和“表

示"并未直译。)

**例 4** "解这个方程的方法是选择变换参数。"道地英语说成:

The equation is solved by choosing suitable transformation parameter. (注意"方法"并未直译。)

**例 5** "近年来模糊数学的应用有了迅速发展。"道地英语说成:

Recent years have witnessed a very rapid growth of developments of Fuzzy mathematics. (注意主语变成了"Recent years",这句可看成是特殊句型。)

对于初学者,要掌握道地的英语是一件难事,必须在学习中注意比较和积累,才能逐步提高英语写作水平。

4. 注意地区与团体对英语表达的影响

英国英语与美国英语主要差别在于发音,不过在拼写、用词和表达方式上也有少许差异(见附录4)。不同的学术团体所采用的专业术语稍有一点不同,不同作者的写作风格也会有所差别。例如,作为数学术语的"边界",多数英语论著采用"boundary",但以英国著名函数论大师 Haymann 为首的学派则采用外来语"frontier"表示;作为数学术语的"扫除",在英语中采用 sweeping out 表示,但似乎更多专家喜欢采用法语 balayage(原是法国数学家首创),其中的道理是:把它当成外来语,不仅引人注目,而且不会与 sweeping out 的通常含义混淆。

不过,我们在写作一篇论文时,应自始至终采用统一的某个地区或团体的表示方式;避免时而这样,时而那样,甚至前后不一、杂乱无章。

## §4.3 英语数学论文的精练要求

英语科技论文都讲究精练,数学论文更是如此。下面列举一些精练的要求。

1. 论文的内容和句子要尽量减少不必要的重复。

**例** If function $f(x)$ is a continuous function on $[a,b]$, then there exists a maximum point $c$ in $[a,b]$. 句子没错,但后一个 function 明显是重复,但不能简单地把它去掉就算了,应改为:

If function $f(x)$ is continuous on $[a,b]$, then there exists a maximum point $c$ in $[a,b]$. (若函数 $f(x)$ 在 $[a,b]$ 连续,则在 $[a,b]$ 有一个最大值点 $c$。)

2. 减少不必要的词语。

**例** The results (which were) obtained show the new method is better. 其中括号内部分可省去。(所得结果表明,新方法较好。)

3. 过多的无人称代词 it 也应省去。

**例** It was concluded that a new method must be devised. 可改为如下更直截了当的表达方式:

A new method must be devised.(必须设计出一种新方法来。)

4. 减少过多的"the"。

现代的科技论文提倡减少冠词"the"的使用,能省则省。

**例 1**　The coating of the fabric took 10 hours.(将纺织品涂层要用 10 个小时。)可改成 Coating the fabric took 10 hours.

应该注意的是,用"the"与不用"the"有时其意义根本不同,需认真加以区别,不可随便省略。

**例 2**　一篇论文的题目若为"The principles of calculus",则表示"微积分的所有原理",若将其中的"the"省去,则表示"微积分的一些原理"。

5. 正确地使用省略(见一般的英语语法书)。

## §4.4　英语标点和数学符号的正确使用

### 4.4.1　标点符号

英语科技论文中标点符号的使用除了遵循一般语法规则外,还具有某些特殊规律。下面以常用的句点". "、逗号","、分号";"与冒号":"为例说明之。

1. 句点

使用场合:

(1) 用在陈述句之句尾。

**例**　The proof is complete.(证明完毕。)

(2) 在日常英语中,句点常用来表示缩写,但作为数学符号的英语单词的缩写不用句点。

**例**　Dr., Prof., Ms., a. m. 为缩写, Mass. 为 Massachusetts(马萨诸塞州——美国的一个州)的省略,均有句点。

数学式 log $A$, cos $A$ 中的 log, cos 分别为 logarithm, cosine 的缩写,注意此时不加句点。

(3) 用于表示省略。

**例**　The question is….(日常英语)

$1+2+3+\cdots+n+\cdots$.(数学表达式)

2. 逗号

英语数学论文中逗号用得很普遍,除了对应于中文用逗号之处外,在中文用顿号之处也常用逗号。

(1) 连接一系列并列的词或短语,要用逗号和 and。

**例**　Positive fractions, negative fractions and zero are called rational numbers.

(正分数、负分数和零称为有理数。)

(2) 副词和副词短语插入时,要用逗号。

**例** The purpose of the investigation, namely to establish optimum conditions, was clearly stated.(研究的目的,即建立最佳的条件已阐述清楚了。)

(3) 一个较长的从句放在主句之前,则该从句之后要用逗号。

**例** If all needed variables and parameters are considered and supposed, the modeling may be started .(如果所有必需的变量和参数均已考虑并设定,就可以开始建模。)

(4) 非限定性从句要用逗号分开,但限定性从句不可用逗号分开。

The method that proved most convenient was employed. 其中 that 从句是限定性的,that 之前不用逗号分开。(被论证为最方便的方法被采用了。)

The third method, which is considered as most convenient, was employed. 其中 that 从句是非限定性,that 之前用逗号分开。(第三种方法被采用了,它被认为是最方便的。)

3. 分号

使用范围:

(1) 分隔两个并列句(不用 and)。

**例** In this paper, conventional algorithm was specified throughout; encryption algorithm was not mentioned.(文中对常规算法从头到尾作了详细论述,而密码算法则未提及。)

(2) 一句中分成若干个主要部分,若主要部分内已用了逗号,则应用分号将各主要部分分隔开来。

**例** PC include CPU (central processing unit); monitor, for output; and keyboard, for input.(个人电脑由 CPU(中央处理器),显示器(用于输出)和键盘(用于输入)组成。)

4. 冒号

使用范围:

(1) 冒号用于一个长的问句之前。

**例** The use of the graph is illustrated as follows: where the function $f(x)$ is decreasing and where $f(x)$ is increasing and there are how many local maximun and minimum points. (这种图可用来说明,函数 $f(x)$ 在哪里是递减的,在哪里是递增的,有多少个局部最大、最小值点.)

(2) 冒号用于一系列枚举之前。

**例** When you define a variable in C++, you must tell the compiler what kind of variable it is: an integer, a character and so forth.(当你定义 C++的一个变量时,

必须告诉编译器它是什么类型:整型、字符型或其他类型。)

(3) 冒号用于分类叙述的提示语之后。

**例** Theorem 1 …. $f(x)$ is a constant function if and only if $f'(x)=0$ for all $x$ in $(a,b)$.

*Proof* "Only if" part: Since $f(x)$ is a constant function, …. (必要性:因为$f(x)$是常数值函数,……。)

"If" part: Let $x_1$, $x_2$ be in $[a,b]$ such that $x_1<x_2$. Then…. (充分性:设$x_1$,$x_2$属于$[a,b]$且满足$x_1<x_2$,则……。)

(4) 冒号用于表示比率和时间。

**例** The operating ratio was 7:9 at 8:30 a. m..

(操作比率在上午8时半是7比9。)

### 4.4.2 数目字和数学符号的书写

1. 一句话以数目字开始时,数目字要用英语单词表示,不要写成阿拉伯数字。

**例** Sixty manhours are required to complete the job. (完成这项工作需要60工时。)

2. 一句话一般不宜用符号开始,特别要避免可能引起混淆的情形。

**例** If $a=b$, then $c$ holds too. (正确)

If $a=b$, $c$ holds too. (不恰当)

If $a=1.23$, $c$ is finite. (不恰当)

3. 不要用等号"="作为一句(特别是主句)的主要动词,但等号可作从句的动词。

**例** When (7) is substituted in (8), one obtains $a=b$. (恰当)

When (7) is substituted in (8), $a=b$. (不恰当)

Since $x+y=1$, we have $x=2$ and $y=-1$ by (3.1). (恰当)

4. 用数学公式作同位语时,不必加括号,它的前面也不要用逗号隔开。

**例** By using the equation $a=b$, we have… (恰当)

By using the equation, $a=b$, we have… (不恰当)

By using the equation $(a=b)$, we have… (不恰当)

## 附录2 参考论文

这是2004年发表于《伦敦数学会通报》(Bulletin of the London Mathematical Society)第36卷(p.516~518)的一篇数学论文。该文短小精练(正文仅由两部分组成),结构完整。作者在引言中用简短的文字把问题的由来及现状说得一

清二楚;正文之后还附上有关的最新信息。转载时编者对格式稍做更动并增补了关键词(原文缺),以便与通用格式一致。

# ON A QUESTION OF HERMAN, BAKER AND RIPPON CONCERNING SIEGEL DISKS

LASSE REMPE

(Math. Institute, Univ. of Warwick, Coventy CV4 7AL. UK)

**Abstract** Consider the family of exponential maps $E_\kappa(z) = \exp(z) + \kappa$. This paper shows that any unbounded Siegel disk $U$ of $E_\kappa$ contains the singular value $\kappa$ on its boundary. By a result of Herman, this implies that $\kappa \in \partial U$ if the rotation number is diophantine.

**Key words** exponential map; Siegel disk; singular value; rotation number; diophantine

**2000 Mathematics Subject Classification** 37F10, 30D05

## 1. Introduction

In the collection [4] of search problems in complex analysis, the following was posed as problem 2.86, attributed to Herman, Baker and Rippon.

Let the function $f_\lambda(z) = \lambda(e^z - 1)$, $|\lambda| = 1$, have a Siegel disk $U$ that contains 0.

(a) Prove that there exists some number $\lambda$ such that $U$ is bounded in $\mathbf{C}$.

(b) If $U$ is unbounded in $\mathbf{C}$, does the singular value $-\lambda$ belong to $\partial U$?

In [9], it was shown that if $\lambda = e^{2\pi i\alpha}$, where $\alpha$ is diophantine, then $U$ is unbounded. Rippon [11] generalized an argument of Carleson and Jones [5, p. 86] to give an elementary proof that $-\lambda \in \partial U$ for almost every $\lambda$. It was also mentioned in [11] that problem (a) could be solved by adapting a method from [7].

In this paper, we give a positive answer to problem (b), with a rather simple proof. This implies in particular that the singular value lies on the boundary for diophantine rotation numbers. We shall prove the result in the following form, which allows Siegel disks of arbitrary period.

THEOREM. Let $\kappa \in \mathbf{C}$, and suppose that the function $E_\kappa(z) := \exp(z) + \kappa$ has an unbounded Siegel disk $U$. Then there exists a $j$ such that $\kappa \in \partial E_\kappa^j(U)$.

Note that $f_\lambda$ is conjugate to $E_\kappa$ for $\kappa = \log\lambda - \lambda$.

## 2. Proof of the theorem

For basic definitions and results, we refer the reader to an expository text on the iteration of entire functions, such as [1] or [3].

To prove the theorem, we shall use the following property of exponential maps.

PROPOSITION 1. Let $\kappa \in \mathbf{C}$. Then there exists a curve $\gamma: [0, \infty) \to J(E_\kappa)$ such that

$$\lim_{t\to\infty} \operatorname{Re}(\gamma(t)) = +\infty \text{ and } \limsup_{t\to\infty} |\operatorname{Im}(\gamma(t))| < \infty.$$

This well-known fact seems to have been first by Devaney, Goldberg and Hubbard [6]. In fact, it is now known that the set of escaping points of $E_\kappa$ consists entirely of such curves [12].

Fix a $\kappa \in \mathbf{C}$ for which $E_\kappa$ has a Siegel disk $U$. The result, proved by Sullivan [13], that rational functions do not have wandering domains, has been generalized to the family of exponential maps by Baker and Rippon [2]. It is well known that

$$\partial U \subset P := \overline{\{E_\kappa^n(\kappa) : n \in \mathbf{N}\}}; \qquad (*)$$

see, for example, [3, Theorem 7]. Thus $\kappa$ belongs to the Julia set of $E_\kappa$. Indeed, otherwise $\kappa$ would eventually map into some periodic component of the Fatou set. Since each such component is an attracting domain, a parabolic domain, a Baker domain or a Siegel disk [3, Theorem 6], $P$ would then intersect the Julia set in at most finitely many points, which contradicts (*). (In fact, Baker domains do not occur for exponential maps, as all escaping points lie in the Julia set [8].)

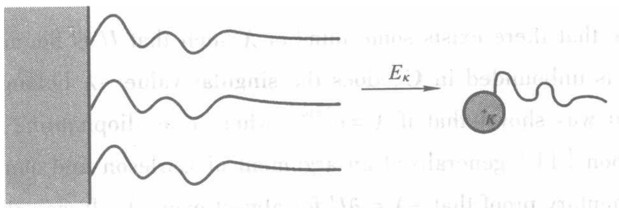


Fig. A-1-1 The set $K$ and its image.

Let us suppose that $\kappa \notin \partial E_\kappa^j(U)$ for every $j \in \mathbf{N}$; we wish to show that $U$ is bounded. Choose $\delta > 0$ such that

$$\overline{D_\delta}(\kappa) := \{z \in \mathbf{C} : |z-\kappa| \leqslant \delta\} \subset \mathbf{C} \setminus \bigcup_j E_\kappa^j(U).$$

Note that this implies that for every $j$, $E_\kappa^j(U) \subset \mathbf{C} \setminus \overline{H}$, where

$$H = E_\kappa^{-1}\, \overline{D_\delta}(\kappa) = \{z \in \mathbf{C} : \operatorname{Re} z < \log \delta\}.$$

PROPOSITION 2. For every $j \in \mathbf{N}$, the set $E_\kappa^j(U)$ has bounded imaginary part.

*Proof.* Let $\gamma$ be the curve by Proposition 1. Since $\kappa \in J(E_\kappa)$, we can find a preimage $g_0$ of $\gamma(0)$ under an iterate of $E_\kappa$ such that $|g_0-\kappa|<\delta$. Taking the appropriate pullback of $\gamma$, we obtain a curve $g:[0,\infty)\to J(E_\kappa)$ with $g(0)=g_0$. (If the orbit of $\kappa$ intersected $\gamma$, we might not be able to take these pullbacks. However, in this case we can pull back $\gamma$ along the orbit of $\kappa$ and obtain a curve actually starting at $\kappa$.) Choose the largest $t_0$ with $|g(t_0)-\kappa|=\delta$, and consider the set

$$K:=E_\kappa^{-1}(\overline{D_\delta}(\kappa)\cup g([t_0,\infty))).$$

This set consists of $\overline{H}$ together with the preimages of $g([t_0,\infty))$ (see Figure A-1-1). Each of these preimages is asymptotic to a line $\{\operatorname{Im} z=2k\pi\}$, where $k\in\mathbf{Z}$, and thus has bounded imaginary part. Therefore every component of $\mathbf{C}\setminus K$ has bounded imaginary part. Since $E_\kappa^j(U)\subset\mathbf{C}\setminus K$, this proves the claim.

*Proof of the theorem.* By the previous proposition, we can find $S>0$ with $|\operatorname{Im} z|<S$ for all $j\in\mathbf{N}$ and $z\in E_\kappa^j(U)$. Choose $R>1$ large enough such that $\exp(R-1)>-\log\delta$ and $\exp(R)\geq\exp(R-1)+S+1+|\kappa|$. Then, if $z\in\mathbf{C}$ with $r:=\operatorname{Re} z\geq R$ and $|\operatorname{Im} E_\kappa(z)|<S$,

$$|\operatorname{Re} E_\kappa(z)|>|E_\kappa(z)|-S\geq\exp(r)-|\kappa|-S\geq\exp(r-1)+1.$$

If also $\operatorname{Re} E_\kappa(z)>\log\delta$, then in particular $\operatorname{Re}(E_\kappa(z))-1>\exp^n(r-1)$.

Now suppose that there were a $z\in U$ with $r=\operatorname{Re}(z)\geq R$. It would then follow by induction that

$$\operatorname{Re}(E_\kappa(z))-1>\exp^n(r-1),$$

which is a contradiction, since points in a Siegel disk do not escape to $\infty$. □

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NOTE ADDED IN PROOF (March 2004): N. Fagella has kindly informed me that she and X. Buff have independently obtained a proof of our theorem.

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## 附录3 来稿须知(英文,附译文)

*Journal of Mathematical Analysis and Applications*

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# 附录 4 美国英语和英国英语对数学表述的影响

美国英语(American English,简称 AE)和英国英语(British English,简称 BE)基本上是一致的,但也存在一定的差异。这种差异主要体现在语音上,而在拼写、语法、词汇和惯用法方面也有少量的反映。后者自然要影响到数学专业英语。特别是 20 世纪中叶以来,美国科技在世界上居领先地位并发挥巨大的作用;美国作为数学大国,对数学发展起着重要的作用,这也在某种程度上加大了 AE 对专业英语的影响。因此,我们有必要了解一下 AE 和 BE 在数学专业英语中的差异,以利于阅读、翻译和写作。

值得注意的是,写作一篇英语的数学论文时,应自始至终只采用一种语体:AE 或 BE,尽量避免交叉使用。最简单的例子是,在同一篇文章中,若用米作为长度单位时,不宜时而使用 meter(AE),时而使用 metre(BE)。

参照文献[10]和[11]的观点,我们把 AE 和 BE 在数学专业英语中的主要差异作如下简单介绍。与此同时,我们应知道,由于数学科学是国际性的,各国之间交流非常频繁且广泛,因此,AE 和 BE 的差异在数学专业中表现得较不明显。

**一、拼写上的差异:**

AE 和 BE 在拼写形式上基本上一致,但有些词 AE 的拼写与 BE 的拼写不同。主要表现在:

(a) 美语单词有时省略某些字母

BE 中词尾的字母组合 *ement*,在 AE 中省略成 *ment*,如

judgement (BE)→judgment (AE);(判断)

BE 中的字母组合 ll,在 AE 中有时省略成 l,如

labeller (BE)→labeler (AE);(标志)

BE 中词尾的字母组合 our,在 AE 中有时省略成 or,如

neighbourhood (BE)→neighborhood (AE).(邻域)

(b) 字母组合拼写的变化

BE 中的字母组合 se,有时在 AE 中以 ze 出现,如

analyse (BE)→analyze (AE), (分析)

generalise (BE)→ generalize(AE); (推广)

BE 中的字母组合 xion 有时在 AE 中改为 ction,如

connexion (BE)→ connection (AE); (连通)

BE 中的字母组合 re,在 AE 中常改为 er,如

metre (BE)→ meter (AE), (米)

centre (BE)→ center (AE). (中心、圆心)

(c) 连字符"-"的采用与否

个别前缀之后 BE 采用连字符"-",而 AE 常不用。如

non-negative (BE)→ nonnegative (AE),(非负的)

ultra-filter (BE)→ ultrafilter (AE).(超滤子)

## 二、语法上的差异

AE 与 BE 在语法上的差异很有限,主要表现在词法与句法上。这种区别常带有强烈的相对性,且正在趋于一致。从总体上看,灵活与自由是 AE 的特点。

### (一) 词法上的区别

(a) 动词

to have 表示"拥有"的含义时,AE 和 BE 的问句形式在结构上不同。例如,BE 用"Has the function property A?"表示"该函数是否具有性质 A?", AE 同时也常用"Does the function have property A?"来表示同一个问句。

另外,在个别动词的使用上,AE 与 BE 不同,例如 prove(证明、证实)的过去分词有 proved 和 proven 两种,AE 中两种形式通用,但在 BE 中 proven 只作形容词用。

(b) 名词的数

个别抽象名词,如 committee 在 BE 中可看成单数以表示团体,也可看成复数以表示团体的多数成员;而在美语中只当成单数使用。

(c) 冠词的用法

对于在 of structure 之前出现的名词(其中 structure 代表不用冠词的名词),BE 必须加冠词,但 AE 不做要求,特别当这一名词为抽象名词时,常不用冠词。下面是 AE 的例子,其中 Substitution 不用冠词:

Substitution of iron for wood hastened the decline of the sailing vessel.

反过来,BE 中一些不用冠词的短语,如 at table, in hospital, at school 等,AE 常在其中名词之前加定冠词 the。另外,在一些短语中,冠词的位置也不同。例如 BE 只用 half an hour 表示"半小时",AE 也可采用 a half hour 表示同一意思。

(d) 代词的用法

BE 使用不定代词 one 的地方,美语常用 he, his, himself 分别代替 one, one's 和oneself,这种区别较明显。

(e) 介词的用法

AE 常用 on a train (boat)(在火车(船)上), on the street(在街上),BE 则用 in a train(boat), in the street 表示同一个意义。在数学中,表示"在一个空间(区域)上"时,也有 on a space (domain)与 in a space(domain)两种方式。

另外,BE 用 all those(所有这些),AE 则采用 all of those 表示。BE 用 to approximate to the value(逼近这个值)，AE 则省去动词 approximate 之后的 to. 类似的例子说明,美语将不少动词从间接动词化为直接动词。

(f) 连词的用法

BE 中表示结果的连词 so that, so…that 以及 such…that 等,其中的 that 在 AE 中常被省略。

**(二) 句法上的区别**

(a) AE 中状语的位置比较灵活,如副词可放在助动词之前或之后,而 BE 中副词一般都按规则放在助动词之后。如

AE 的句子:Those curves certainly don't intersect at point $O$. BE 写成 Those curves don't certainly intersect at point $O$.(这些曲线肯定不会在 $O$ 点相交。)

(b) 有相当部分在 BE 中采用现在完成时表示的句子在 AE 中改为一般过去式。

(c) AE 表述数学内容时,被动语态频率较 BE 为低,采用主动语态的频率相对较高。

**三、词义和用语的差异**

由于 BE 与 AE 的不同惯用法,有不少同词异义或同义异词的现象,这些不同之处在美语对英语影响逐渐加大的过程中已经趋于一致。在数学英语上也有少量的同义异词,例如,"集合"一词的标准用词是 set,这一点 AE 与 BE 无异;但对于由一些集构成的"族",BE 常用 class 表示,而 AE 比较灵活,可用 family 或 collection 等表示,"一类方程"BE 表示为 a sort of equations,而 AE 常用 kind 代替 sort.

当然,有的同义词的采用与团体及个人的习惯有关,不宜以 AE 和 BE 来截然划分。

**两点说明:**

1. 这里只是关于 BE 与 AE 在数学表达上的差异的初步介绍。初学者无需在细节上耗费太多精力。有兴趣者可参见书末参考文献[10]和[11]。

2. 查看英美两国作者出版的论著,可发现不同作者的写作风格各异。大体上说,AE 的论著表达较自由、灵活、简单;BE 的论著则较严谨和复杂些。其他国家的作者有的采用 AE,有的采用 BE,写作的风格更是因人而异。对初学者,提倡广泛阅读、博采长处;在写作效仿上,则应采用相对固定的一种模式,循序渐进。

# 第五章　查阅英语数学文献的基本知识

21 世纪已被公认为信息时代。善于获取信息和处理信息是人们在这时代的竞争中得以立足并获得胜利的基本条件之一。查阅数学文献是数学专业人士广泛获取准确信息、了解学科发展动态、积累科研资料的主要渠道,也是数学各专业学生(包括研究生)学习和写作毕业论文的需要,因此也是他们必须训练的基本功之一。由于世界上 70% 左右的数学文献是用英文书写的,所以本书在提供专业英语的阅读材料和写作基础知识之后,也在本章简要介绍查阅外文(主要是英文)数学文献的基本知识,包括介绍上网检索。

本章的资料多数取材或引用自文献[2]和[8]。

## §5.1　英语数学文献简介

数学文献泛指记载数学知识的一切载体(介质)。

### 5.1.1　数学文献的类型、作用与检索数学文献的重要性

文献的**载体类型**有纸张、磁带[MT]、磁盘[DK]、光盘[CD]和联机网络[OL]等,其中括号内的英文字母组合为载体**类型标识符**,如 OL 是联机网络的标识符,在引用文献时必须标上(除纸张型无标识符外)。

文献除了载体类型外,根据其内容和形式也可划分类型(称为**文献类型**),各种类型也对应有标识符,下面类型后括号中的英文字母或字母组合就表示该类型的标识符:

图书[M]、论文集[C]、期刊(杂志)[J]、学位论文[D]、报告[R]、标准[S]、析出文献[A]、数据库[DB]、计算机程序[CP]和电子公告[EB]等。

上面两种类型可联合使用,通常先写载体类型,后写文献类型,但在引用文献书写标识符时,要求先写文献类型标识符,加一斜线"/"后接着写载体类型标识符。如磁盘数据库[DB/MT],网上期刊[J/OL]等。

数学文献是人们学习数学知识的基本工具、了解数学发展信息和动态的主要媒体、从事数学研究的根据和重要手段。特别是数学研究论文的撰写，对数学文献的依赖性远远超过其他自然科学学科（比如做物理论文，通常更重要的是实验）。因此了解数学文献的基本知识，学会查阅数学文献的技能对于数学专业的学生（包括研究生）很有必要。特别是现在，数学文献急剧膨胀，浩如烟海，每个人只能掌握和使用其中一小部分。要从中获取自己需要的资料，一定要了解检索方法，掌握必要的检索工具，才能做到及时和有效；否则犹如大海捞针，奏效甚微。

从学习的角度来说，由于国际上数学文献多数用英文书写，了解英语数学文献的基础知识，会为数学专业英语的应用和水平的提高创造更多的机会。

## 5.1.2 数学图书的基本类型

（1）教科书（Textbook）也称课本，属于教材。本科的基础课教材多数不作为科研的主要参考资料，但研究生教材，特别是像“斯普林格”（Springer-Verlag）等国外重要出版社的研究生教科书都是数学研究的重要参考资料之一。

（2）专著（Monograph）是某一个学科的专题著作，专门就数学某一学科或分支的课题进行系统、全面的论述，即系统介绍该专题的理论基础研究成果、发展趋势以及尚待解决的问题。

（3）丛书（Series）是编印各种单独著作并冠以总名的连续出版物（Serial）。世界著名的出版机构，如美国的“学术出版社”（Academic Publish House），德国的“斯普林格”，荷兰的“克鲁卫尔”（Kluwer Publisher），英国“剑桥大学出版社”（Cambridge Univ. Press）等出版了大量的数学丛书。这些丛书涉及数学及其边缘学科的各个领域，内容精深广博，对数学研究所起的作用很大。

《数学讲演集》（Lecture Notes in Mathematics），《剑桥数学专题丛书》（Cambridge Tracts in Math），《数学研究纪事》（Annals of Mathematics Studies）等就是上述出版社出版的最有代表性的高水平数学丛书。

（4）成套著作是全面介绍某学科系统知识而成套出版的图书，常分成若干册，每册系统介绍一个专题。美国 Addison-Wesley 出版社出版的《数学及其应用大全》（Encyclopedia of Mathematics & its Applications）是最大型的成套著作。

（5）工具书，又称参考工具书，包括百科全书（Encyclopedia）、辞典（Dictionary）、年鉴（Yearbook）、手册（Handbook）、指南（Directary, Guide）、数学表（Mathematical Table）、标准（Standard）、书目（Bibliography）、传记资料（Biography）、文摘（Abstract）和索引（Index）等。

### 5.1.3 数学杂志简介

期刊(Periodical)又名杂志(Journal),是定期或不定期连续出版物,每期版式基本相同,有固定名称,用卷期或年、月顺序编号出版。

数学期刊是最主要的数学研究信息资源。与图书相比,它出版的周期短、刊载论文速度快、数量大、内容新、品种多、范围广,且发行面宽,流通量大,能及时反映世界科学技术水平。因此,期刊对研究数学的人(包括习作毕业论文的本科生和研究生)来说特别重要。可以说,不阅读新期刊就不能了解科研前沿状态,难于进行创造性研究。

1. 常用数学杂志按性质可划分为八种类型。

(1) 纯数学杂志;(2) 数学文摘型杂志;(3) 含有数学论文的自然科学杂志;(4) 数学边缘学科杂志;(5) 数学教学与初等数学杂志;(6) 数学史以及含有数学史、数学哲学、思想方法论的自然科学史等有关杂志;(7) 数学以及含有数学的自然科学会议报道型杂志;(8) 其他学科的杂志。

2. 纯数学杂志的类型

(1) 综合数学杂志,如美国的《美国数学会会报》(Proceeding of American Mathematical Society);(2) 仅刊登某些数学分支的杂志,如美国《微分方程杂志》(Journal of Differential Equations)只刊登常微分方程、偏微分方程、泛函微分方程以及与之相关的积分方程、变分法的研究论文;(3) 独篇杂志,如美国的《美国数学会文集》(Memoirs of the American Mathematical Society),每期只登一篇数学论文;(4) 翻译杂志,如英国出版的《苏联近代数学分析杂志》(Soviet Journal of Contemporary Mathematical Analysis)便是有选择地翻译、另行编辑的刊物。

国外的主要数学杂志可参见附件5。

## §5.2 英语数学文献的著录(编排)格式

数学论文、专著等科研成果必须在正文之后给出该成果中引用过的所有重要文献的有关信息,并在参考文献(References)的标题下依序按照一定的格式列出(参见§4.1.8)。这种格式称为"文后参考文献著录格式"[14](以下简称为"著录格式")或"文后参考文献表编排格式"[15]。

一般说来,国外各出版机构规定的著录格式大体上是统一的,但也有细微差别,而国内与国外出版机构要求的格式差别显著;在同一个出版机构的规定中,不同类型的文献,比如专著、论文集中的论文与杂志上的论文等的著录格式也有明显的差异。投稿时,作者应根据出版机构(这里指用稿单位编辑部与出版社

等)规定的格式来书写参考文献,这种格式规定可在该出版机构的有关的出版物(包括网页上)查到(参见第四章附录2《来稿须知》)。1987~1998年,国内多数出版机构采用国家标准局(以下简称为GB)1987年发布的《文后参考文献著录规则》[14];1999年1月,我国新闻出版署印发了《中国学术期刊(光盘版)检索与评价数据规范》(以下简称为CAJ-CD)[15],对GB规定的格式做了修订。目前国内大多数学术期刊(包括所有大学学报)采用CAJ-CD的格式。

一般读者了解著录格式的主要目的是:一、写作;二、查找文献;三、交流的需要。这里主要根据写作英语数学论文和投稿的需要来详细说明著录格式。

### 5.2.1 杂志上论文的著录格式

描述一篇发表在杂志上的论文通常按如下次序列出各项主要信息:作者姓名、论文题目、杂志名称、卷号、出版年代和起止页码。必要时还要列出期号。不同出版机构的要求稍有不同,如有的规定略去"论文题目"这一项。

1. 目前国外多数出版机构规定的著录格式为:

作者姓名,论文题目,杂志名称 卷号,出版年,起止页码.

例如:

[1] E. S. Noussair, On the existence of solutions of nonlinear elliptic boundary problems, J. diff. Eqns. **34**(1999), 482-495.

[2] H. Berestycki, P. L. Lions, Nonlinear scalar field equations, Arch. Rat. Mech. Anal. **82**, No. 3(1998), 313-345.

下面做进一步说明。

(1) 这一格式中除了杂志名称和卷号之间只用空格分开外(注意,在这两例中,卷号之前的句点是杂志名缩写的要求),各项信息之间以逗号或圆括号来分开,最后用句点表示结束。

(2) 格式中方括号[ ]内通常应填入所摘文献的序号,这里假定序号为1和2。国际上大多数杂志统一规定用方括号,不能随便更改。另外,有的出版机构允许参考文献不按阿拉伯数字排序,而用作者姓名的缩写按字典次序编排。但不论如何排序,此处每个文献的序号应与正文的标引序号一致。

(3) 姓名的书写顺序有两种:一是姓在前,名在后,与多数中国人的习惯相同,国外20世纪90年代以前较流行。二是名在前,姓在后,近年来欧美的杂志较通用。这两篇论文的引用采用了第二种书写顺序。[1]中作者的姓为Noussair,名字的缩写为E. S.。在这里,名字的缩写是不可少的。这种顺序与论文或著作中行文的引用正好一致,但行文中在不引起混淆时,可略去名字而仅写出作者的姓Noussair。倘若一篇论文的作者超过一个,则依此列出作者姓名,作者之间用逗号隔开,见[2]。当作者只有两人时,作者之间可用 and 或 & 连接,代替

逗号。

(4) 在论文题目之后排列的是杂志名称的缩写。要强调的是,杂志名称一定要缩写。为了查阅方便,本书在附录4汇集了英文刊名常用词及国际省略法供初学者参照。

(5) 杂志名称之后(空一格,不加逗号)用粗体阿拉伯数字写出卷号;之后是出版年,为了明显起见,并避免与前后的数据相混,一般要求年份加圆括号,其前面不加标点。有的出版机构规定,当年份后接起止页码时,如文献[1],圆括号后不加逗号,直接写起止页码。

(6) 如杂志是按卷编页的,则期号可以不出现;如杂志是按期编页的,则期号是必须的。加上"期号"时,如要指明系该卷的第三期,可用 No.3 放在出版年前或后,也可紧接在表出版年的圆括号之后写上3。例如,文献[2]中的", No. 3(1998)"可能根据出版机构的要求换成"(1998),No.3"或"(1998)3"。

(7) 最后一项信息是起止页码,其作用主要是便于读者查阅,顺便估计篇幅,对购置、复印单行本估价等都有好处。书写时格式是"起始页码-终止页码"(用连字符连接)。

(8) 结束时一般应加句点,另有规定除外。

2. 我国 GB 1987 年发布的著录格式为[14]:

作者姓名.论文题目.杂志名称,出版年,卷号(期号):起止页码

例如,按 GB 规定,上面列出的文献[2]要改写为:

[2] Berestycki H, Lions P L. Nonlinear scalar field equations, Arch. Rat. Mech. Anal., 1998,82(3):313-345

我国 CAJ-CD 采用的著录格式为:

作者姓名.论文题目.杂志名称[J],出版年,卷号(期号):起止页码.

例如,按 CAJ-CD 的规定,上面列出的文献[2]要改写为:

[2] Berestycki H, Lions P L. Nonlinear scalar field equations [J]. Arch. Rat. Mech. Anal.,1998, 82 (3):313-345.

与国外的例子相比,可发现 GB 的要求有许多不同之处。例如:(1) 姓在前,名在后;姓之后不加逗号,(2) 省略的名之后也不加句点,(3) 作者列举完毕后要加句点,(4) 出版年代不加圆括号,排在卷号之前,(5) 期号加圆括号紧接在卷号之后,(6) 结束时不加句点。

与 GB 的例子相比,可发现 CAJ-CD 采用的格式比 GB 的格式多了两个要求:(1) 每一条文献结束时要加句点;(2) 要加杂志的文献类型的标识符为 J,并放在方括号中(若论文发表在论文集上,则必须在论文名称后加标识符[A],所在论文集名称后加标识符[C](见§5.2.4))。

**注** 国内许多杂志是按期编页的,这时一定要列出期数。

### 5.2.2 有关杂志名称著录的几个问题

（1）缩写格式

通常应当根据国际上公认的"ISO"建议，按语法上的要求缩写。对一些著名的老杂志，则可以用名称中各名词第一个字母的大写拼成。如前苏联的《应用数学和力学》，原名称为"Прикладная Математика И Механика"可以缩写为"Прик. Матем. И Механ."或者"ПММ"。

（2）带副名的期刊

带副名的期刊在正名与副名之间用分号"；"连接，二者一定要同时列出。如印度的杂志"Mathematica; Journal of Meerut University Mathematical Society"（《数学；密拉特大学数学会杂志》）。

（3）带附刊的杂志

如荷兰出版的"Mathematical Programming, with Mathematical Programing Studies"（《数学规划，附数学规划研究》）。这里刊名"数学规划"与附刊名"数学规划研究"均应列出，中间用"，"分开。

### 5.2.3 专著和其他图书的著录格式

专著和其他图书的著录格式与杂志略有差别。

1. 国外多数出版机构规定的著录格式为：

作者姓名，书名或其他文献标题，出版者，出版地，出版年.

其中书名必须全文列出，图书和其他文献的出版者指的是出版社、出版公司或某大学、某研究机构等等，即未必为出版社。地点一般列出城市即可。必要时也可在出版年之后加逗号，增列起止页码。每一条文献的末尾加不加标点也根据出版机构的要求而定。例如：

[3] R. Bellman & K. L. Cooke, Differential-Difference Equations, Academic Press, New York, 1963.

2. CAJ-CD 采用的著录格式为：

作者姓名. 书名或其他文献标题[文献标识符]. 出版地：出版者，出版年. 起止页码（任选）.

被引用的书名之后加文献标识符[M]；被引用的学位论文名之后加文献标识符[D]，报告之后加文献标识符[R]。注意先写出版地，加冒号后接出版者；其余要求参照§5.2.1之2。例如，

[4] Doob J L. Classical potential theory and its probabilistic counterpart [M]. Berlin: Springer, 1983.

**注1** "（任选）"放在格式中某一项（这里是"起止页码"）之后表示该项内

容可写出或不写出。

**注 2**　对专著和其他图书的著录格式和以下各项格式,GB 的格式中均不出现文献标识符[M],文献的末尾不加标点;GB 对报告和学位论文的规定稍有不同。详见书末参考文献[14]。

### 5.2.4　丛书、论文集及其中的论文的著录格式

1. 国外多数出版机构规定的丛书著录格式为:

作者姓名,书名,丛书名称,卷号,出版者,出版地,年代,起止页码(任选).

这一格式就是在图书著录格式的基础上增加了丛书名称和卷号这两栏。例如:

[5] J. E. Marsden & M. F. Mac Cracken, *The Hopf Bifurcation and Its Applications*, Appl. Math. Sci., Vol. 19, Springer-Verlag, New York, 1976.

其中"Appl. Math. Sci."是丛书名称(应用数理科学)的缩写。其中书名排成斜体只是为了明显区别于前后项的内容,有的出版机构不作要求。

论文集的种类很多。著录论文集的格式与图书的著录格式基本一样,只是把书的作者改为论文集的编者(主编),把书名改为论文集的名称。例如:

[7] M. Kishi, *Proceedings of the International Conference on Potential Theory 1990, Nagoya*, Walter de Gruyter, Berlin, 1992.(其中论文集的名称为:"1990 年名古屋国际位势论会议论文集",除了指明会议的主题、范围外,还列出开会时间、地点)。论文集的名称还可用 Proceedings of the International Conference on Potential Theory 1990, Nagoya 作为副标题而另加一个主标题(比如:Potential Theory);若有主标题,著录时也可以只写出主标题。例如:

[7] M. Kishi, *Potential theory*, Walter de Gruyter, Berlin, 1992.

在涉及论文集的引用中,更多的是引用其中的论文(称为**析出文献**)。论文集中的析出文献的著录格式可与丛书的著录格式相类比,其主要改变是:把书名改为论文题目,丛书名称改为论文集名称,论文集名称之前加"in:"表示所引论文在这本论文集之中。例如:

[8] J. Wu, Q. Gao, *On potential extension and capacity on harmonic spaces*, in: Potential theory, Walter de Gruyter, Berlin, 1992, 361-366.

必要时还要列出论文集的编辑(主编)。如上例 Potential theory 的主编是 M. Kishi,应列在论文集名称之前,改为:

[8] J. Wu, Q. Gao, *On potential extension and capacity on harmonic spaces*, in M. Kishi, Potential theory, Walter de Gruyter, Berlin, 1992, 361-366.

2. CAJ-CD 采用的著录格式为:丛书和论文集著录格式**可按图书处理**;著录论文集中的析出文献的格式为:

论文作者. 论文标题[A]. 论文集主编(任选). 论文集名称[C]. 出版地:出版者,出版年. 论文起止页码.

其中[A]是析出文献的标识符,[C]是论文集的标识符。作者姓名等书写格式参照§5.2.1和§5.2.1的国内部分。论文集主编可以省略。上一文献按国内要求应改为(注意标点符号的变化,in 改为 In):

[6] Wu J, Gao Q. *On potential extension and capacity on harmonic spaces* [A]. In: Kishi M. Potential theory [C]. Berlin: Walter de Gruyter 1992, 361-366.

### 5.2.5　电子文献著录格式

目前,电子文献日益增加。国外许多杂志都设有电子版。

CAJ-CD 规定的电子文献的著录格式为:

作者姓名. 电子文献题名,电子文献及载体类型标识. 电子文献的出处或可获得地址,发表或更新日期/引用日期(任选).

例如

[7] Wang M. On Lipshitz-Zhang problem in metric measure spaces [EB/OL]. http://www.cajcd.edu.cn/pub/wml.tex/980810-2html, 2002-8-18.

其中[EB/OL]是电子文献及载体类型标识。

国外的著录格式相仿,只要做相应修改。此处不再赘述。

## §5.3　英语数学文摘杂志

数学文摘型杂志对数学科研工作极其重要,而其中美国的《数学评论》较为完整且多数读者比较熟悉英文,所以这一节对《数学评论》作系统介绍,并讲述其使用方法。

### 5.3.1　《数学评论》概况

《数学评论》(Mathematical Reviews)由美国数学会出版(美国数学会设在罗得岛州普罗维登斯市的布朗大学),但撰写评论条目的评论员则由该刊编辑部直接聘请,来自世界各国。每一个条目的最后附有评论员姓名及其所在的城市。

这个刊物为月刊,原来每6期为一卷。但近年来每期厚度大大增加,实际上变成1期订为一本。每年年底还有作者索引、内容索引各一册。原刊每份年订价高达1200多美元,是少有的一种高价刊物。自1940年创刊以来,开本、格式和篇幅变动较大。每一期的首页都重复印出目录表,此目录表由《数学评论》与德国的《数学及其边缘学科文摘》编辑部共同编订,它是在《数学评论》原来使用

的分类表的基础上于1980年编订的"Mathematics Subject Classification"(数学主题分类)的总目(1985,1990和2000年均做修订)。

目录表中把数学及其边缘学科划为11类,每类的项目多少不等,总共有60个项目。这里所谓项目即为数学的一个分支。

所有项目统一用两位数码编号。第1个数码表示相关的各项目,可能为11类中的1类,也可能跨越邻近的一至两类。第2个数码表示类中的项目序号。目录表中的最后15项是计算机科学、力学、生物数学、经济数学等边缘学科的文摘。现将目录表翻译如下,其中用横线把每一类分开。

**目录表(TABLE OF CONTENTS)**

00 General(通论)

01 History and biography (数学史与传记)

---

03 Mathematical logic and foundations(数理逻辑与数学基础)

04 Set theory(集合论)

05 Combinatorics(组合学)

06 Order, lattices, ordered algebraic structures(序,格,有序代数结构)

08 General mathematical systems(一般数学系统)

---

11 Number theory(数论)

12 Field theory and polynomials(域论与多项式)

13 Commutative rings and algebras(交换环与交换代数)

14 Algebraic geometry(代数几何)

15 Linear and multilinear algebra, matrix theory(线性代数与多重线性代数,矩阵论)

16 Associative rings and algebras(结合环与结合代数)

17 Nonassociative rings and algebras(非结合环与非结合代数)

18 Category theory, homological algebra(范畴论,同调代数)

---

20 Group theory and generalization(群论及其推广)

22 Topological groups, Lie groups(拓扑群,李群)

26 Real functions(实函数)

28 Measure and integration(测度与积分)

30 Functions of a complex variable(单复变函数)

31 Potential theory(位势论)

32 Several complex variables and analytic space(多复变与解析空间)
33 Special functions(特殊函数)
34 Ordinary differential equations(常微分方程)
35 Partial differential equations(偏微分方程)
39 Finite differences and functional equations(有限差分方程与泛函方程)

---

40 Sequences, series, summability(序列,级数,可求和性)
41 Approximations and expansions(逼近与展开)
42 Fourier analysis(傅里叶分析)
43 Abstract harmonic analysis(抽象调和分析)
44 Integral transforms, operational calculus(积分变换,算子演算)
45 Integral equations(积分方程)
46 Functional analysis(泛函分析)
47 Operator theory(算子理论)
49 Calculus of variations and optimal control, optimization(变分法与最优控制,最优化)

---

51 Geometry(几何)
52 Convex sets and related geometric topics(凸集与有关几何命题)
53 Differential geometry(微分几何)
54 General topology(一般拓扑)
55 Algebraic topology(代数拓扑)
57 Manifolds and cell complexes(流形与胞腔复形)
58 Global analysis, analysis on manifolds(大范围分析,流形上的分析)

---

60 Probability theory and stochastic processes(概率论与随机过程)
62 Statistics(统计)
65 Numerical analysis(数值分析)
68 Computer science(计算机科学)

---

70 Mechanics of particles and systems(质点力学与系统)
73 Mechanics of solids(固体力学)
76 Fluid mechanics(流体力学)
78 Optics, electromagnetic theory(光学,电磁理论)

---

| | |
|---|---|
| 80 | Classical thermodynamics, heat transfer(经典热力学,热传导) |
| 81 | Quantum mechanics(量子力学) |
| 82 | Statistical physics, structure of matter(统计物理,物质结构) |
| 83 | Relativity(相对论) |
| 85 | Astronomy and astrophysics(天文学与天体物理学) |
| 86 | Geophysics(地球物理学) |
| 90 | Economics, operations research, programming, games(经济学,运筹学,规划,对策论) |
| 92 | Biology and behavioral sciences(生物科学与行为科学) |
| 93 | Systems theory, control(系统理论,控制) |
| 94 | Information and communication, circuits(信息与通讯,电路) |

其中序号编码空缺部分是原来编辑是有意跳过的,并非疏忽失误。需要知道"数学主题分类"细节的读者,可以参看 2000 年《数学评论》的"数学主题分类"。

每一个项目之下还分若干个子项目,而子项目之下又有细目(更细的子项目)。每一个细条目用一个五位的字符串表示,其前两位是数字,代表项目序号,如前面已经指出的那样,"34"指常微分方程这个项目;第三位是一个大写英文字母或连字符"-",表示子项目;第四、五位都是数字,用以表示细目,其具体含义由下面的例子说明。

34-00 表示常微分方程手册、词典及有关工具书;

34-01 表示常微分方程课本、基础释文;

……

34Axx 表示常微分方程一般理论(即 A 表示一般理论),其中 xx 将表以两位数字编号,用来表明这种一般理论中的更细的研究方面。如 34A15 则表示常微分方程一般理论中"初值问题,解的延拓"两个方面。又如 34Dxx 表示常微分方程稳定性理论,而 34D20 便表示"李雅普诺夫稳定性"。

## 5.3.2 《数学评论》条目简介

这里指的是条目格式。由于摘引的内容不同,格式也略有区别,我们择要指出几点,见下表:

条目序号 主题分类号

作者姓名

论文题或书名 所用文字(非英语),外文提要。

杂志名称,卷,年代,期,页数。

内容摘评……

摘评者姓名(地址)

其中各项说明如下:**作者姓名**不论人数多少一律姓在前,名在后。**条目序号**历年来略有变动。例如 85m:34006 表示 1985 年,m 是第 12 个英文字母,这里表示第 12 期(类似地 a,b,…分别表示第 1 期,第 2 期,……),34 是常微分方程,006 表示本期常微分方程的第 6 篇摘评。**论文题目或书名**一律译为英文。如论文不是用英文发表,则用圆括号标明原文是用什么文字发表的,接着注明该文用什么文字附上摘要。**内容摘评**可以是转载其他文摘,翻译非英文文摘,也可以由该刊在世界各地的评论员撰写。摘评者**姓名地址**这一栏的地址通常只标明城市。如为转载或翻译的文摘,则改为“译自……”(或“转载自……”)。

特别要指出,现在摘评中的杂志名称都用原文(非英文)的拉丁拼音书写再缩写,所以要特别小心。

## §5.4 上网查阅数学文献和有关信息

### 5.4.1 上网查阅文摘杂志和国外出版的数学杂志

(1) 上网查阅文摘杂志

随着计算机和网络的迅速发展,许多杂志都增加了网络版。美国《数学评论》和德国《数学及其边缘学科文摘》都有网络版,可通过上网(交费)查阅,既方便又及时。上网查美国《数学评论》,应进入 MathSciNet 数据库;查德国《数学及其边缘学科文摘》,应进入 Zentralblatt Math.

以下是在网上查到的《数学评论》对吴炯圻所发表的一篇论文的评论。

**MR2317493 (2008d:35071)** 35J60 (35B40 35B45)

**Wu, Jiong Qi** (PRC-ZNU)

**Bounded positive entire solutions of singular quasilinear elliptic equations.**

**(English summary)**

*J. Differential Equations* **235** (2007), *no.* 2, 510–526.

The author investigates the equation

(1) $$\Delta u+f(x,u,\nabla u)\,u^{-\beta}=0$$

in $\mathbb{R}^N$, $N>2$, whenever $\beta \geqslant 0$, $f$ is locally Hölder continuous and such that

(2) $$|f(x,u,p)|\leqslant C(1+|p|^2)$$

for all $x\in \overline{D}$, $D$ is a bounded subset of $\mathbb{R}^N$, $0<u\leqslant M$ and $p\in\mathbb{R}^N$. In particular, under further conditions on $f$, it is proved that if $\beta\in[0,1)$, then, equation (1) admits at least one positive entire solution satisfying either

(3) $$u(x)\geqslant \varepsilon\varphi(|x|),\varepsilon>0,\ \lim_{|x|\to\infty}u(x)=0,$$

or

(4) $$\varepsilon\varphi(|x|)\leqslant u(x)\leqslant \varepsilon^{-1}\varphi(|x|),\ 0<\varepsilon<1,$$

in $\mathbb{R}^N$, where

$$\varphi(|x|)=\min\{1,|x|^{2-N}\}.$$

Finally, the author proves sufficient conditions under which, for every $\beta\geqslant 0$, the equation (1) admits infinitely many positive entire bounded solutions $u$ such that each solution $u$ has a positive lower bound.

Reviewed by *Roberta Filippucci*

(2) 查找一般的数学期刊

通过出版社的网页都可以查到该社出版的数学图书与期刊杂志。

德国的 Springer 出版社(网址为 http://www.springerpub.com/)和 Kluwer 出版社(Kluwer Academic Publishers,北美总部网址为 http://www.wkap.com/,荷兰总部网址为 http://www.wkap.nl/)都是著名的出版商,多年来出版了大量数学图书资料与期刊杂志。

号称世界上最大的出版商 Elsevier 是一家历史悠久的跨国科学出版公司,其出版的期刊是世界公认的高品位学术期刊,且大多数为核心期刊。该公司在网上提供了 ScienceDirect 资料库,内容涉及数学、物理、化学、生命科学,覆盖 24 个学科领域的 1800 多种电子期刊。

如果要查 Elsevier 出版的图书资料,可输入网址 http://www.elsevier.com/ 进入 Elsevier 网页。如要查杂志,再进入 ScienceDirect 资料库,从中找到我们所需要的杂志。进入该杂志的网页后,可以查到现刊和近年来各期的目录、每篇论文的摘要、全文和相关文献。除了查阅或下载全文需要输入用户名和密码(适用于已通过有关渠道注册者),其余资料都可免费自由查阅。

目前许多网络公司,如 google、yahoo、百度等都提供免费搜索工具。读者也

可通过这些搜索工具先查找 ScienceDirect，从显示的网址 ScienceDirect-home 进入，逐步达到所要查的杂志的网页。

如果只查一种杂志，不论是否知道其出版商，都可通过网络搜索工具，在查找栏(search)中输入杂志的名称，如输入 Journal of Mathematical Analysis and Applications，网上会弹出查找的结果，显示 Journal of Mathematical Analysis and Applications 或 Journal of Mathematical Analysis and Applications-elsevier(2002 年以后归入 elsevier)这一条目，点击该处就可以进入杂志的网页。

网络上有些杂志是可以免费阅读的，特别是北欧一些国家出版的杂志就是如此。例如，芬兰科学院出版的《Annales Academiæ Scientiarum Fennicæ—Mathematica》是 SCI 刊源杂志，可按上述最后一种办法通过网络搜索工具查找该杂志的网页，进入后可以免费阅读和下载。

(3) 利用大学或科研机构的网络系统查找数学资料

国内多所大学或科研机构的网络系统都通过向资源库缴费的方式，为读者提供查找图书资料的方便。特别是重点大学图书馆提供的网上资源更丰富，例如，国内外享有盛名的南开大学数学研究院就是一个典型。读者可通过

http://www.math.nankai.edu.cn/

进入南开大学数学研究院的网站，然后通过点击图标就可进入其中的"数学图书馆"。这时可看到，馆内的"网络资源"包括全国期刊联合目录、MathSciNet 数据库、Zentralblatt Math、Spinger Link 镜像、Kluwer Online 期刊、数学信息镜像系统等 12 种资料库。通过点击这些窗口就能直接进入相应的资源库。其中有部分资料可以自由阅读。不过，多数资料，特别是较有学术价值的资料，必须先到该院有关部门注册才能查阅。

## 5.4.2 上网查阅科学引文索引

1960 年以来，美国科学信息研究所(Institute for Scientific Information，简称 ISI)创建了系列引文索引资料库，现已归 Thomson Reuters 所有。科学引文索引(Science Citation Index，网址：http://www.isinet.com)是 ISI 中最重要的一种资料库，被公认为世界范围最权威的科学技术文献的索引工具，能够提供科学技术领域所有重要的研究成果。SCI 引文检索的体系更是独一无二，不仅可以从文献引证的角度评估文章的学术价值，还可以迅速方便地组建研究课题的参考文献网络。发表的学术论文被 SCI 收录或引用的数量，已被世界上许多大学作为评价学术水平的一个重要标准。SCI 也是检索最新数学文献和有关信息的一个有效工具。

此外，ISI 每年还出版 JCR(《期刊引用报告》，全称 Journal Citation Reports)。JCR 对包括 SCI 收录的 3500 种期刊在内的 4700 种期刊之间的引用和被引用数

据进行统计、运算，并对每种期刊影响因子（Impact Factor）等指数加以报道。一种期刊的影响因子，指的是该刊前两年发表的文献在当前年的平均被引用次数。一种刊物的影响因子越高，也即其刊载的文献被引用率越高，一方面说明这些文献报道的研究成果影响力大，另一方面也反映该刊物的学术水平高。因此，JCR以其大量的期刊统计数据及计算的影响因子等指数，而成为一种期刊评价工具。图书馆可根据 JCR 提供的数据制定期刊引进政策；论文作者可根据期刊的影响因子排名决定投稿方向。

目前，清华大学图书馆设立了 SCI 咨询中心，其工作内容主要包括：指导教师、研究生进行 SCI 收录期刊论文及其引用的检索，按学科发布 SCI 收录期刊的影响因子等信息，对师生在 SCI 收录的期刊上发表文章提供咨询；为学校科学技术处提供当月 SCI 网络版收录国内著名高校论文的统计信息；发布 SCI、Ei 所收录国内外期刊的信息，发布国内中文核心期刊的相关信息等。

有关查找 SCI Web 版、Ei、ISTP、JCR 的问题，都可以从清华大学 SCI 咨询中心得到有效的解答。有兴趣的读者可以进入下面网站作进一步了解：

http://www.lib.tsinghua.edu.cn/service/SCIcenter/SCIcenter.html。

# 附录 5　数学期刊常用英语词汇与略语表

为了帮助读者迅速掌握英文期刊缩写和其他各种附注，这里有选择地列出有关英文词汇的略词表（采用 ISO 建议）。关于其他语言的情形可参看[2]。应该指出的是，传统的英文缩写词后要加点号，但现代英语常省略这一点。如“Mr.”变成“Mr”。

## 附 5.1　英文刊名常用词及国际省略法

| | | |
|---|---|---|
| Abstracts | 文摘 | (Abstr.) |
| Acta | 学报 | (Acta) |
| Advances | 进展 | (Advanc.) |
| Annales | 纪事，年刊 | (Ann.) |
| Annual | 年刊 | (Ann.) |
| Annual Review | 年评 | (Ann. Rev.) |
| Annual Reports | 年报 | (Ann. Rep.) |
| Archives | 文献集 | (Arch.) |
| Bibliography | 参考文献目录，图书目录 | (Bibliogr.) |
| Biography | 传记 | (Biogr.) |
| Bulletin | 通报，公报 | (Bull.) |
| Chronicle | 纪事(报) | (Chron.) |

| | | |
|---|---|---|
| Communications | 通告,通信,快报 | (Commun.) |
| Contributions | 论文集 | (Contr.) |
| Courier | 快报 | (Cour.) |
| Digest | 辑要,文摘 | (Dig.) |
| Gazette | 报,公报 | (Gaz.) |
| Journal | 杂志,会志 | (J.) |
| Magazine | 杂志 | (Mag.) |
| Memoirs | 会志,纪要,研究报告 | (Mem.) |
| Proceedings | 会议录,会报 | (Proc.) |
| Progress (in) | 进展 | (Prog.) |
| Record | 记事,记录 | (Rec.) |
| Report | 报告,报道 | (Rep.) |
| Review | 评论 | (Rev.) |
| Symposium | 论文集,论丛,讨论会 | (Symp.) |
| Transactions | 汇刊,汇报,会议录 | (Tran.) |

下面几个常用词不省略:

| | |
|---|---|
| Current | 近期 |
| Index | 索引 |
| Letter | 快报 |
| Note | 札记 |
| Press | 出版社,报 |
| Survey | 概览,综述 |

## 附 5.2 英文有关词汇略语

| | | |
|---|---|---|
| Abstracts | 文摘 | (Abstr.) |
| Acta | 学报 | (Acta) |
| Advances | 进展 | (Advanc.) |
| Annales | 纪事,年刊 | (Ann.) |
| Annual | 年刊 | (Ann.) |
| Annual Review | 年评 | (Ann. Rev.) |
| Annual Reports | 年报 | (Ann. Rep.) |
| Archives | 文献集 | (Arch.) |
| Bibliography | 参考文献目录,图书目录 | (Bibliogr.) |
| Biography | 传记 | (Biogr.) |
| Bulletin | 通报,公报 | (Bull.) |
| Chronicle | 纪事(报) | (Chron.) |
| Communications | 通告,通信,快报 | (Commun.) |
| Contributions | 论文集 | (Contr.) |

| | | |
|---|---|---|
| Courier | 快报 | (Cour.) |
| Digest | 辑要,文摘 | (Dig.) |
| Gazette | 报,公报 | (Gaz.) |
| Abridged | 删节 | (Abr.) |
| Addenda | 补遗 | (Add.) |
| Address | 地址 | (Add.) |
| Appendix | 附录 | (App.) |
| Approximately | 大约 | (Approx.) |
| as it was | 原为 | (as.) |
| Association | 协会 | (Assn.) |
| Avenue | 路(街道) | (Ave.) |
| Cataloque | 目录 | (Cat. 或 Caltg.) |
| Company | 公司 | (Co.) |
| Compile | 编者,编辑 | (Comp.) |
| Editor | 编者,版 | (Ed.) |
| Edition | 版次 | (Ed.) |
| et cetera = and so on | 等等 | (etc.) |
| Except | 除……外 | (Exc.) |
| Example | 例如,例 | (Ex. 或 Exx.) |
| Including | 包括 | (Incl.) |
| Institute | 学会,研究所 | (Inst.) |
| International | 国际 | (Int.) |
| Irregular | 不定期 | (Irr.) |
| Member | 会员,成员 | (Mem.) |
| Monograph | 专题论文,单行本 | (Monog.) |
| Master of Science | 理科硕士 | (M.S.) |
| (Numbers) | 期,册 | (No(s)或 no) |
| Not Yet Published | 尚未出版 | (NYP) |
| Organization | 组织 | (Orgn.) |
| Docter of Philosophy | 哲学博士 | (Ph. D.) |
| Page, Pages | 页,页数 | (P. PP.) |
| Per, annum | 每年 | (P. a.) |
| Publication(s) | 出版物 | (Publ.) |
| Research | 研究,研究工作 | (Res.) |
| Research and Development | 研究与发展 | (R&D) |
| Royal Society | 皇家学会 | (RS) |
| Series | 辑 | (Ser.) |
| Semimonthly = twice monthly | 半月刊 | (S-M) |

| | | |
|---|---|---|
| Society | 学会 | (Soc.) |
| Street, Strasse | 街道 | (St. 或 Str.) |
| Supplement(s) | 附刊 | (Suppl.) |
| Title | 标题,书刊名称 | (Tit.) |
| Volume | 卷 | (Vol.) |
| Who's Who | 名人录,名人传略集 | |
| Year-book | 年刊、年撰 | (Y. b.) |
| 3 times a year | 每年 3 期 | (3/Yr.) |

## 附录 6 国外重要数学杂志①

Acta Arithmetica 算术学报[波兰]

Acta Mathematica 数学学报[瑞典]

Acta Mathematica Hungarica 数学学报[匈牙利]

Advances in Applied Probability 应用概率论进展[英国]

Advances in Mathematics 数学进展[美国]

American Journal of Mathematics 美国数学杂志[美国]

Annales Academae Scientiarum Fennicæ. Series A, I: Mathematica 芬兰科学院纪事 A 辑:数学杂志[芬兰]

Annales de l' Institut Fourier 傅里叶研究所纪事[法国]

Annali di Mathematica Pure ed Applicata 理论与应用数学纪事[意大利]

Annals of Mathematics 数学年刊[美国]

Annals of Probability 概率论年刊[美国]

Annals of Statistics 统计学纪事[美国]

Annals of the Institute of Statistical Mathematics 统计数学研究所年刊[日本]

Archive der Mathematik 数学文献[瑞士]

Archive for Rational Mechanics and Analysis 理论力学与分析文献[德国]

Asterisque 星[法国]

Bulletin de la Societe Mathematique de France 法国数学会通报[法国]

Bulletin of the American Mathematical Society (New Series) 美国数学会通报(新辑)[美国]

Bulletin of the London Mathematical Society 伦敦数学会通报[英国]

Canadian Journal of Mathematics 加拿大数学杂志[加拿大]

Commentarii Mathematici Helvetici 瑞士数学通讯[瑞士]

Communications in Algebra 代数通讯[美国]

Communications in Mathematical Physics 数学物理通讯[德国]

① 本附录摘自参考文献[12]。

Communications in Partial Differential Equations 偏微分方程通讯[美国]

Communications on Pure and Applied Mathematics 纯粹与应用数学通讯[美国]

Compositio Mathematica 数学论文集[荷兰]

Comptes Rendus de l', Academie des Sciences Series Ⅰ:Mathematique 法国科学院报告辑Ⅰ:数学[法国]

Computing 计算[奥地利]

Differential Equations 微分方程[美国]

Discrete Mathematics 离散数学[荷兰]

Duke Mathematical Journal 杜克数学杂志[美国]

Functional Analysis and Its Applications 泛函分析及其应用[美国]

Fundamenta Mathematicae 基础数学[波兰]

Illinois Journal of Mathematics 伊利诺数学杂志[美国]

Indiana University Mathematics Journal 印第安纳大学数学杂志[美国]

Information and Computation 信息与计算[美国]

Inventiones Mathematicae 数学创造[德国]

Israel Journal of Mathematics 以色列数学杂志[以色列]

Journal d' Analyse Mathematique 分析数学杂志[以色列]

Journal de Mathematiques Pures et Appliquees 纯粹与应用数学杂志[法国]

Journal fur die Reine und Angewandte Mathematik 理论与应用数学杂志[德国]

Journal of Algebra 代数杂志[美国]

Journal of Algorithms 算法杂志[美国]

Journal of Applied Probability 应用概率论杂志[英国]

Journal of Approximation Theory 近似理论杂志[美国]

Journal of Combinatorial Theory Series A 组合论杂志A辑[美国]

Journal of Combinatorial Theory Series B 组合论杂志B辑[美国]

Journal of Differential Equations 微分方程杂志[美国]

Journal of Differential Geometry 微分几何杂志[美国]

Journal of Functional Analysis 泛函分析杂志[美国]

Journal of Mathematical Analysis and Applications 数学分析与应用杂志[美国]

Journal of Multivariate Analysis 多元分析杂志[美国]

Journal of Number Theory 数论杂志[美国]

Journal of Pure and Applied Algebra 纯粹与应用代数杂志[荷兰]

Journal of Symbolic Logic 符号逻辑杂志[美国]

Journal of the London Mathematical Society 伦敦数学会杂志[英国]

Journal of the Mathematical Society of Japan 日本数学会杂志[日本]

Letters in Mathematical Physics 数学物理快报[荷兰]

Linear Algebra and Its Applications 线性代数及其应用[美国]

Manuscripta Mathematica 数学论文集[德国]

Mathematical Proceedings of the Cambridge Philosophical Society 剑桥哲学会数学汇刊[英国]

Mathematical Programming 数学规划[荷兰]

Mathematica Scandinavica 斯堪的纳维亚数学[丹麦]

Mathematics of Computation 计算数学[美国]

Mathematics of the USSR-Sbornik 苏联数学汇编[美国]

Mathematische Annalen 数学纪事[德国]

Mathematische Nachrichten 数学通讯[德国]

Mathematische Zeitschrift 数学杂志[德国]

Memoirs of the American Mathematical Society 美国数学会文集[美国]

Michigan Mathematical Journal 密西根数学杂志[美国]

Nagoya Mathematical Journal 名古屋数学杂志[日本]

Nonlinear Analyses: Theory Methods & Applications 非线性分析:理论方法与应用[英国]

Numerische Mathematik 数值数学[德国]

Pacific Journal of Mathematics 太平洋数学杂志[美国]

Proceedings of the American Mathematical Society 美国数学会会报[美国]

Proceedings of the London Mathematical Society 伦敦数学会会报[英国]

Proceedings of the Royal Society of Edinburgh Section A: Mathematics 爱丁堡皇家学会会报 A 辑:数学[英国]

Publications Mathematiques 数学丛刊[法国]

Publications of the Research Institute for Mathematical Sciences (京都大学)数学科学研究所纪要[日本]

Quarterly Journal of Mathematics 数学季刊[英国]

Quarterly of Applied Mathematics 应用数学季刊[美国]

Russian Mathematical Surveys 俄罗斯数学概观[英国]

SIAM Journal on Applied Mathematics 工业与应用数学会应用数学杂志[美国]

SIAM Journal on Computing 工业与应用数学会计算杂志[美国]

SIAM Journal on Control and Optimization 工业与应用数学会控制与最优化杂志[美国]

SIAM Journal on Mathematical Analysis 工业与应用数学会数学分析杂志[美国]

SIAM Journal on Numerical Analysis 工业与应用数学会数值分析杂志[美国]

SIAM Journal on Scientific and Statistical Computing 工业与应用数学会科学与统计计算杂志[美国]

SIAM Review 工业与应用数学会评论[美国]

Siberian Mathematical Journal 西伯利亚数学杂志[美国]

Stochastic Processes and Their Applications 随机过程及其应用[荷兰]

Studia Mathematica 数学研究[波兰]

Studies in Applied Mathematics 应用数学研究[美国]

Theory of Probability and Its Applications 概率论及其应用[美国]

Tohoku Mathematical Journal　东北数学杂志[日本]

Topology　拓扑学[英国]

Transactions of the American Mathematican Society　美国数学会汇刊[美国]

USSR Computational Mathematics and Mathematical Physics　苏联计算数学与数学物理[英国]

Zeitschrift fur Angewandte Mathematik und Mechanik　应用数学与力学杂志[德国]

# 第六章　数学文献常用英语词汇

本章列出英文版的数学文献(主要是本科与研究生教材)中常用的部分词汇,共约2200条,其中大多是数学术语(含专业术语和半专业术语),也包括了少量常见的一般动词、形容词及相关词组等。读者若能熟练掌握它们,将对学习本书第二、三章和阅读数学文献带来较大的方便,而且能为进一步的学习和深造奠定良好的基础。

## A

aposteriori [ˈeipɔsˌteriˈəurai]　adj. 归纳的,经验的

apriori [eipraiˈɔːrai]　先验的,演绎的

apriori distribution　先验分布

apriori error bounds　先验误差界

abac [ˈæbək]　n. 算图

abacus [ˈæbəkəs]　n. 算盘

abbreviation [əˌbriːviˈeiʃən]　n. 缩写,省略

Abelian function　阿贝尔函数

Abelian group　交换群,阿贝尔群

Abelian integral　阿贝尔积分

abound [əˈbaund]　v. 大量存在

abscissa [æbˈsisə]　n. 横坐标

absolute [ˈæbsəluːt]　adj. 绝对的

absolute address　绝对地址

absolute continuity　绝对连续性

absolute convergence　绝对收敛性

absolute error　绝对误差

absolute value　绝对值

absolute-value function　绝对值函数

absolutely summable series　绝对可和级数

absolutely unbiased estimator　绝对无偏估计量

abstract [ˈæbstrækt]　n. 抽象,摘要

abstract algebra　抽象代数学

abstract code　抽象码

abstract harmonic analysis　抽象调和分析

abstract integral　抽象积分

absurd [əbˈsəːd]　adj. 荒谬的

acceleration [ækˌseləˈreiʃən]　n. 加速度

accessible [əkˈsesəbl] adj. 可达的

accessible boundary point　可达边界点

accident error　偶然误差

accumulated error　累积误差

accuracy [ˈækjurəsi]　n. 准确性,精确度

acnode [ˈæknəud]　n. 孤立点

active constraint　起作用的约束

actual infinity　实无穷

acute angle　锐角

acute triangle　锐角三角形

acyclic set　非循环序集

adaptive optimization　自适应最优化

add [æd] v. 加
addend [ə'dend] n. 加数
adder ['ædə] n. 加法器
adder-subtracter n. 加减器
addition [ə'diʃən] n. 加,加法
additive ['æditiv] adj. 加法的,加性的
additive congruential method 加同余法
additive group 加法群
additive operation 加法运算
adherence [əd'hiərəns] n. 接触
adhesion [əd'hiːʒən] n. 附着
adjacent sides 邻边
adjoint [ə'dʒɔint] adj. 伴随的
adjoint determinant 伴随行列式
adjoint difference equation 伴随差分方程
adjoint differential equation 伴随微分方程
adjoint matrix 伴随矩阵
adjugate determinant 转置伴随行列式
adjugate matrix 转置伴随矩阵
admissible [əd'misəbl] adj. 容许的
admissible curve 容许曲线
admissible error 容许误差
admissible estimate 容许估计
admissible function 容许函数
affine [ə'fain] n. 仿射
affine connection 仿射联络
affine coordinate 仿射坐标
affine differential geometry 仿射微分几何
affine geometry 仿射几何
aggregate ['ægrigit] n. 集合
airfoil ['ɛəfɔil] n. 翼剖面
algebra ['ældʒibrə] n. 代数学,代数
algebra of logic 逻辑代数
algebra of matrices 矩阵代数
algebraic [ˌældʒi'breiik] adj. 代数的
algebraic adder 代数加法器
algebraic curve 代数曲线
algebraic function 代数函数
algebraic geometry 代数几何
algebraic plane curve 代数平面曲线
algebraical [ˌældʒi'breiikəl] adj. 代数学的,代数的
algebraical difference 代数差
algebraically closed 代数封闭
algorithm ['ælgəriðəm] n. 算法,规则系统
algorithmic language 算法语言
alignment chart 列线图,算图
aliquot part 整除部分
allege [ə'ledʒ] v. 宣称,断言
all-purpose computer 通用计算机
almost everywhere 几乎处处
almost everywhere convergent 几乎处处收敛的
almost periodic function 殆周期函数
alpha code 字母码
alphabet ['æːlfəbit] n. 字母表
alpha-numeric digit 字母-数字数码
alternate angle 交错角,外错角
alternate determinant 交错行列式
alternate series 交错级数
alternative [ɔːl'təːnətiv] adj. 互斥的,交错的
altitude ['æltitjuːd] n. 高度,高线
altitude of a triangle 三角形的高线
ambiguous [æm'bigjuəs] adj. 二义的,不清楚的
amicable number 亲和数
amount [ə'maunt] n. 总数,本利和
amount of information 信息量
amplify ['æmplifai] v. 扩大,放大
amplitude ['æmplitjuːd] n. 幅度,振幅
amplitude modulation 调幅
an ordered pair 一个有序对
analogue ['ænəlɔg] n. 模拟,类似物
analogy [ə'nælədʒi] n. 类似,类比

*analysis* [ə'nælisis] *n.* 分析
analysis of covariance 协方差分析
analysis of variance 方差分析
analytic curve 解析曲线
analytic function 解析函数
analytic geometry 解析几何
analytic sheaf 解析层
analytical [ˌænə'litikəl] adj. 分析的，解析的
analytical model 解析模型
analytically irreducible 解析不可约的
analytic extension 解析开拓
angle ['æŋgl] n. 角，角度
angular bisector 角平分线
angular domain 角域
angular velocity 角速度
*anharmonic ratio* 交比，非调和比
annihilating ideal 零化理想
annihilation operator 零化算子
annular domain 圆环域
annular region 环形区域
antecedent [ˌænti'siːdənt] n. 前项，前件
antidifferential [ˌæntidifə'renʃəl] adj. 反微分的
*anti-homomorphism* [ˌænti-hɔmə'mɔːfizm] n. 反同态
anti-sine [ˌænti-'sain] n. 反正弦
anti-symmetric [ˌænti-si'metrik] adj. 反对称的
antitone ['æntitəun] n. 反序
apex ['eipeks] n. 顶点
apolar conics 从配极二次曲线
applicable surface 可贴曲面
application [ˌæpli'keiʃən] n. 应用
applied mathematics 应用数学
appraisal [ə'preizəl] n. 估计
approach [ə'prəutʃ] v. 趋于，逼近；n. 接近，逼近
*approach zero* 趋于 0
approximate [ə'prɔksimeit] adj. 近似的
approximate calculation 近似计算
approximate error 近似误差
approximate evaluations 近似估计
approximate expansion 近似展开式
approximate value 近似值
approximation [əˌprɔksi'meiʃən] n. 近似，逼近
approximation by least squares 用最小二乘方的近似
approximation of 1st degree 一次近似
approximation of 2nd degree 二次近似
approximation of root 根的近似
Arabian cypher 阿拉伯数码
Arabic ['ærəbik] adj. 阿拉伯的
Arabic numeral 阿拉伯数字
arbitrary ['ɑːbitrəri] adj. 随意的，任意的
arc [ɑːk] n. 弧
arc length 弧长
arc of a graph 图的弧
arc-hyperbolic function 反双曲函数
arch [ɑːtʃ] n. 弓形
Archimedian valuation 阿基米德赋值
architect ['ɑːkitekt] n. 建筑师，设计师
arcsine [ˌɑːk'sain] n. 反正弦
arcwise connectedness 弧的连通性
area ['ɛəriə] n. 面积，区域
area of a curved surface 曲面面积
areal coordinates 重心坐标
argument ['ɑːgjumənt] n. 幅度，自变数，幅角
argument function 辐角函数，变函数
argument of a function 函数的自变量
arithmetic [ə'riθmətik] adj. 算术的；n. 算术
arithmetic continuum 算数连续统
arithmetic division 算术除法
*arithmetic expression* 算术表达式

arithmetic mean 算术中项,算术平均
arithmetic operation 算术运算
arithmetic progression 算术级数
arithmetical [ˌæriθ'metikəl] adj. 算术的,算术上的
arithmetical invariant 算术不变量
arrangement [ə'reindʒmənt] n. 排列
array [ə'rei] n. 数组
array identifier 数组标识符
array list 数组表
array of difference 差分格式
artificial intelligence 人工智能
ascending order 递增升序
assemblage [ə'semblidʒ] n. 集合
assembly program 汇编程序
assertion [ə'səːʃən] n. 断定,断言,论断
assign [ə'sain] v. 分配,指定
assignment statement 赋值语句
associate [ə'səuʃieit] adj. 相伴的 v. 使……联系,使与……关联
associated ideal 相伴理想
association [əˌsəuʃi'eiʃən] n. 结合
association scheme 结合方案
associative algebra 结合代数
associative law 结合律
associative law of addition 加法结合律
associative law of multiplication 乘法结合律
assume [ə'sjuːm] v. 假定,取(值)
assumption [ə'sʌmpʃən] n. 假定
asterisk ['æstərisk] n. 星号
asteroid ['æstərɔid] n. 星形线
astronomy [ə'strɔnəmi] n. 天文学
asymmetrical [ˌæsi'metrikəl] adj. 不对称的
asymmetry [æ'simətri] n. 不对称
asymptote ['æsimptəut] n. 渐近线
asymptotic [ˌæsimp'tɔtik] adj. 渐近的
asymptotic behavior 渐近状态
asymptotic curve 渐近曲线
asymptotic line 渐近线
asynchronous [ei'siŋkrənəs] adj. 非同步的
atomic event 原子事件
atomless set function 缺原子的集函数
attainability [əˌteinə'biləti] n. 到达性
attenuation [əˌtenju'eiʃən] n. 衰减
attraction [ə'trækʃən] n. 引力,吸引
attribute [ə'tribjuːt] n. 属性
augend ['ɔːdʒend] n. 被加数
auto coding 自动编码
autocorrelation [ˌɔːtəukɔri'leiʃən] n. 自相关
auto-covariance [ˌɔːtəukəu'vɛəriəns] n. 自协方差
automata ['ɔːtəmətə] n. 自动机
automatic coding 自动编码
automatic data processing 自动数据处理
automorphic function 自守函数
automorphism [ˌɔːtə'mɔːfizm] n. 自同构
autonomous differential equation 自治微分方程,自控微分方程
autonomous system 自治系统
auxiliary [ɔːg'ziljəri] adj. 辅助的
auxiliary circle 辅助圆
auxiliary problem 辅助问题
average ['ævəridʒ] n. 平均;adj. 平均的
average error 平均误差
average rate 平均变化率
average rate of convergence 平均收敛速率
average value 平均值
axial ['æksiəl] adj. 轴向的
axiom ['æksiəm] n. 公理
axiom of choice 选择公理
axiom of completeness 完备性公理
axiom of parallels 平行公理
axiomatic method 公理法
axiomatic set theory 公理化集合论
axis ['æksis] n. 轴
axis of 实轴 (=real axis)

axisymmetric [ˌæksisiˈmetrik] adj. 轴对称的

## B

balance equation 平衡方程
balayage method 扫除法
Banach algebra 巴拿赫代数
Banach space 巴拿赫空间
bar graph 条形图
barrier function 障碍函数
barycenter [ˈbæriˌsentə] n. 重心
barycentric coordinates 重心坐标
base [beis] n. 底,基
base angle 底角
base of a triangle 三角形的底
base of exponential function 指数函数的底
base vector 基向量
Bernoulli (人名)伯努利
Bessel-Clifford equation 贝塞尔-克利福德方程
Bessel's inequality 贝塞尔不等式
best approximation 最佳逼近
best estimator 最佳估计量
beta function β-函数
betti group 贝蒂群
biased [ˈbaiəst] adj. 有偏的
biased error 有偏误差
bicontinuous function 双连续函数
bicubic interpolation 双三次插值
bifurcation [ˌbaifəˈkeiʃən] n. 歧点
biharmonic equation 双调和方程
bijection [baiˈdʒekʃən] n. 双射,一一映射
bilinear [baiˈliniə] adj. 双线性的
billion [ˈbiljən] num. 十亿
binary [ˈbainəri] adj. 二进制的,二元的
binary arithmetic 二进制算术
binary digit 二进制数字
binary operation 二元运算
binary quadric form 二元二次形式
binary relation 二元关系
binary code 二进制代码
binomial correlation 二项相关
binomial equation 二项方程
biprojective space 双射影空间
biquadratic equation 双二次方程
biquadratic form 双二次形式
biquadratic interpolation 双二次插值
bisector [baiˈsektə] n. 等分线,平分线
bisymmetric [ˌbaisiˈmetrik] adj. 双对称的
bit [bit] n. 位
blank [blæŋk] adj. 空白的
block diagonal matrix 分块对角矩阵
block diagram 框图
block multiplication 分块相乘
block relaxation 块松弛
block search 分批试验法
body of rotation 旋转体
bondage [ˈbɔndidʒ] n. 约束
Boolean algebra 布尔代数
Boolean formula 布尔公式
Boolean variable 布尔变量
Borel set 博雷尔集
bound [baund] n. 界,限
boundary [ˈbaundəri] n. 界,边界
boundary point 边界点
boundary value problem 边值问题
bounded [ˈbaundid] adj. 有界的
bounded above 有上界的
bounded below 有下界的
bounded set 有界集
bounded variable 有界变量
bounded variation 有界变差
box [bɔks] n. 框,单元
brace [breis] n. 大括号
bracket [ˈbrækit] n. 括号,方括号

branch [braːntʃ] n. 分支
branching point 支点
breadth [bredθ] n. 宽度
breakpoint information 分割点信息
bridge [bridʒ] n. 桥,桥牌
bridging order 返回指令
broken line 折线
Brownian motion process 布朗运动过程
bundle space 丛空间
by induction on $n$ (用数学归纳法)对 $n$ 进行归纳
byproduct [baiˈprɔdəkt] n. 副产品

## C

calculable [ˈkælkjuləbl] adj. 可计算的
calculation [ˌkælkjuˈleiʃən] n. 计算
calculator [ˈkælkjuleitə] n. 计算器
calculus [ˈkælkjuləs] n. 微积分[学],演算
calculus of variation 变分法
call in 调入
call on 访问
cancel [ˈkænsəl] v. 取消,约简
canonical [kəˈnɔnikəl] adj. 典型的
canonical form 典范型,标准型
cap [kæp] n. 交,求交运算
cap product 卡积
capacity [kəˈpæsiti] n. 容量
capacity dimension 容量维数
cardinal [ˈkaːdinəl] adj. 基数的;n. 基数
cardinal number 基数
carry over to 使继续下去
carry [ˈkæri] n. 进位
Cartesian [kaːˈtiːzjən] adj. 笛卡儿的,卡氏的
Cartesian basis 笛卡儿基
Cartesian coordinates 笛卡儿坐标
Cartesian geometry 笛卡儿几何
Cartesian product 笛卡儿乘积[学]
cartography [kaːˈtɔgrəfi] n. 制图学
cascade carry 逐位进位
catalogue [ˈkætəlɔg] n. 目录
categorical syllogism 直言三段论
category [ˈkætigəri] n. 范畴,类型
catenary [kəˈtiːnəri] n. 悬连线
Cauchy inequality 柯西不等式
Cauchy-Schwarz inequality 柯西-施瓦茨不等式
center [ˈsentə] n. 中心,圆心
center focus 中心焦点
center of a circle 圆心
center of gravity 重心
center of inversion 反演中心
center of similarity 相似中心
center of sphere 球心
center of symmetry 对称中心
centered affine space 中心仿射空间
centered conic 有心二次曲线
centered difference 中心差分
central confidence interval 重心置信区间
central quadric 有心二次曲面
certain event 必然事件
chain [tʃein] n. 链
chain condition 链条件
chain homomorphism 链同态
chain rule 链式法则
chance move 随机游走,随机移动
change [tʃeindʒ] n. 变更,改变
change of scale 变换尺度
character [ˈkæriktə] n. 特征[标],字符
characteristic [ˌkæriktəˈristik] adj. 特征的
characteristic basic manifold 特征基流形
characteristic class 示性类
characteristic curve 特征曲线
characteristic root 特征根

characterisic value 本征值,特征值
characterisic vector 特征向量
chart [tʃɑːt] n. 表格,图表
check [tʃek] v. 校对
check formula 验算公式
chi square criterion $\chi^2$检验
Chinese remainder theorem 孙子剩余定理,中国剩余定理
chord [kɔːd] n. 弦
chord at contact 切点弦
circle [ˈsəːkl] n. 圆周,圆
circle at infinity 无穷远[虚]圆
circuit [ˈsəːkit] n. 环路
circular [ˈsəːkjulə] adj. 圆的,圆周的,圆形的
circular arc 圆弧
circular cone 圆锥
circular cylinder 圆柱
circular cylindrical coordinates 圆柱坐标
circular disc 圆盘
circular frequency 周频
circular points 虚圆点
circular region 圆域
circular ring 圆环
circulating decimal 循环小数
circumcircle [ˈsəːkəmˈsəːkl] n. 外接圆
*circumference* [səˈkʌmfərəns] n. 圆周,周长
circumradius [ˌsəːkəmˈreidiəs] n. 外接圆半径
circumscribe [ˈsəːkəmskraib] v. 在……周围画线,使外切
*circumscribed circle* 外接圆
circumscribed triangle 外切三角形
class [klɑːs] n. 类,族
classification [ˌklæsifiˈkeiʃən] n. 分类,分组
classification statistic 分类统计
clockwise sense 顺时针方向
close [kləuz] n. 封,闭
closed [kləuzd] adj. 闭的
closed aggregate 闭集 (=closed set)
closed curve 闭曲线
closed operator 闭算子
clothoid [ˈkləuθɔid] n. 回旋曲线
closure [ˈkləuʒə] n. 闭包,闭合
cluster point 聚点
cluster point of a sequence 序列的聚点
cluster set 聚值集
code [kəud] n. 码
code data 编码的数据
code machine 编码机
coefficient [ˌkəuiˈfiʃənt] n. 系数
coefficient of alienation 不相关系数
coefficient of autocorrelation 自相关系数
coefficient of regression 回归系数
coercive [kəuˈəːsiv] adj. 强制的
cofactor [kəuˈfæktə] n. 余因子,余子式
cofinal [kəuˈfainəl] adj. 共尾的
cofunction [kəuˈfʌŋkʃən] n. 余函数
cogradient *matrices* 同步矩阵,相合矩阵
coincide [ˌkəuinˈsaid] v. 一致,符合
coincident [kəuˈinsidənt] adj. 一致的,符合的
collection [kəˈlekʃən] n. 集合,聚集
collinear [kɔˈlinjə] adj. 共线的
collinear vectors 共线向量
collineation [ˌkəliniˈeiʃən] n. 直射,共线
collocation [ˌkɔləˈkeiʃən] n. 配置
collocation method 配置法
column [ˈkɔləm] n. 柱,列
column matrix 列矩阵
combination [ˌkɔmbiˈneiʃən] n. 组合
combinatorial analysis 组合分析
combinatorial topology 组合拓扑
combined method 组合方法
combining estimate of correlation 相关的合并估计

comentropy [kə'mentrəpi] n. 信息熵
command [kə'mɑ:nd] n. 指令
commensurable [kə'menʃərəbl] adj. 可公度的
comment ['kɔment] n./v. 注释,评论
common ['kɔmən] adj. 公共的,普通的
common denominator 公分母
common difference 公差
common factor 公因子
common multiple 公倍数
common ratio 公比
commonsense reasoning 常识推理
commutative [kə'mju:tətiv] adj. 可交换的
commutative algebra 交换代数
commutative group 交换群
commutative law 交换律
compact [kəm'pækt] adj. 紧的
compact set 紧集
comparable ['kɔmpərəbl] adj. 可比较的
comparative [kəm'pærətiv] adj. 比较的,相当的
compass ['kʌmpəs] n. 指南针,圆规
compatible [kəm'pætəbl] adj. 相容的
compatible event 相容的事件
compatibility [kəmˌpæti'biliti] n. 相容性
compiler [kəm'pailə] n. 编译程序,编译
complement ['kɔmplimənt] n. 余,补
complement form 补码形式
complementarity law 互余律
complementary [kəmˌplə'mentəri] adj. [互]余的,[互]补的
complementary angle 余角
complementary event 补事件
complementary space 余空间
complete [kəm'pli:t] adj. 完全的,完备的
complete axiom system 完备公理系统
complete induction 完全归纳法
completely additive 完全可加的
completely continuous operator 全连续算子
complex ['kɔmpleks] adj. 复的,复杂的
complex analysis 复分析
complex analytic curve 复解析曲线
complex conjugate 复共轭
complex integration 复积分
complex number n. 复数
complex plane 复平面
complexity [kəm'pleksiti] n. 复杂性
complex-valued sequence 复值序列
component [kəm'pəunənt] n. 成分,分量,分支
component ideal 支理想
component of a vector 向量的分量
composite ['kɔmpəzit] adj. 复合的
composite mapping 复合映射
composite matrix 合成矩阵
compound function 复合函数
computability [kəmˌpju:tə'biliti] n. 可计算性
computable [kəm'pju:təbl] adj. 可计算的
computation [ˌkɔmpju'teiʃən] n. 计算
computational efficiency 计算效率
computational instability 计算的不稳定性
computational method 计算方法
computer [kəm'pju:tə] n. 计算机
computer graphics 计算机制图
computer program 计算机程序
computer programming language 计算机程序语言
computerize [kəm'pju:təraiz] v. 计算机化
concave [kɔn'keiv] adj. 凹的
concave curve 凹曲线
concave up 凹向上
concavity [kɔn'kæviti] n. 凹性
concentric circles 同心圆

concept [ˈkɔnsept] n. 概念
conception [kənˈsepʃən] n. 概念
conclusion [kənˈkluːʒən] n. 结论
concurrent [kənˈkʌrənt] adj. 共点的
condensation test 并项检验法
condition [kənˈdiʃən] n. 条件
conditional distribution 条件分布
conditional probability density 条件概率密度
conditionally convergent 条件收敛
cone [kəun] n. 圆锥
confidence interval 置信区间
confidence level 置信水平
configuration [kənˌfigjuˈreiʃən] n. 构形，布局，格局，组态
confine [kənˈfain] v. 限制，限定
conformable matrices 可相乘矩阵
conformal [kɔnˈfɔːml] adj. 保角的，共形的
conformal geometry 保形几何，共形几何
conformal transformation 保形变换，共形变换
congruence [ˈkɔŋgruəns] n. 同余，全等
congruent [ˈkɔŋgruənt] adj. 同余的，全等的
congruent integer 同余整数
congruent to each other 互相全等
congruent transformation 全等变换
congruent triangle 全等三角形
conic [ˈkɔnik] n. 二次曲线
conic polar 极二次曲线
conic section 圆锥截线，二次曲线
conical [ˈkɔnikəl] adj. 圆锥的，圆锥形的
conicoid [ˈkɔnikɔid] n. 二次曲面
Conics [ˈkɔniks] n. 圆锥曲线论，锥线论
conjecture [kənˈdʒektʃə] n. 猜想，猜测
conjugate [ˈkɔndʒugit] n. 共轭；adj. 共轭的
conjugate compex number 共轭复数
conjugate conics 共轭圆锥曲线
conjunction [kənˈdʒʌŋkʃən] n. 合取
conjunction of propositions 命题的合取
conjunctive matrix 共轭矩阵
connect [kəˈnekt] v. 连接，联合，关连
connected [kəˈnektid] adj. 连通的
connected region 连通区域
connected space 连通空间
connection [kəˈnekʃən] n. 联络
connective [kəˈnektiv] n. 连结词，联接词；adj. 联接的
connectivity [kənekˈtiviti] n. 连通性
consecutive [kənˈsekjutiv] adj. 依次的，邻接的
consequence [ˈkɔnsikwəns] n. 推论
consequent [ˈkɔnsikwənt] n. 后项，后件
conservation [ˌkɔnsəːˈveiʃən] n. 守恒
conservation laws 守恒定律
consistence [kənˈsistəns] n. 相容，一致
consistency principle 一致原则
consistent [kənˈsistənt] adj. 相容的，一致的
constant [ˈkɔnstənt] n. 常数
constant coefficient 常系数
constant curvature 常曲率
constant input 常数输入
constant term 常数项
constructible [kənˈstrʌktəbl] adj. 可构成的
construction [kənˈstrʌkʃən] n. 构造
construction problem 作图题
constructive definition 构造性定义
consult [kənˈsʌlt] v. 请教，参考
contact [ˈkɔntækt] v. 接触
contain [tənˈtein] v. 包含
content [ˈkɔntent] n. 容度
content function 容度函数

content zero　零容度
context [ˈkɔntekst]　n. 前后关系,上下文
context-free grammar　上下文无关语法
continuable　adj. 可延拓的,可延伸的
continued equality　连等式
continued fraction　连分数
continued multiplication　连乘法
continuity from above　上连续
continuity from below　下连续
continuity in the mean　均方连续
continuous [kənˈtinjuəs]　adj. 连续的
continuous function　连续函数
continuous mapping　连续映射
continuum [kənˈtinjuəm]　n. 连续统
contour [ˈkɔntuə]　n. 周线,围道,回路
contravariant tensor　反变张量
contraction [kənˈtrækʃən]　n. 收缩,缩并
contraction mapping　压缩映射
contradiction [ˌkɔntrəˈdikʃən]　n. 矛盾
contradictory [kɔntrəˈdiktəri]　adj. 矛盾的
contrary [ˈkɔntrəri]　adj. 相反的,逆的
contrary proposition　相反命题
control [kənˈtrəul]　n. 控制
control circuit　控制线路
control theory　控制理论
converge [kənˈvəːdʒ]　v. 收敛
converge absolutely　绝对收敛
converge uniformly　一致收敛
convergence [kənˈvəːdʒəns]　n. 收敛
convergent [kənˈvəːdʒənt]　adj. 收敛的
convergent sequence　收敛序列
converse [kənˈvəːs]　n. 逆;adj. 逆的
conversely [ˈkɔnvəːsli]　adv. 反之,反过来
convex [ˈkɔnˈveks]　adj. 凸的
convexity [kɔnˈveksiti]　n. 凸性
convey [kənˈvei]　v. 表达,传递
coordinate [kəuˈɔːdinit]　n. 坐标
coordinate axis　坐标轴
coordinate system　坐标系
coplanar [kəuˈpleinə]　adj. 共面的
coprime [kəuˈpraim]　n. 互质,互素
corner [ˈkɔːnə]　n. 隅角
corollary [kəˈrɔləri]　n. 系,推论
correction [kəˈrekʃən]　n. 校正
correspond [ˌkɔrisˈpɔnd]　v. 对应
correspondence [ˌkɔrisˈpɔndəns]　n. 对应
corresponding [ˌkɔrisˈpɔndiŋ]　adj. 对应的,相应的
corresponding angles　同位角,对应角
corresponding point　对应点
cosine [kəuˈsain]　n. 余弦
cosine law　余弦定理
cotangent [ˈkəuˈtændʒənt]　n. 余切
count [kaunt]　v. 计数
countability [ˌkauntəˈbiləti]　n. 可数性
countable [ˈkauntəbl]　adj. 可计算的,可数的
countable base　可数基
countable infinite　可数无穷的
countable set　可数集
countably addtive　可数加性的
countably addtive measure　可数加性测度
counter [ˈkauntə]　n. 计数器
counter clockwise　反时针方向
counterexample [ˈkauntərigˈzɑːmpl]　n. 反例
counting principle　计数原理
couple [ˈkʌpl]　n. 偶,耦合
covariance [kəuˈvɛəriəns]　n. 协方差
covariant [kəuˈvɛəriənt]　adj. 共变的
covariant tensor　共变张量
cover [ˈkʌvə]　v. 覆盖
criterion [kraiˈtiəriən]　n. 准则,判别法
criterion chi square　$\chi^2$检验
critical [ˈkritikəl]　adj. 临界的,判定的
critical point　临界点
cross check　相互检验

cross correlation function 互相关函数
cross cut 交,正交
cross line 正交线
cross product 向量积,叉积
cross ratio 交比,非调和比
crossed products 交叉乘积
crossing off 删除
cryptography [krip'tɔgrəfi] n. 密码学
cube [kjuːb] n. 立方,立方体;v. 求立方
cube root 方立根
cubic ['kjuːbik] adj. 立方体的,立方的,三次的
cubic binary quantic 二元三次代数形式
cubic curve 三次曲线
cuboid ['kjuːbɔid] n. 长方体
cumulative error 累计误差
cup [kʌp] n. 求并运算
curl [kəːl] n. 旋度
curvature ['kəːvətʃə] n. 曲率
curvature of a curve 曲线的曲率
curve [kəːv] n. 曲线;v. 弯曲
curved [kəːvd] adj. 弯曲的
curved space 弯曲空间
curved surface 曲面
curvilinear [kəːvi'liniə] adj. 曲线的
curvilinear integral 线积分
curvilinear motion 曲线运动
cusp [kʌsp] n. 尖点,歧点
cuspidal cubic 尖点三次曲线
customary ['kʌstəməri] adj. 习惯的,惯例的
cut [kʌt] v. 截,剖,割
cutting plane 割平面
cycle ['saikl] n. 循环
cycle limit 循环极限
cyclic graph 循环图
cycloid ['saiklɔid] n. 摆线
cylinder ['silində] n. 柱体
cylinder functions 柱函数
cylindrical coordinates 柱面坐标
cylindrical helix 柱面螺旋线
cylindrical polar coordinates 柱面极坐标

## D

damped vibration 阻尼振动
data ['deitə] n. 数据(datum 的复数)
data compression 数据压缩
data handling 数据处理
data processing 数据处理
data reduction system 数据简化系统
datum ['deitəm] n. 数据
debug [ˌdiː'bʌg] n. 调试,调程序
decade ['dekeid] adj. 十进制的
decagon ['dekəgən] n. 十边形
decay [di'kei] n. 衰变,衰减
deceleration [diːˌselə'reiʃən] n. 减速度
decidable [di'saidəbl] adj. 可判定的
decile ['desil] n. 十分位数
decimal ['desiməl] n. 十进制小数
decimal-binary ['desiməl-'bainəri] n. 十进二
decimal code 十进数码
decimal notation 十进记数法
decimal number 十进制数
decimal part 小数部分
decimal point 小数点
decimal system 十进制
decimal system of counting 十进制
decision [di'siʒən] n. 判定,决策,决定
decision model 决策模式
decision process 决策过程
decision theory 决策论
declaration [ˌdeklə'reiʃən] n. 说明
declarator [di'klærətə] n. 说明符
decode [diː'kəud] n. 译码,译出指令
decomposition [ˌdiːkɔmpə'ziʃən] n. 分解

decomposition into direct sum　直和分解
decrease [diː'kriːs]　v. 减少
decreasing function　减函数
decreasing sequence　下降序列
deduce [di'djuːs]　v. 推导
deducible [di'djuːsəbl]　可推导的
deduction [di'dʌkʃən]　n. 推论,演绎
deductive [di'dʌktiv]　adj. 推论的,演绎的
deductive method　演绎法
deficiency [di'fiʃənsi]　n. 亏数,亏量
define [di'fain]　v. 定义
definite ['definit]　adj. 确定的
definite form　定型
definite integral　定积分
definite quadratic form　定二次型
definition [ˌdefi'niʃən]　n. 定义
deflation [di'fleiʃən]　n. 收缩,压缩
deflection [di'flekʃən]　n. 绕曲
deformation [ˌdiːfɔː'meiʃən]　n. 变形
degeneracy [di'dʒenərəsi]　n. 退化
degenerate conic　退化二次曲线
degenerate differential equation　退化微分方程
degree [di'griː]　n. 次,度,阶
degree of an angle　角的度数
degree of freedom　自由度
degree of a polynomial　多项式的次数
degree of precision　精密度
delete [di'liːt]　v. 删除
deletion [di'liːʃən]　n. 删除
demand [di'mɑːnd]　n./v. 需要
demand function　需求函数
demi-continuous mapping　次连续映射
denary logarithm　以十为底的对数
denary notation　十进制记数法
denominator [di'nɔmineitə]　n. 分母
denote [di'nəut]　v. 表示,记
dense [dens]　adj. 稠密的
dense everywhere　处处稠密
density ['densiti]　n. 稠密性,密度
denumerable [di'njuːmərəbəl]　adj. 可数的
dependence [di'pendəns]　n. 相关
dependent [di'pendənt]　adj. 相关的
dependent event　相关事件
dependent linear equation　相关线性方程
dependent variable　因变量
depression of order　降价法
derangement [di'reindʒmənt]　n. 重排
derivate ['deriveit]　n. 导数
derivation [ˌderi'veiʃən]　n. 求导[法]
derivative[di'rivətiv]　n. 导数,微商
derivative on the left　左导数
derivative on the right　右导数
derivative order　微分阶
derivatives of higher order　高阶导数
derive [di'raiv]　v. 得到,导出
derived function　导函数,导出函数
Descartes,R.（人名）R. 笛卡儿
describe [dis'kraib]　v. 描写,记述
descriptive geometry　画法几何[学]
design [di'zain]　v. 设计
designate ['dezigneit]　v. 标记,指定,任命
designated ['dezigneitid]　adj. 特指的
detached coefficient　分离系数
detection of error　误差的检查
determinacy [di'təːminəsi]　n. 确定性
determinant [di'təːminənt]　n. 行列式
determinant divisor　行列式因子
determinant factor　行列式因子
determinant of coefficient　系数行列式
determinant rank　行列式的秩
determinantal expansion　行列式展开式
deterministic policy　确定性决策
deterministically　确定性地
develop [di'veləp]　v. 展开,开发,发展

developable surface 可展曲面
deviation [ˌdiːviˈeiʃən] n. 偏差
diagonal [daiˈægənl] n. 对角线
diagram [ˈdaiəgræm] n. 图形,图解,图表
diameter [daiˈæmitə] n. 直径
diamond [ˈdaiəmənd] n. 菱形
dichotomy [daiˈkɔtəmi] n. 二分法
difference [ˈdifərəns] n. 差,差分
difference coefficient 差商,差分系数
difference equation 差分方程
difference method 差分法
difference of sets 集的差
difference quotient 差商
differentiability [ˈdifəˌrenʃiəˈbiləti] n. 可微性
differentiable [ˌdifəˈrenʃiəbl] adj. 可微的
differentiable function 可微函数
differentiable manifold 微分流形
differential [ˌdifəˈrenʃəl] n. 微分;adj. 微分的
differential and integral calculus 微积分[学]
differential calculus 微分学
differential equation 微分方程
differential equation of first order 一阶微分方程
differential equation of higher order 高阶微分方程
differentiate [ˌdifəˈrenʃieit] v. 求导数,求微分
differentiation [ˌdifəˌrenʃiˈeiʃən] n. 微分法
diffusion [diˈfjuːʒən] n. 扩散
diffusion equation 扩散方程
digit [ˈdidʒit] n. 数字
digital [ˈdidʒitl] adj. 数字的
digital approximation 数值逼近
digital computation 数字计算
digital filter 数字滤波
digital bits 数字字节
digital-analogue type 数字模拟型
dilatation [ˌdaileiˈteiʃən] n. 膨胀,伸缩
dilemma [diˈlemə] n. 二难推论,难题
dimension [diˈmenʃən] n. 维数,因次,量纲
Diophantine equation 丢番图方程
dipolar coordinates 双极坐标
direct product 直积
direct ratio 正比
direct sum 直和
directed graph 有向图
directed line 有向直线
directed set 有向集,定向集
direction [diˈrekʃən] n. 方向
directional derivative 方向导数
directory [diˈrektəri] n. 号码表,名册
directrix [diˈrektriks] n. 准线
Dirichlet series 狄利克雷级数
disc [disk] n. 圆盘
discernible [diˈsəːnəbl] adj. 可识别的
disconnected [ˌdiskəˈnektid] adj. 不连通的
discontinuous [ˌdiskənˈtinjuəs] adj. 不连续的
discontinuous solution 间断解
discourse [ˈdiskɔːs] n. 演讲,论述
discrete [disˈkriːt] n. 离散;adj. 离散的
discrete space 离散空间
discrete time 离散时间
discrete variable 离散变量
discretization [disˌkriːtiˈzeiʃən] n. 离散化
disintegrate [disˈintigreit] v. 解体,衰变
disjoint [disˈdʒɔint] adj. 不相交的
disjunction [disˈdʒʌŋkʃən] n. 析取
displacement [disˈpleismənt] n. 位移,置换

displacement method　位移法
dissection [di'sekʃən]　n. 剖分
distance ['distəns]　n. 距离
distance from sth. 到……的距离
distant ['distənt]　adj. 远的,有距离的
distinguished [dis'tiŋgwiʃt]　adj. 注明的,显著的
distinguished subgroup　正规子群
distortion [dis'tɔːʃən]　n. 畸变,失真
distribution [ˌdistri'bjuːʃən]　n. 分布,分配,广义函数
distribution law　分布律
distribution of primes　素数分布
distribution of values　值的分布
distributive [dis'tribjutiv]　adj. 分配的
distributive law　分配律
disturbance [dis'təːbəns]　n. 扰动
diverge [dai'vəːdʒ]　v. 发散,分叉,分歧
divergence [dai'vəːdʒəns]　n. 发散
divergent [dai'vəːdʒənt]　adj. 发散的
divide [di'vaid]　v. 除
divided difference　均差
dividend ['dividend]　n. 被除数
divisible [di'vizəbl]　adj. 可除的,整除的
division [di'viʒən]　n. 除,除法
division algorithm　带余除法
divisibility [diˌvizi'biləti]　n. 整除性
divisor [di'vaizə]　n. 除数,因子
dodecagon [dəu'dekəgən]　n. 十二边形
domain [dəu'mein]　n. 区域,定义域
dominant ['dɔminənt]　adj. 支配的,控制的
dominant function　强函数
dominant series　强级数
dominanted convergence theorem　控制收敛定理
dot [dɔt]　n. 点
dot product　点乘
double ['dʌbl]　adj. 双,二
double angle formula　倍角公式
double integral　二重积分
double interpolation　二重插值
double limit　二重极限
double point　二重点
double sampling　双重抽样
draftsman ['drɑːftsmən]　n. 制图员
dual ['djuːəl]　n. 对偶;adj. 对偶的
duality principle　对偶原理
dummy ['dʌmi]　adj. 哑的
dummy index　哑指标,哑标,跑标
dummy order　虚指令
dummy suffix　哑下标
dump [dʌmp]　n. 信息转储
duplication formula　倍角公式
dynamic programming algorithm　动态规划算法
dynamic system　动态系统
dynamics [dai'næmiks]　n. 动力学

## E

e. g. 例如(拉丁语的缩写,相当于 for example)
eccentric circle　离心圆
edge [edʒ]　n. 棱,边
effect [i'fekt]　n. 效应
effectiveness [i'fektivnis]　n. 能行性
effectiveness theory　能行性理论
efficiency [i'fiʃənsi]　n. 有效性
eigenfunction ['aigənˌfʌŋkʃən]　n. 特征函数
eigenvalue ['aigənˌvæljuː]　n. 特征值,本征值
electronic digital computer　电子数字计算机
element ['elimənt]　n. 元素
element of arc length　弧长元素

element of area 面积元素
elementary [ˌeli'mentəri] adj. 初等的
elementary algebra 初等代数[学]
elementary geometry 初等几何[学]
elementary matrix 初等矩阵
eliminate [i'limineit] v. 排除,消除
elimination [iˌlimi'neiʃən] n. 消元法,消去
elimination by addition or subtraction 加减消元法
elimination method 消元法,消去法
ellipse [i'lips] n. 椭圆
elliptic cone 椭圆锥面
elliptic function 椭圆函数
elliptic partial differential equation 椭圆型偏微分方程
elucidate [i'lju:sideit] v. 阐明,解释
embedding [em'bediŋ] n. 嵌入
empirical [em'pirikəl] adj. 经验主义的
empty ['empti] adj. 空的
empty product 空乘积
empty set 空集
endless ['endlis] adj. 无止境的,无穷的
endomorphism [ˌendəu'mɔ:fizm] n. 自同态
endpoint ['endpɔint] n. 端点,终点
entire function 整函数
entirely complete 完整的
entity ['entiti] n. 实物
entry ['entri] n. 进入,通路,表中的值
entry of matrix 矩阵的元
enumerable [i'nju:mərəbəl] adj. 可枚举的
enumerable set 可枚举集,可数集
enumerate [i'nju:məreit] v. 列举
enumeration [iˌnju:mə'reiʃən] n. 枚举,计数
enumeration function 枚举函数
enumerator [i'nju:məreitə] n. 计数器
envelop [in'veləp] n. 包络
equicontinuity [ˌi:kwikən'tinju:əti] n. 等度连续性
equal ['i:kwəl] adj. 相等的;v. 等于
equality [i:'kwɔliti] n. 相等,等式
equally likely 同等可能的
equation [i'kweiʃən] n. 方程,等式
equation of condition 条件等式,条件方程
equidistant [ˌi:kwi'distənt] adj. 等距离的
equilateral [ˌi:kwi'lætərəl] adj. 等边的
equilateral polygon 等边多边形
equilibration [ˌi:kwilai'breiʃən] n. 平衡
equipotential line 等位线
equivalence [i'kwivələns] n. 等价,等势,等积
equivalence relation 等价关系
equivalent [i'kwivələnt] adj. 等价的
ergodic hypothesis 遍历性假说
error ['erə] n. 误差
error control 误差控制
error estimate 误差估计
establishment [is'tæbliʃmənt] n. 建立
estimate ['estimeit] v. 估计;n. 估值,估价
estimate value 估值
etc. [et'setərə] adv. 等等(et cetera 的缩写)
Euclidean algorithm 欧几里得算法
Euler (人名)欧拉
Euler number 欧拉数
Euler's angles 欧拉角
evaluate [i'væljueit] v. 评价,估计,求……的值
evaluation [iˌvælju'eiʃən] n. 赋值,求值
even ['i:vən] adv. 甚至,也;adj. 偶数的
even function 偶函数
even integer 偶数 (=even number)

event [iˈvent] n. 事件
evolution [ˌiːvəˈluːʃən] n. 开方
exact differential equation 恰当微分方程
exception [ikˈsepʃən] n. 例外
exceptional value 例外值
excess [ˈekses] n. 超出量
exclusive [iksˈkluːsiv] adj. 不可兼的
execute [ˈeksikjuːt] v. 执行
exemplify [igˈzemplifai] v. 例证，例示，作为……例子
exhaustion [igˈzɔːstʃən] n. 穷举，耗尽
exhaustive [igˈzɔːstiv] adj. 穷竭的，穷尽的
existence [igˈzistəns] n. 存在
existential quantification 存在量词化
existential quantifier 存在量词
expansion [iksˈpænʃən] n. 展开式，展开
expansion in series 级数展开
expected value 期望值
expenditure [iksˈpenditʃə] n. 消费，支出
experiment [iksˈperimənt] n. 实验，试验
experiment design 试验设计
expert system 专家系统
explicit definition 显定义
exponent [eksˈpəunənt] n. 指数
exponential [ˌekspəuˈnenʃəl] adj. 指数的，幂数的
exponential equation 指数方程
exponential function 指数函数
exponential law 指数律
exponential time 指数时间
express [iksˈpres] v. 表达，用符号表示
expression [iksˈpreʃən] n. 式，表达式
extend [iksˈtend] v. 扩充，延拓，伸展
extended real number 广义实数
extension [iksˈtenʃən] n. 外延，扩张，开拓
extension of field 域的扩张
extension of mapping 映射的扩张
extension ratio 伸张比
exterior angle 外角
exterior differential 外微分
exterior product 外积
extract [iksˈtrækt] v. 开方，求根
extract a root 求根，开方
extrapolate [eksˈtræpəleit] v. 推广，外推
extrapolation [ˌekstrəpəuˈleiʃən] n. 外插
extrapolation method 外插法
extremal length 极值长度
extreme point 极值点，极端点
extreme value 极值

## F

face [feis] n. 面
face of a dihedral angle 二面角的面
factor [ˈfæktə] n. 因子，因式；v. 分解因式
factor module 商模
factor ring 商环
factoring [ˈfæktəriŋ] n. 因式分解
factorization [ˌfæktəraiˈzeiʃən] n. 因子分解
false [fɔːls] adj. 假的，不成立的
family [ˈfæmili] n. 家庭，族，系
family of circles 圆族
family of sets 集族
fast access 快速存取
feasible [ˈfiːzəbl] adj. 可行的
feature [ˈfiːtʃə] n. 特征
feedback [ˈfiːdbæk] n. 反馈
Fermat Conjecture 费马猜想
Fibonacci number 菲波那契数
fibre bundle 纤维丛
fiducial [fiˈdjuːʃəl] n. 置信
field [fiːld] n. 域，场

field axiom 域公理
figure [ˈfigə] n. 图,图形,表,数字
filter [ˈfiltə] n. 滤子,滤波器;v. 过滤
fine [fain] adj. 细的
finite [ˈfainait] adj. 有限的
finite basis 有限基
finite difference 有限差分
finite dimensional 有限维的
finite element 有限元
finite iteration 有限迭代
finite set 有限集
finitely additive measure 有限可加测度
first derivation 一阶求导
first order difference equation 一阶差分方程
first order equation 一阶方程
first term 首项
fit [fit] v. 适合
fixed point 不动点,定点
fixed value 不变值
flag [flæg] n. 标志
floating number 浮点数(=float number)
flow chart 框图
flow diagram 流向图
flux [flʌks] n. 流量,通量
focal distance 焦距
focal point 焦点
focus [ˈfəukəs] n. 焦点
forcing function 强制函数
forecasting function 预报函数
form [fɔːm] n. 形,形状,型;v. 形成
formal inference 形式推理
formalize [ˈfɔːməlaiz] v. 正式化,形式化
format [ˈfɔːmæt] n. 格式
formula [ˈfɔːmjulə] n. 公式
formulate [ˈfɔːmjuleit] v. 用公式表示,明确地表达
forward error analysis 前向误差分析
foundation of mathematics 数学基础
four color problem 四色问题
four dimensional geometry 四维几何[学]
Fourier analysis 傅里叶分析
Fourier series 傅里叶级数
fractal [ˈfræktəl] n. 分形;adj. 分形的
fractal geometry 分形几何[学]
fraction [ˈfrækʃən] n. 分数,分式
fraction in lowest terms 最简分数
fractional [ˈfrækʃənl] adj. 分数的,小数的
fractional part 分数部分,小数部分
frame [freim] n. 框架,标架
free vector 自由向量
frequency [ˈfriːkwənsi] n. 频率,频数
full group 完全群
full rank 满秩
function [ˈfʌŋkʃən] n. 函数
function digit 操作数
function idea 函数思想
function theory 函数论
functional [ˈfʌŋkʃənl] n. 泛函;adj. 泛函的
functional analysis 泛函分析
functional space 函数空间
fundamental [ˌfʌndəˈmentl] adj. 基础的,基本的
fundamental group 基本群
fuzzy [ˈfʌzi] adj. 模糊的,不分明的
fuzzy mathematics 模糊数学

## G

gain [gein] v. 获得,增益;n. 收益
Gallup polls 盖鲁普民意测验
Galois equation 伽罗瓦方程
Galois group 伽罗瓦群
game [geim] n. 对策,博弈
gauge [geidʒ] n. 度规,规范
Gaussian distribution 高斯分布

Gaussian elimination　高斯消去法
general continuum hypothesis　广义连续统假设
general solution　通解
general term　通项
generalization [ˌdʒenərəlaiˈzeiʃən]　n. 一般化，推广
generalized [ˈdʒenərəlaizd]　adj. 广义的，推广的
generalized binomial distribution　广义二项分布
generated ring　生成环
generating [ˈdʒenəˌreitiŋ]　n. 生成；adj. 生成的
generating curve　母曲线
generating line　母线
generator [ˈdʒenəreitə]　n. 生成元，母线
genus [ˈdʒiːnəs]　n. 亏格
geodesic [ˌdʒiːəuˈdesik]　n. 测地线
geometric [dʒiəˈmetrik]　adj. 几何的，几何学的
geometric interpretation　几何解释
geometric invariant　几何不变式
geometric mean　几何平均
geometrical [dʒiəˈmetrikəl]　adj. 几何学的，几何的
geometry [dʒiˈɔmitri]　n. 几何学
geometry of plane　平面几何学
gigantic [dʒaiˈgæntik]　adj. 巨人般的，巨大的
global [ˈgləubəl]　adj. 全局的
global error　全局误差
golden section　黄金分割
gradient [ˈgreidiənt]　n. 斜率，梯度
graph [grɑːf]　n. 图，图形
graph theory　图论
graphical [ˈgræfikəl]　adj. 图解的
graphical method　图解法
great number　大数
group [gruːp]　n. 组，群
group of algebra　代数群

## H

half [hɑːf]　n. 一半，二分之一
half line　半直线，射线
half period　半周期
hardware design　硬件设计
harmonic [hɑːˈmɔnik]　adj. 调和的
harmonic function　调和函数
harmonic series　调和级数
height [hait]　n. 高度，高
helicoid [ˈhelikɔid]　n. 螺旋面
heptagon [ˈheptəgən]　n. 七边形
hereditary [hiˈreditəri]　adj. 遗传的
hereditary property　遗传性质
hexadecimal [heksəˈdesim(ə)l]　n. 十六进制的
hexagon [ˈheksəgən]　n. 六边形
high degree polynomial　高阶多项式
higher algebra　高等代数
higher mathematics　高等数学
higher plane curve　高阶平面曲线
highlight [ˈhailait]　v. 强调，凸显；n. 最显著部分
Hilbert problem　希尔伯特问题
histogram [ˈhistəugræm]　n. 直方图，柱形图
hold [həuld]　v. 成立，有效，适用
holomorphic [ˌhɔləuˈmɔːfik]　adj. 全纯的
holomorphic function　全纯函数，解析函数
homeomorphic [ˌhəumiəuˈmɔːfik]　adj. 同胚的
homogeneity [ˌhɔməudʒeˈniːiti]　n. 齐性
homogeneous [ˌhɔməuˈdʒiːnjəs]　adj. 齐次的
homogeneous differential equation　齐次微分

方程
homologous [hɔ'mɔləgəs] adj. 同调的
homomorph ['həuməmɔ:f] n. 同态象
homomorphism [ˌhɔmə'mɔ:fizm] n. 同态
homotopic [ˌhɔmə'tɔ:pik] adj. 同伦的
homotopy [ˌhɔmə'tɔ:pi] n. 同伦
Hooke, R. (人名) R. 胡克
Hooke law 胡克定律
horizontal [ˌhɔri'zɔntl] adj. 地平线的, 水平的; n. 水平线
horizontal line 水平线
hyperbola [hai'pə:bələ] n. 双曲线
hyperboloid [hai'pə:bəlɔid] n. 双曲面
hypergeometric distribution 超几何分布
hyperplane ['haipəplein] n. 超平面
hypersurface [ˌhaipə'sə:fis] n. 超曲面
hypotenuse [hai'pɔtinju:s] n. 斜边
hypothesis [hai'pɔθisis] n. 假设

## I

icosahedral [ˌaikəusə'hedrəl] adj. 二十面体的
idea [ai'diə] n. 想法, 思想
ideal [ai'diəl] n. 理想, 理想数
ideal element 理想元素
idempotent ['aidəmˌpəutənt] adj. 幂等的
identical [ai'dentikəl] adj. 恒等的
identical element 单位元素, 幺元
identification code 识别码
identify [ai'dentifai] v. 识别, 认同
identity [ai'dentiti] n. 恒等, 恒等式
identity element 单位元[素], 幺元
identity function 恒等函数
identity law 同一律
identity matrix 单位矩阵
if and only if 当且仅当
illuminating [i'lju:miˌneitiŋ] adj. 明朗的, 清晰的
illustrate ['iləstreit] v. 例解, 图解, 阐明
image ['imidʒ] n. 像, 像点; v. 想像
imaginary [i'mædʒinəri] adj. 虚的, 虚数的
imaginary axis 虚轴
imaginary number 虚数
imaginary part 虚部
imitate ['imiteit] v. 模拟
implementation [ˌimplimen'teiʃən] n. 实现
implication [ˌimpli'keiʃən] n. 蕴涵, 推出
implicit [im'plisit] adj. 隐的; n. 隐式
imply [im'plai] v. 蕴涵, 推出
impossible event 不可能事件
improper fraction 假分数, 可约分数
improper subset 非真子集
inclination [ˌinkli'neiʃən] n. 倾向
incline [in'klain] v. 倾向于, 倾斜
include [in'klu:d] v. 包含
inclusion mapping 包含映射
inclusion of sets 集合的包含关系
inclusive [in'klu:siv] adj. 可兼的
incomparable [in'kɔmpərəbl] adj. 不可比的
incompatibility ['inkəmˌpætə'biliti] n. 不相容性
incorrect [ˌinkə'rekt] adj. 错误的, 不正确的
increasing function 增函数
increment ['inkrimənt] n. 增加, 增量
indefinite equation 不定方程
indefinite integral 不定积分
indefinitely [in'definitli] adv. 不确定地
independence [ˌindi'pendəns] n. 无关[性], 独立[性]
independent [ˌindi'pendənt] adj. 无关的, 独立的
independent variable 自变量

independent event 独立事件
indeterminate [ˌindiˈtəːminit] adj. 不确定的;n. 未定元
index [ˈindeks] n. 指数,指标
index set 指标集
indicate [ˈindikeit] v. 指明,指出,指定
indivisible [ˌindiˈvizəbl] adj. 除不尽的
induced mapping 诱导映射
induction [inˈdʌkʃən] n. [数学]归纳法
induction hypothesis 归纳法假设
induction principle 归纳法原则
inductive set 归纳集
inequality [ˌiniˈkwɔliti] n. 不等,不等式
inequality constraint 不等式约束
inertia [iˈnəːʃjə] n. 惯性
inessential singularity 非本性奇点
infer [inˈfəː] v. 推理,推断
inference [ˈinfərəns] n. 推理,推断
infimum [inˈfaiməm] n. 下确界
infinite [ˈinfinit] n. 无穷的,无限的
infinite decimal 无尽小数
infinite dimensional 无限维的
infinite product 无穷乘积
infinite sequence 无穷序列
infinite series 无穷级数
infinite set 无穷集
infinitely [ˈinfinitli] adv. 无穷地,无限地
infinitesimal [ˌinfiniˈtesiməl] n. 无穷小[量];adj. 无穷小的
infinitesimal calculus 微积分[学]
infinitesimal element 无穷小元素
infinity [inˈfiniti] n. 无穷大
inflection point 拐点,回折点
information [ˌinfəˈmeiʃən] n. 信息
information channel 信息通道
initial condition 初始条件
initial data 初始数据
initial value 初值
initial value problem 初值问题
injection [inˈdʒekʃən] n. 内射,单射
innumerable [iˈnjuːmərəbl] adj. 无数的,数不清的
input data 输入数据
input output analysis 投入产出分析
inscribe [inˈskraib] v. 使内接,使内切
inscribed cone 内接圆锥
inscribed polygon 内接多边形
insertion algorithm 插入算法
insolvable [inˈsɔlvəbl] n. 不可解的
instantaneous [ˌinstənˈteinjəs] adj. 即时的,瞬时的
instantaneous velocity 即时速度
instruction [inˈstrʌkʃən] n. 指令
instrument error 仪器误差
integer [ˈintidʒə] n. 整数
integrability [ˌintigrəˈbiliti] n. 可积性
integrable [ˈintigrəbl] adj. 可积的
integral [ˈintigrəl] adj. 整数的,积分的;n. 积分
integral calculus 积分学
integrate [ˈintigreit] v. 对……积分
integrating factor 积分因子
integration [ˌintiˈgreiʃən] n. 积分,积分法
intensity [inˈtensiti] n. 强度
interchange [ˌintəˈtʃeindʒ] v. 交换,相互交换
interchangeably [ˌintəˈtʃeindʒəbli] adv. 可互相交换
interdependence coefficient 依存系数
interior [inˈtiəriə] n. 内部
interior angle 内角
intermediate value 介值
internal regression 内回归
interpolation formula 插值公式
interpret [inˈtəːprit] v. 解释,说明
interpretive order 解释指令

interrelationship 相互关系

intersect [ˌintəˈsekt] v. 相交

intersection [intəˈsekʃən] n. 交，交集，相交

interval [ˈintəvəl] n. 区间，线节

interval estimation 区间估计

intransitive [inˈtrænsitiv] adj. 非可递的，反传递的

intrinsic [inˈtrinsik] adj. 内在的

intuitionism [ˌintjuːˈiʃənizm] n. 直觉主义

intuitive [inˈtjuitiv] adj. 直观的

invariant set 不变集

inverse [ˈinˈvəːs] n. 反，逆；adj. 逆的

inverse circular function 反三角函数

inverse correlation 逆相关

inverse function 反函数

inverse image 逆像，原像

inverse mapping 逆映射

inverse matrix 逆矩阵

inverse point 逆演点，反演点

inverse sine 反正弦

inverse trigonometric function 反三角函数

inversion [inˈvəːʃən] n. 反演，求逆，反转

invertible matrix 可逆矩阵

involution [ˌinvəˈluːʃən] n. 乘方，对合

involve [inˈvɔlv] v. 包含

inward normal 内向法线

irrational [iˈræʃənəl] adj. 无理的，无理数的

irrational number 无理数

irreducible [ˌiriˈdjuːsəbl] adj. 不可约的

irreducible fraction 不可约分数

irreflexive [ˌiriˈfleksiv] adj. 反自反的，非自反的

irregularity [iˌregjuˈlæriti] n. 不正规性

isentropic [aisenˈtrɔpik] adj. 等熵的

isogonal line 等角线

isolated [ˈaisəleitid] adj. 孤立的

isometric [ˌaisəuˈmetrik] adj. 等距的

isometry [aiˈsɔmitri] n. 等距

isomorph [ˈaisəumɔːf] n. 同构

isomorphism [ˈaisəuˈmɔːfizm] n. 同构

isoperimetric inequality 等周不等式

isopotential [ˌaisəpəˈtenʃəl] n. 等位势线

isosceles trapezoid 等腰梯形

isosceles triangle 等腰三角形

isotone [ˈaisəutəun] n. 保序

isotropic [aisəuˈtrɔpik] adj. 迷向，各向同性的

item of information 信息项

iterated interpolation method 迭代插值法

iteration [ˌitəˈreiʃən] n. 迭代

iterative solution 迭代解

## J

Jacobi algorithm 雅克比算法

Jacobian [dʒæˈkəubiən] n. 雅克比行列式；adj. 雅克比理论的

join [dʒɔin] v. 参加，连接；n. 连接，联合

joint [dʒɔint] adj. 连接的，联合的

joint estimate 联合估计

joint observation 联合观测

Jordan arc 若尔当弧

jump function 跳跃函数

## K

Kelvin transformation 开尔文变换

kernel [ˈkəːnl] n. 核

Klein bottle 克莱因瓶

knowledge-based system 基于知识的系统

known [nəun] adj. 已知的

known function 已知函数

## L

label [ˈleibəl] n. 标号,标签; v. 标记
Lagrange multiplier 拉格朗日乘子
Lagrange's interpolation formula 拉格朗日插值公式
Laplace equation 拉普拉斯方程
Laplace transform 拉普拉斯变换
lateral face 侧面
lattice [ˈlætis] n. 格
lattice point 格点
law of association 结合律
law of commutation 交换律
law of contradiction 矛盾律
law of distribution 分配律
law of identity 同一律
law of large number 大数定律
law of reciprocity 互反律
leading diagonal 主对角线
least common multiple 最小公倍数
least square method 最小二乘法
left ideal 左理想
Leibniz, G. W. (人名)G. W. 莱布尼茨
lemma [ˈlemə] n. 引理
lemniscate [ˈlemniˌskeit] n. 双纽线
length [leŋθ] n. 长度
length of arc 弧长
level line 等高线
lexicographic [ˌleksikəˈgræfik] adj. 字典式的
library of subroutine 子程序序库
like terms 同类项
limit function 极限函数
limitation [ˌlimiˈteiʃən] n. 限制,局限性
line [lain] n. 线,直线
line of reference 参考线
line segment 直线段,线段
linear combination 线性组合
linear correlation 线性相关
linear differential equation 线性微分方程
linear equation 线性方程
linear estimation 线性估计
linear function 线性函数
linear operator 线性算子
linear regression 线性回归
linear space 线性空间
linear transform 线性变换
linearly dependent 线性相关的
linearly independent 线性无关的
literal constant 文字常量
literal equation 文字方程
local coordinates 局部坐标
locally bounded 局部有界的
locally convex 局部凸的
location of root 寻根法
location test 位置检验
locus [ˈləukəs] n. 轨迹
logarithm [ˈlɔgəˌriθm] n. 对数
logarithm function 对数函数
logarithmic [ˌlɔgəˈriθmik] adj. 对数的
logarithmic table 对数表
logical deduction 逻辑推理
logical order 逻辑指令
logically equivalent 逻辑等价的
loss of information 信息损失
lower bound 下界
lower extreme 下端
lower limit 下限
lozenge [ˈlɔzindʒ] n. 菱形

## M

machine arithmetic 机器运算
machine computation 机器计算
machinist [məˈʃiːnist] n. 机械师,机工

macro instruction　宏指令
magnitude [ˈmægnitjuːd]　n. 量,数量
main diagonal　主对角线
major arc　优弧
major axis　长轴
many sorted predicate calculus　多种类谓词演算
many-valued function　多值函数
many-valued logic　多值逻辑
map into　映入
map onto　映上,映到
mapping [ˈmæpiŋ]　n. 映射
margin analysis　边缘分析
marginal classification　边缘分类
mark [mɑːk]　n. 痕迹,记号;v. 表示
Markovian process　马尔可夫过程
match [mætʃ]　v. 配对,匹配
mathematical analysis　数学分析
mathematical induction　数学归纳法
mathematical model　数学模型
mathematical physics　数学物理
mathematical statistics　数理统计
mathematician [ˌmæθiməˈtiʃən]　n. 数学家
mathematics [ˌmæθiˈmætiks]　n. 数学
matrix [ˈmeitriks]　n. 矩阵,母式
matrix trace　矩阵的迹
maximal [ˈmæksiməl]　adj. 极大的,最大的
maximum [ˈmæksiməm]　n. 极大值,最大值
maximum slope　最大斜率
maximum value　极大值,最大值
mean [miːn]　n. 平均,平均值
mean center difference　平均中心差分
mean deviation　平均偏差
mean proportional　比例中项
mean square error　均方误差
mean value　平均值
measurable [ˈmeʒərəbl]　adj. 可测的
measure [ˈmeʒə]　v. 测量,度量;n. 测度
measure space　测度空间
measurement [ˈmeʒəmənt]　n. 测量,观测
mechanics [miˈkæniks]　n. 力学
median line　中线
memory dump　信息转储
memory space　存储空间,存储量
mental arithmetic　心算,智力运算
meromorphic function　亚纯函数
method of concomitant variation　共变法
method of exhaustion　穷竭法
method of induction　归纳法
method of undetermined coefficient　待定系数法
metric [ˈmetrik]　n. 度量;adj. 度量的
metric space　度量空间
million [ˈmiljən]　num. 百万,百万个;n. 无数
minimal [ˈminiməl]　adj. 极小的,最小的
minimal surface　极小曲面
*minimal value*　极小值
minimax method　极小化极大法
minimum [ˈminiməm]　n. 最小值,极小值
minor [ˈmainə]　adj. 较小的;n. 子式
minor arc　劣弧
minor axis　短轴
minor determinant　子行列式
minus [ˈmainəs]　prep. 减; adj. 负的; n. 减号,负号,亏损
minus sign　减号
missing [ˈmisiŋ]　adj. 不见的,缺少的
mixed fraction　带分数
mixed initial value　混合初值
model [ˈmɔdl]　n. 模型
modeling [ˈmɔdliŋ]　n. 建模
modeling process　建模过程
modern algebra　近世代数

modified formula　修正公式
modular [ˈmɔdjulə]　adj. 模的,模数的
moment [ˈməumənt]　n. 矩,片刻
momentum [məuˈmentəm]　n. 动量
monogamy [məˈnɔgəmi]　n. 一一对应
monoid [ˈmɔnɔid]　n. 幺半群
monotone [ˈmɔnətəun]　adj. 单调的
monotone class　单调类
monotonic [mɔnəˈtɔnik]　adj. 单调的
monotonicity [ˌmɔnətəˈnisiti]　n. 单调性
monotonicity principle　单调性原理
multiform function　多值函数
multilevel [ˌmʌltiˈlevəl]　n. 多水平; adj. 多水平的
multiple [ˈmʌltipl]　n. 倍数; adj. 多样的
multiple error　多级误差
multiple sample plan　多重抽样方案
multiple valued logic　多值逻辑
multiplication [ˌmʌltipliˈkeiʃən]　n. 乘法,乘
multiply [ˈmʌltiplai]　v. 乘,倍增,扩大; adv. 以多种形式地
multiply connected　多连通的
multiply connected region　多连通区域
multipole [ˈmʌltipəul]　n. 多极
multivalent function　多叶函数
mutually exclusive　互斥的
mutually inverse　互逆的

## N

naive assumption　朴素的假设
natural boundary of a function　函数的自然边界
natural frequency　固有频率
natural system　自然对数系
nature [ˈneitʃə]　n. 性质,自然
$n$-dimensional vectors　$n$ 维向量
necessary and sufficient condition　充要条件
necessary condition　必要条件
negative [ˈnegətiv]　adj. 负
negative definite　负定的
negative exponent　负指数
negative sign　负号
neighborhood [ˈneibəhud]　n. 邻域
nested interval　区间套
net [net]　n. 网,网格
net force　净力,纯力
neural network　神经网络
neurology [njuəˈrɔlədʒi]　n. 神经学,神经病学
neutral [ˈnjuːtrəl]　n. 中性,零元; adj. 中间的
neutral element　零元素,中性元
Newton, I. (人名) I. 牛顿
Newton's second law of motion　牛顿第二运动定律
nodal cubic　结点三次曲线
node [nəud]　n. 结点
non-algebraic [ˈnɔn-ˌældʒiˈbreiik]　adj. 非代数的
non-countable [ˈnɔn-ˈkauntəbl]　adj. 不可数的
non-Euclidean geometry　非欧几何[学]
non-existence [ˌnɔnigˈzistəns]　n. 不存在性
non-integral dimension　非整数维
non-overlap [ˈnɔn-ˌouvəˈlæp]　v. 不相重叠
non-vacuous [ˈnɔn-ˈvækjuəs]　adj. 非空的
non-vanishing [ˈnɔn-ˈvæniʃiŋ]　adj. 非零的
noncentral conic　无心二次曲面
nondegenerate [ˌnɔndiˈdʒenəreit]　adj. 非退化的
nondense [ˈnɔnˈdens]　adj. 疏朗的,无处稠密的
nonhomogeneous [ˈnɔnˌhɔməˈdʒiːnjəs]

adj. 非齐次的

nonidentical [ 'nɔnai'dentikəl ] adj. 不相同的,不恒等的

nonlinear [ 'nɔn'liniə ] adj. 非线性的

nonlinear equation 非线性方程

nonnegative [ 'nɔn'negətiv ] adj. 非负的

*nonnegative type* 非负型

nonnumerical [ 'nɔnnjuː'merikəl ] adj. 非数值的

nonpositive [ 'nɔn'pɔzitiv ] adj. 非正的

nonsingular [ 'nɔn'siŋgjulə ] adj. 非奇异的

norm [ nɔːm ] n. 范数

normal [ 'nɔːməl ] adj. 正常的,正交的,正态的;n. 法线

normal derivative 法向导数

normal distribution 正态分布

normal family 正规族

*normal process* 正态过程

normal set 良序集

normalize [ 'nɔːməlaiz ] v. 正规化

normalized form 标准型

notation [ nəu'teiʃən ] n. 符号,记法

note [ nəut ] n. 附注,注记

nought [ nɔːt ] n. 零,空

*n*-tuple [ 'en'tjuːpl ] n. *n*-元组

null [ nʌl ] adj. 零,空

*null set* 零集,空集

null vector 零向量

number [ 'nʌmbə ] n. 数,号码

number of terms 项数

number system 数系

number theory 数论

numeral [ 'njuːmərəl ] n. 数字; adj. 数字的

numerator [ 'njuːməreitə ] n. 分子

numeric [ njuː'merik ] adj. 数字的,数值的

numerical [ njuː'merikəl ] adj. 数字的,数值的

numerical analysis 数值分析

numerical coding 数字编码

numerical computation 数值计算

numerical experimentation 数值实验

numerically adv. 用数字,在数字上

*numerous* [ 'njuːmərəs ] adj. 众多的,许多的

## O

objective function 目标函数

oblateness [ 'ɔbleitnis ] n. 扁率

oblique [ ə'bliːk ] adj. 倾斜的

oblique prism 斜棱柱

observation [ ˌɔbzə'veiʃən ] n. 观察,观察值

obtuse angle 钝角

obtuse triangle 钝角三角形

octadic [ ɔk'tædik ] adj. 八进制的

octal system 八进制

odd [ ɔd ] adj. 奇数的,奇怪的

odd function 奇函数

odd number 奇数

odd permutation 奇排列,奇置换

odevity [ əu'deviti ] n. 奇偶性

*one third* 三分之一

one-one correspondence 一一对应

one-to-one 一对一[的]

open interval 开区间

operation [ ˌɔpə'reiʃən ] n. 运算,运作

operational code 操作码

operations research 运筹学 ( = operational research)

operative symbol 运算符号

operator [ 'ɔpəreitə ] *n.* 运算符,算子

opposite [ 'ɔpəzit ] adj. 相对的,相反的

opposite sides 对边

optima [ˈɔptimə] n. 最优(optimum 的复数)
optimal approximation 最佳逼近
optimal basic feasible solution 最优基本可行解
optimal control 最优控制
optimal solution 最优解
optimization [ˌɔptimaiˈzeiʃən] n. 最优化
optimum [ˈɔptiməm] n. 最优
orbit space 轨道空间
order [ˈɔːdə] n. 阶,次序
order code 指令码
order of a determinant 行列式的阶
ordered [ˈɔːdəd] adj. 有序的
ordered basis 有序基
ordered *n*-tuple 有序 *n*-元组
ordered set 有序集
ordered topological 序拓扑
orderliness [ˈɔːdəlinis] n. 井然有序
order relation 序的关系
ordinal [ˈɔːdinl] adj. 序数的
ordinal number 序数
ordinary defferential equation 常微分方程
ordinary solution 通常解
ordinate [ˈɔːdinit] n. 纵坐标
oriented [ˈɔːrientid] adj. 定向的,有向的
origin [ˈɔridʒin] n. 原点
originality [əˌridʒiˈnæliti] n. 创造力
orthogonal [ɔːˈθɔgənl] adj. 正交的
orthogonal coordinate system 正交坐标系,直角坐标系
orthogonal trajectory 正交轨线
orthogonality [ɔːθɔgəˈnæliti] n. 正交性,相互垂直
oscillate [ˈɔsileit] v. 振动
oscillation [ˌɔsiˈleiʃən] n. 振荡,振动
outcome [ˈautkʌm] n. 结果,结局
outer product 外积
outline [ˈautlain] v. 描画轮廓,略述
output [ˈautput] n. 输出;v. 输出

## P

pairwise orthogonal 两两正交
parabola [pəˈræbələ] n. 抛物线
parabolic [ˌpærəˈbɔlik] adj. 抛物的
parabolic curve 抛物曲线
parabolic segment 抛物弓形,抛物线段
paraboloid [pəˈræbəlɔid] n. 抛物面
paradox [ˈpærədɔks] n. 悖论
parallel [ˈpærəlel] adj. 平行的,并行的;n. 平行线
parallel computer 并行计算机
parallel lines 平行线
parallel postulate 平行公设
parallelogram [ˌpærəˈleləgræm] n. 平行四边形
parameter [pəˈræmitə] n. 参数
parity [ˈpæriti] n. 奇偶性
partial derivative 偏导数,偏微商
partial differential equation 偏微分方程
partial ordering 偏序
particular solution 特解
partition [pɑːˈtiʃən] n. 划分,细分,分类
path [pɑːθ] n. 道路,轨道
pathological [ˌpæθəˈlɔdʒikəl] adj. 病态的
pattern recognition 类型识别
pencil of circles 圆束
pentagon [ˈpentəgən] n. 五边形
perfect set 完备集,完满集
perfect square 完全平方
performance [pəˈfɔːməns] n. 实行,完成
perimeter [pəˈrimitə] n. 周长
period [ˈpiəriəd] n. 周期
periodic [ˌpiəriˈɔdik] adj. 周期的
periodic decimal 循环小数

permutation [ ˌpəːmjuˈteiʃən ] n. 排列，置换
perpendicular [ ˌpəːpənˈdikjulə ] adj. [互相]垂直的；n. 垂线
Perron method 庇隆方法
perspective [ pəˈspektiv ] n. 透视；adj. 透视的
phase space 相空间
phrase [ freiz ] n. 短语，词组，惯用语；v. 用语言表示
pie charts 饼分图
piecewise polynomial interpolation 分段多项式插值
piecewise smooth function 逐段光滑函数
plane [ plein ] n. 平面
plane analytic geometry 平面解析几何
plane of symmetry 对称平面
player [ ˈpleiə ] n. 局中人
plot [ plɔt ] v. 划分，绘图，画(草图)
plural [ ˈpluərəl ] adj. 复数的(名词的数)
plurality [ pluəˈræliti ] n. 过半数，多数
plus [ plʌs ] prep. 加；adj. 正的；n. 正号，加号，盈余
plus sign 加号
point [ pɔint ] n. 点
point of discontinuity 不连续点
point of inflection 拐点
Poisson equation 泊松方程
polar [ ˈpəulə ] adj. 极的；n. 极线，极面
polar coordinates 极坐标
polarity [ pəuˈlæriti ] n. 配极[变换]
pole [ pəul ] n. 极，极点
pole of order 1 一阶极点
polygon [ ˈpɔligən ] n. 多边形
polygonal [ pɔˈligənəl ] adj. 多边形的
polygonal line 折线
polygonal region 多边形区域
polynomial [ ˌpɔliˈnəumiəl ] n. 多项式
a polynomial in $x$ 关于 $x$ 的多项式
polynomial interpolation 多形式插值
polynomial time algorithm 多项式时间算法
polytropic [ ˌpɔliˈtrɔpik ] adj. 多变的
population [ ˌpɔpjuˈleiʃən ] n. 总体，人口
position [ pəˈziʃən ] n. 位置，状态
position function 位置函数
positive [ ˈpɔzətiv ] adj. 正的，肯定的，阳性的
positive definite 正定的
positive number 正数
positively homogeneous 正齐次的
positivity [ ˈpɔzitiviti ] n. 正性
possibility [ ˌpɔsəˈbiliti ] n. 可能性
postulate [ ˈpɔstjuleit ] v. 假设，假定；n. 公设
potential [ pəˈtenʃəl ] n. 位势；a. 潜在的
power [ ˈpauə ] n. 乘幂，乘方，势，权
power series 幂级数
precision [ priˈsiʒən ] n. 明确，精密
predicate [ ˈpredikit ] n. 谓词，谓语
predicate calculus 谓词演算
preferable [ ˈprefərəbl ] adj. 更可取的，更好的
prescribe [ priˈskraib ] v. 命令，规定
primary [ ˈpraiməri ] adj. 主要的，初步的
prime [ praim ] n. 素数(质数)；adj. 素数的
prime ideal 素理想
prime number 素数
primitive form 基本形
primitive function 原函数
principal [ ˈprinsəpəl ] adj. 主要的
principal direction 主方向
principal value of an integral 积分的主值
principle [ ˈprinsəpl ] n. 原理
principle of argument 辐角原理
principle of induction 归纳法原理

principle of the excluded middle　排中律
principle of the least element　最小数原理
priority [prai'ɔriti]　n. 优先权
probabilistic [ˌprɔbəbi'listik]　adj. 概率性的
probabilistic method　概率方法
probability [ˌprɔbə'biliti]　n. 概率
probability of occurrence　事件概率
probability sample　概率样本
probability space　概率空间
probability theory　概率论
probability zero　零概率
probe [prəub]　n. 试验值
procedure chart　程序图
process of iteration　迭代法
product ['prɔdəkt]　n. 乘积
product formula　乘积公式
　repeated product　连乘积
program ['prəugræm]　n. 程序
program design　程序设计
programmer ['prəugræmə]　n. 程序设计者，程序员
programming ['prəugræmiŋ]　n. 规划，编程序
programming language　程序语言
programming skill　编程技巧
progression [prə'greʃən]　n. 级数
projection [prə'dʒekʃən]　n. 射影，投影，投射
projective method　射影法，投射法
prolongation [ˌprəulɔŋ'geiʃən]　n. 延长，拓展
proof [pruːf]　n. 证明
proof by induction　[用数学]归纳法证明
proof-generating technique　证明生成技术
proper ['prɔpə]　n. 正常；adj. 正常的，真的
proper factor　真因子
proper vector　特征向量
proportion [prə'pɔːʃən]　n. 比例
proportional [prə'pɔːʃənəl]　adj. 成比例的
proposition [ˌprɔpə'ziʃən]　n. 命题
propositional [ˌprɔpə'ziʃənəl]　adj. 命题的
propositional connective　命题连词
propositional function　命题函数
prove [pruːv]　v. 证明，被证明是
pseudo area　伪面积
pseudo code n. 伪码，伪代码
pseudo-programming language　伪程序语言
psychoanalysis [ˌsaikəuə'næləsis]　n. 心理分析
public key　公开密钥
pure imaginary　纯虚数
pyramid ['pirəmid]　n. 棱锥，金字塔
Pythagorean identity　毕达哥拉斯等式

## Q

Q. E. D. (外来词 quod erat demon-strandum 的缩写)谨此作答，证完
quadrangle ['kwɔdræŋgl]　n. 四角形
quadrant ['kwɔdrənt]　n. 象限
quadratic curve　二次曲线
quadratic differential form　二次微分形式
quadratic equation　二次方程
quadratic function　二次函数
quadratic surface　二次曲面
quadrature ['kwɔdrətʃə]　n. 求[面]积，求积分，积圆法
quadrature formula　求积公式
quadric form　二次形式，二次型
qualitative ['kwɔlitətiv]　adj. 性质上的，定性的
quality ['kwɔliti]　n. 质量，品质
quantification [ˌkwɔntifi'keiʃən]　n. 量词化

quantifier [ˈkwɔntifaiə] n. 量词
quantitative [ˈkwɔntitətiv] adj. 数量的，定量的
quantity [ˈkwɔntiti] n. 量
quantizer [ˈkwɔntaizə] n. 数字转换器
quarter [ˈkwɔːtə] n. 四分之一
quasi-divisor [ˈkwɑːzidiˈvaizə] n. 拟因子
quaternion [kwəˈtəːniən] n. 四元数
queue [kjuː] v. 排队；n. 排队
queueing theory 排队论
quotient [ˈkwəuʃənt] n. 商
quotient set 商集
quotient space 商空间

## R

radial [ˈreidiəl] adj. 径向的
radian [ˈreidjən] n. 弧度
radical [ˈrædikəl] n. 根式，根号
radicand [ˈrædikænd] n. 被开方数
radius [ˈreidjəs] n. 半径(复数为：radii)
raise [reiz] v. 使自乘，提出(问题)
raise to a power 使自乘到……幂
random [ˈrændəm] adj. 随机的
random event 随机事件
random occurrence 随机事件
random walk 随机游动
randomized computation 随机化计算
range [reindʒ] n. 区域，范围，值域，极差
range of values 值的范围
rank [ræŋk] n. 秩
ratio [ˈreiʃiəu] n. 比，比率
ratio of equality 等比
rational [ˈræʃənl] adj. 有理的
rational function 有理函数
rational number 有理数
ray [rei] n. 射线，半直线
real [riəl] adj. 实的
real axis 实轴
real line 实直线
real root 实根
real variable 实变量，实变函数
realizable [ˈriəlaizəbl] adj. 可实现的
real-valued function 实值函数
real-valued sequence 实值序列
rearrangement [ˌriːəˈreindʒmənt] n. 重排
reasonable constraint 合理的约束
reasoning [ˈriːzəniŋ] n. 推理，评理，论证；adj. 推理的
reasoning method 推理方法
reciprocal [riˈsiprəkəl] adj. 倒数的，互反的
reciprocal ratio 反比
recommendation [ˌrekəmenˈdeiʃən] n. 推荐，褒奖，忠告
rectangle [ˈrektæŋgl] n. 矩形
rectangle axes 直角轴
rectangular [rekˈtæŋgjulə] adj. 矩形的，成直角的
rectifiable [ˈrektifaiəbl] adj. 可求长的
rectifiable curve 可求长的曲线
rectilinear [ˌrektiˈliniə] adj. 直线的，直的
rectilinear motion 直线运动
recurrence formula 循环公式，递推公式
recurring decimal 循环小数
recurring period 小数的循环节
recursion [riˈkəːʃən] n. 递归式，递推，循环
recursion formula 递推公式，递归公式
reduce [riˈdjuːs] v. 简化，转化，化简
reducibility [riˌdjuːsəˈbiliti] n. 可约性
reducible [riˈdjuːsəbl] adj. 可简化的，可归约的
reducible fraction 可约分数
reduction [riˈdʌkʃən] n. 简化，归约，化简
redundant [riˈdʌndənt] adj. 多余的

reference [ˈrefrəns] n. 参考，涉及
reference system 参考系
reflection [riˈflekʃən] n. 反射，镜射
reflex angle 优角
reflexive [riˈfleksiv] adj. 自反的，反身的
region [ˈriːdʒən] n. 区域
regional coding 局部编码
regression analysis 回归分析
regular [ˈregjulə] adj. 正则的
regular curve 正则曲线
reject [riˈdʒekt] v. 丢弃，拒绝
relation [riˈleiʃən] 关系
relation of inclusion 包含关系
relational algebra 关系代数
relative coding 相对编码
relative complement 相对补
relatively complemented 互补的
relativity [ˌreləˈtiviti] n. 相对性，相对论
 the theory of relativity 相对论
relaxation factor 松弛因子
relevant [ˈreləvənt] adj. 有关的，关联的
relief [riˈliːf] n. 缓和，减轻
remain [riˈmein] v. 剩下，余留
remainder [riˈmeində] n. 余数
repeated integral 累次积分
repeated products 连乘积
repeated root 重根
repeated summation 累积求和法
repeating decimal 循环小数
replacement [riˈpleismənt] n. 替换，更新
representation [ˌreprizenˈteiʃən] n. 表示
required value 预期值
rescaling [riːˈskeiliŋ] n. 换算
residual [riˈzidjuəl] n. 剩余，残差，余项
resolution of a unit 单位的分解
result [riˈzʌlt] n. 结果
reverse order 逆序
reversible process 可逆过程
rhombus [ˈrɔmbəs] n. 菱形
Riemann surface 黎曼曲面
right angle 直角
right circular cone 正圆锥，直立圆锥
right circular cylinder 正圆柱
right line 直线
right triangle 直角三角形
right-handed system 右手系
rigid body 刚体
rigid law 刚性定律
rigid motion 刚体运动
ring [riŋ] n. 环
ring surface 环面
Robot control 机器人控制
root [ruːt] n. 根
rotation [rəuˈteiʃn] n. 旋转
round angle 周角
rounding error 舍入误差
rounding off 舍入
row [rəu] n. 行
rule [ruːl] n. 规则，准则
rule of combination 组合规则
rule of inference 推演规则
rule out 排除，否决
ruler [ˈruːlə] n. 尺，直尺

## S

saddle point 鞍点
saltus [ˈsæltəs] n. 振幅，跃度
sample [ˈsæmpl] n. 样本
sample space 样本空间
sampling plan 抽样方案
sampling unit 样本单位，样本个体
satisfiability [ˌsætisfaiəˈbiliti] n. 可满足性
satisfiable [ˈsætisfaiəbl] adj. 可满足的
satisfy [ˈsætisfai] v. 适合，满足

scalar [ˈskeilə] n. 标量,纯量;adj. 标量的,数量的
scalar field 标量场
scalar multiple 标量倍数
scalar product 标量积,数量积,内积
scale [skeil] n. 尺度,标度,刻度
scaling law 换算律,定标律
schematic representation 图解表示
schematically [ˌskiˈmætikəli] adv. 图解式地
scope [skəup] n. 范围,领域
search method 搜索法
secant [ˈsiːkənt] adj. 切的,割的;n. 割线,正割
second order difference 二阶差分
secret code 密码
secret key 密钥
sectionally continuous 分段连续的
sectionally smooth 分段光滑的
segment [ˈsegmənt] n. 线段,线节
segment of a circle 弓形
self-adjoint [self-ˈædˌdʒɔint] adj. 自伴的
semi-circle [ˈsemi-ˈsəkl] n. 半圆
semi-closed [ˈsemi-ˈkləuzd] adj. 半封闭的
semi-continuous function 半连续函数
separable [ˈsepərəbl] adj. 可分的
separable space 可分空间
separated [ˈsepəreitid] adj. 分离的,被分的
separated variable differrential equation 变量分离微分方程
sequence [ˈsiːkwəns] n. 序列,数列
series [ˈsiəriːz] n. 级数,一连串
set [set] n. 集,组,套;v. 令
set function 集函数
sheaf [ʃiːf] n. 层,簇
sheet [ʃiːt] n. 叶,片
shift [ʃift] v. 转变,更换;n. 转变,更换
shifting function 移位函数
side [said] n. 边
sieve of Eratosthenes 厄拉多色筛法
sign [sain] n. 记号,符号
sign of evolution 根号
significant [sigˈnifikənt] adj. 有效的,有影响的
significant digit 有效数字,有效位
significant figure 有效数[字]
similar polygon 相似多边形
similarity [ˌsimiˈlæriti] n. 相似性
similarity ratio 相似比
simple curve 简单曲线
simple root 单根
simplex [ˈsimpleks] n. 单形,单纯形
simply connected region 单连通区域
simulation [ˌsimjuˈleiʃən] n. 模拟
sine [sain] n. 正弦
single cycle 单循环
single valued function 单值函数
singular [ˈsiŋgjulə] adj. 奇异的,奇的
singular equation 奇异方程
singular point 奇异点
singularity [ˌsiŋgjuˈlæriti] adj. 奇异性,奇点
size [saiz] n. 大小,体积
slope [sləup] n. 斜度,斜面,斜率
smoothed curve 光滑曲线
software package 软件包
solid [ˈsɔlid] adj. 立体的;n. 立体
solid angle 立体角
solution [səˈluːʃən] n. 解,解法
solution of a triangle 三角形的解法
solution of equation 方程的解
solvable [ˈsɔlvəbl] adj. 可解的
solve [ˈsɔlv] v. 解
sorting problem 分类问题

space [speis] n. 空间
span [spæn] v. 张成,支撑
special function 特殊函数
specification [ˌspesifiˈkeiʃən] n. 详述,规格,说明书
specify [ˈspesifai] v. 明确地陈述,指定
spectral [ˈspektrəl] adj. 谱的,光谱的
spectral analysis 谱分析
spectrum [ˈspektrəm] n. 谱
sphere [sfiə] n. 球形,球面
spherical [ˈsferikəl] adj. 球的,球形的
spherical polar coordinates 球极坐标
spring-mass system 弹簧质量系统
square [skwɛə] n. 正方,平方,二次幂
square root 平方根
stability [stəˈbiliti] n. 稳定性
standard deviation 标准差
standard error 标准误差
star domain 星形域
state space 状态空间
statement [ˈsteitmənt] n. 叙述,表达方式,语句
stationary [ˈsteiʃənəri] adj. 平稳的
statistical control 统计控制
statistical inference 统计推断
statistics [stəˈtistiks] n. 统计,统计学
steepness [ˈsti:pnis] n. 陡峭
step counting function 计步函数
storage [ˈstɔ:ridʒ] n. 存储器,存储量
store [stɔ:] n. 存储,存储器
straight angle 平角
straight line 直线
straightforward approach 直接的方法
strategy [ˈstrætidʒi] n. 策略,对策
strictly monotone function 严格单调函数
string [striŋ] n. 行,串
strip [strip] n. 带
strong law of large numbers 强大数定律

subadditivity [sʌbˌædiˈtiviti] n. 次可加性
subbasis [ˈsʌbˈbeisis] n. 子基
subclass [ˈsʌbklɑ:s] n. 子类
subdivision [ˈsʌbdiˌviʒən] n. 细分
subfield [ˈsʌbfi:ld] n. 子域
subgroup [ˈsʌbgru:p] n. 子群
subinterval [ˈsʌbˈintəvəl] n. 子区间
submanifold [ˈsʌbˈmænifəuld] n. 子流形
subscript [ˈsʌbskript] n. 下标
subset [ˈsʌbset] n. 子集
subsist [səbˈsist] v. 生存,存在
subspace [ˈsʌbˌspeis] n. 子空间
  the subspace spanned by $S$ 由 $S$ 生成的子空间
substitution [ˌsʌbstiˈtju:ʃən] n. 代换,代入
subtract [səbˈtrækt] v. 减,从……减去
subtracter [səbˈtræktə] n. 减法器
subtraction [səbˈtrækʃən] n. 减,减法
subtrahend [ˈsʌbtrəhend] n. 减数
succession [səkˈseʃən] n. 逐次性,连贯性
successive [səkˈsesiv] adj. 逐次的,相继的
successive integration 逐次积分
successive substitution 逐次代换法,递代法
successor [səkˈsesə] n. 后继者,后继
suggest [səˈdʒest] v. 建议,暗示
sum [sʌm] n. 和
summary [ˈsʌməri] n. 摘要,概要
summation [sʌˈmeiʃən] n. 总和,和,求和法
superfluous [sju:ˈpə:fluəs] adj. 多余的,过量的
superposition [ˌsju:pəpəˈziʃən] n. 叠加
superscript [ˈsju:pəskript] n. 上标
supplementary angle 补角
supremum [sju:ˈpri:məm] n. 上确界
surd [sə:d] n. 根式,不尽根

surface [ˈsəːfis] n. 面，曲面
surjection [səːˈdʒekʃən] n. 满射
surveyor [səˈveiə] n. 测量者
sustain [səˈstein] v. 持续，坚持
symbol [ˈsimbəl] n. 符号
symbol structure 符号结构
symbol system 符号系统
symbolic logic 符号逻辑
symmetric [siˈmetrik] adj. 对称的
symmetric algebra 对称代数
symmetric figure 对称图形
symmetrical [siˈmetrikəl] adj. 对称的，均匀的
symplectic geometry 辛几何[学]
synonymous [siˈnɔniməs] adj. 同义的
system [ˈsistəm] n. 体系，制度
system design 系统设计
system of equations 方程组
systematic [ˌsistiˈmætik] adj. 系统的，体系的
systematize [ˈsistimətaiz] v. 系统化
systems analysis 系统分析

## T

table of difference 差分表
tabular [ˈtæbjulə] adj. 表格的
tabulate [ˈtæbjuleit] v. 把……制成表格
tally [ˈtæli] v. 计数，计算，得分
tangent [ˈtændʒənt] n. 正切，切线；adj. 切线的
tangent space 切空间
tangential equation 切线方程
tautology [tɔːˈtɔlədʒi] n. 重言式
tensor [ˈtensə] n. 张量
tensor field 张量场
term [təːm] n. 术语，项
terminate [ˈtəːmineit] v. 终止
terminology [ˌtəːmiˈnɔlədʒi] n. 术语
termwise differentiation 逐项微分
ternary quadratic form 三元二次形式
testing of hypothesis 假设检验
text processing 文本处理
the first derivative 一阶导数
the intuitive meaning 直观意义
theorem [ˈθiərəm] n. 定理，法则
theorem of mean 中值定理，平均值定理
theory [ˈθiəri] n. 理论
theory of fields 场论，域论
theory of substitution 置换论
thickness [ˈθiknis] n. 厚度
three-dimensional [ˈθriː diˈmenʃənəl] adj. 三维的
time [taim] n. 时间，倍数，次数；v. 乘以
time-delay system 时滞系统
tiny [ˈtaini] adj. 很少的，微小的
to complete square 配方
tolerance error 容许误差
top [tɔp] n. 顶部；adj. 最高的，主要的
topological algebra 拓扑代数
topological basis 拓扑基
topological space 拓扑空间
topological vector space 拓扑向量空间
topology [təˈpɔlədʒi] n. 拓扑[学]
total derivative 全导数
total differential 全微分
total variation 全变差
totality [təuˈtæliti] n. 全部，全体，总体
trace [treis] n. 迹，追迹；v. 追踪
trait [treit] n. 特性，特征
trajectory [ˈtrædʒiktəri] n. [样本]轨道
transcendental [ˌtrænsenˈdentəl] adj. 超越的
transfer [trænsˈfəː] v. 转移，迁移
[ˈtrænsfəː] n. 转移，迁移
transfinite diameter 超限直径

transient [ 'trænziənt ]　adj. 瞬时的，过渡的
transition law　转移律
transitive [ 'trænsitiv ]　adj. 可递的，可迁的
transitivity [ ˌtrænsi'tiviti ]　n. 可传递性
translate [ træns'leit ]　v. 平移，翻译
translation [ træns'leiʃən ]　n. 平移，翻译
transonic flow　超音速流
transport [ træns'pɔːt ]　n. 迁移
transpose [ træns'pəuz ]　v. 移项；n. 转置
transversal [ trænz'vəːsəl ]　adj. 横截的
trapezium rule　梯性法则
traversing [ 'trævəsiŋ ]　n. 遍历
tree algorithm　树算法
trend analysis　趋势分析
trial [ 'traiəl ]　n. 试验，试用
triangle [ 'traiæŋgl ]　n. 三角形
triangle inequality　三角不等式
trigonometric [ ˌtrigənə'metrik ]　adj. 三角[学]的
trigonometric function　三角函数
trigonometry [ trigə'nɔmitri ]　n. 三角法，三角[学]
trinomial [ trai'nəumjəl ]　n. 三项式
triple [ tripl ]　adj. 三倍的
triple integral　三重积分
trisect [ trai'sekt ]　v. 把……分成三份，三等分
trisection [ trai'sekʃən ]　n. 三等分
truck [ trʌk ]　n. 信息通路
true [ truː ]　adj. 真的，成立的
truncate [ 'trʌŋkeit ]　v. 截断，截尾
truth table　真值表
truth value　真假值
Turing machine　图灵机
twofold [ 'tuːˌfəuld ]　adj. 双重的
type [ taip ]　n. 类型，序型

## U

ultimate [ 'ʌltimit ]　adj. 最终的，最后的
ultrafilter [ ˌʌltrə'filtə ]　n. 超滤子
unambiguous [ 'ʌnæm'bigjuəs ]　adj. 明确的，清楚的
unbiassed estimate　无偏估计
unbounded [ ˌʌn'baundid ]　adj. 无界的
unbounded function　无界函数
unbounded set　无界集
unchanged [ 'ʌn'tʃeindʒd ]　adj. 无变化的，未改变的
undecidable ring　不可判定环
underlying set　基础集
underlying space　承载空间，基础空间
undirected graph　无向图
ungrouped data　未分类数据
unicursal curve　单行曲线，有理曲线
uniform [ 'juːnifɔːm ]　adj. 一致的
uniform boundness　一致有界性
uniform convergence　一致收敛
uniform divergence　一致发散
uniformly bounded　一致有界的
uniformly continuous　一致连续的
union [ 'juːnjən ]　n. 并，并集
union of sets　集的并
unique [ juː'niːk ]　adj. 唯一的
uniquely [ juː'niːkli ]　adv. 唯一地
uniqueness [ juː'niːknis ]　n. 唯一性
unit [ 'juːnit ]　n. 单位
unit circle　单位圆
unit coordinate vector　单位坐标向量
unit distance　单位长度
unit interval　单位区间
unitary matrix　酉矩阵
universal quantification　全称量词化
universal quantifier　全称量词

universal set 全集,宇宙集,通用集
universal set 通集,宇宙集
universe [ˈjuːnivəːs] n. 全域,底集
universe of discourse 论域
unknown [ˈʌnˈnəun] adj. 未知的;n. 未知数
unknown term 未知项
unlimited [ʌnˈlimitid] adj. 无限的,无限制的
unproved [ʌnˈpruːvd] adj. 未证明的,未经证实的
unrealistic [ˈʌnriəˈlistik] adj. 非现实的
unsolvable [ˈʌnˈsɔlvəbl] adj. 不可解的
unsolved [ˈʌnˈsɔlvd] adj. 未解答的,未解决的
unstable [ˈʌnˈsteibl] adj. 不稳定的
untenable [ˈʌnˈtenəbl] adj. 不可达到的
untrue [ˈʌnˈtruː] adj. 不真,不成立
updated [ʌpˈdeitid] adj. 适时的,校正的
upper bound 上界,最大值
upper boundary 上边界
upper extreme 上端
upper limit 上限

## V

valid [ˈvælid] adj. 成立的,有效的
validity [vəˈliditi] n. 有效,有效性
value [ˈvælju] n. 值
vanish [ˈvæniʃ] v. 消失,变成零
variable [ˈvɛəriəbl] adj. 变化的;n. 变量
variable range 变量范围
variability [ˌvɛəriəˈbiliti] n. 可变性,差异
variance analysis 方差分析
variation [ˌvɛəriˈeiʃən] n. 变分,变化
variation principle 变分原理
variety [vəˈraiəti] n. 簇
vector [ˈvektə] n. 向量,矢量
vector field 向量场
vector space 向量空间
velocity [viˈlɔsiti] n. 速度
Venn diagram 文氏图
verification [ˌverifiˈkeiʃən] n. 确认,证明
verifier [ˈverifaiə] n. 检验器,验证法
verify [ˈverifai] v. 检验,验证,核实
vertex [ˈvəːteks] n. 顶点
vertex angle 顶角
vertical [ˈvəːtikəl] adj. 垂直的,竖直的;n. 垂直线
vertical angle 对顶角
vertical side 竖直边
vibration [vaiˈbreiʃən] n. 振动
visual [ˈvizjuəl] adj. 可视的,看得见的
visualization [ˌvizjuəlaiˈzeiʃən] n. 可视化
void set 空集 (=empty set)
volume [ˈvɔljuːm] n. 体积,容积,卷

## W

wave equation 波动方程
wavelet [ˈweivlit] n. 小波
Weierstrass elliptic function 魏尔斯特拉斯椭圆方程
weight function 权函数
weighted means 加权平均值
well-defined [ˈwel diˈfaind] adj. 很好地定义的
well-ordered [ˈwel ˈɔːdəd] adj. 良序的
well-ordered set 良序集
whole number [非负]整数
width [widθ] n. 宽度,宽广
without bound 无界的(=boundless)

## X

$x$-axis [ˈeks ˌæksis] n. $x$ 轴

$x$-component [ˈeks ˌkəmˈpəunənt] n. $x$ 分量

$x$-coordinate [ˌeks kəuˈɔːdinit] n. $x$ 坐标

$x$-direction [ˌeks diˈrekʃən] n. $x$ 方向

$x$-intercept [ˌeks ˌintəˈsept] n. $x$ 截距

## Y

$y$-axis [ˈwai ˌæksis] n. $y$ 轴

$y$-coordinate [ˈwai kəuˈɔːdinit] n. $y$ 坐标

yield [jiːld] n. 产品,产量;v. 产生,给出

yield condition 生长条件,屈服条件

## Z

zero [ˈziərəu] n. 零,零点

zero ideal 零理想

zero-one law 零一律

zero-vector [ˈziərəu ˈvektə] n. 零向量

zeta function $\zeta$ 函数

zonal sampling 区域抽样

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